1. We start assuming that the premisses and the negation of the conclusion are all true:

(1) \((R \lor P) \Rightarrow T\)
(2) \((R \lor Q) \Rightarrow T\)
(3) \(\neg(P \lor Q) \Rightarrow T\)

A negated disjunction is true (i.e. the disjunction is false) if each disjunct is false, i.e.

(4) \(\neg P \Rightarrow T\)
(5) \(\neg Q \Rightarrow T\)

The truth of the disjunction (1) gives us the alternatives

(6) \(R \Rightarrow T\) \(P \Rightarrow T\)

The right-hand branch immediately yields a contradiction: then we can use the disjunction (2) to get

(7) \(R \Rightarrow T\) \(Q \Rightarrow T\)

Which leaves the open branch where \(P \Rightarrow F, Q \Rightarrow F\) and \(R \Rightarrow T\) (which indeed makes the original premisses of the argument true and conclusion false – check that!). **Invalid argument**

2. Again, assume the premisses and the negation of the conclusion are all true:

(1) \(\neg(P \land \neg Q) \Rightarrow T\)
(2) \(P \Rightarrow T\)
(3) \(\neg Q \Rightarrow T\)

A negated conjunction is true (i.e. the conjunction is false) if one of the conjuncts is false, i.e.

(4) \(\neg P \Rightarrow T\) \(\neg \neg Q \Rightarrow T\)

Both branches immediately lead to contradiction, since each branch contains a pair of wffs of the form \(A, \neg A\) each supposed to be true. **Valid argument**

3. As always, assume the premisses and the negation of the conclusion are all true:

(1) \(\neg(P \lor Q) \Rightarrow T\)
(2) \((R \lor P) \Rightarrow T\)
(3) \(\neg(R \land Q) \Rightarrow T\)

There is no right order for ‘unpacking’ these wffs: start, say, with the last .... That yields

(4) \(\neg R \Rightarrow T\) \(\neg Q \Rightarrow T\)

Now we have to explore these alternative ways of making (3) correct; unpacking the first wff, we consider its implications for each alternative:

(5) \(\neg P \Rightarrow T\) \(Q \Rightarrow T\) \(\neg P \Rightarrow T\) \(Q \Rightarrow T\)

We can close off one branch (only one!). Now to explore the remaining three branches, we unpack (2), adding its implications to every open branch:

(6) \(P \Rightarrow T\) \(R \Rightarrow T\) \(P \Rightarrow T\) \(R \Rightarrow T\) \(P \Rightarrow T\) \(R \Rightarrow T\)

That leaves two open branches: one on which \(P \Rightarrow T, Q \Rightarrow T, R \Rightarrow F\), the other where \(P \Rightarrow F, Q \Rightarrow F, R \Rightarrow T\). A quick check confirms that those two valuations make the premisses true and conclusion false. **Invalid argument**
4. 

\[(1) \quad ((P \land \neg Q) \lor R) \Rightarrow T\]
\[(2) \quad (Q \lor S) \Rightarrow T\]
\[(3) \quad \neg(R \land S) \Rightarrow T\]
\[(4) \quad (P \land \neg Q) \Rightarrow T \quad R \Rightarrow T\]
\[(5) \quad P \Rightarrow T\]
\[(6) \quad \neg Q \Rightarrow T\]

Here we’ve unpacked the wff on the right at line (4) [why do we only add those conjuncts to the right-hand branch?]. Now we consider the implications of (2) for each branch

\[(7) \quad Q \Rightarrow T \quad S \Rightarrow T \quad Q \Rightarrow T \quad S \Rightarrow T\]

Which leaves us to unpack (3)

\[(8) \quad \neg R \Rightarrow T \quad \neg S \Rightarrow T \quad \neg R \Rightarrow T \quad \neg S \Rightarrow T \quad \neg R \Rightarrow T \quad \neg S \Rightarrow T\]

Again there are open branches, so this is an invalid argument.

5. This time, let’s leave off the explicit assignments of truth:

\[(1) \quad P\]
\[(2) \quad \neg(P \land \neg Q)\]
\[(3) \quad \neg(Q \land \neg R)\]
\[(4) \quad (\neg R \land S)\]
\[(5) \quad \neg S\]

Unpacking (4) gives us

\[(6) \quad \neg R\]
\[(7) \quad S\]

And we are done already! A wise choice about what to unpack first closes the tree fast! (I doubt that this was intended; a better example would have had \((\neg R \lor S)\) – leading still to a closed tree, but a more complex one.)

6. Again, we start with the premisses and negated conclusion:

\[(1) \quad ((P \land Q) \lor (R \land P))\]
\[(2) \quad \neg((Q \land R) \lor P)\]

Let’s unpack the second first, as this doesn’t immediately lead to a branching tree

\[(3) \quad \neg(Q \land R)\]
\[(4) \quad \neg P\]
\[(5) \quad (P \land Q) \quad (R \land P)\]
\[(6) \quad P \quad R\]
\[(7) \quad Q \quad P\]

A valid argument.