A  

Suppose ‘m’ denotes Socrates, ‘n’ denotes Plato, ‘o’ denotes Aristotle, ‘Fx’ means ‘x is a philosopher’, ‘Gx’ means ‘x is wise’, ‘Mxy’ means ‘x taught y’. Take the domain of discourse to consists of people. And then translate the following into QL:

1. Socrates taught Plato and Plato taught Aristotle
   \((Mmn \land Mno)\)
2. Aristotle taught neither Socrates nor Plato
   \(\neg(Mom \lor Mon)\) or \((\neg Mom \land \neg Mon)\)
3. Plato taught someone
   \(\exists x Mnx\)
4. Some philosophers are wise
   \(\exists x (Fx \land Gx)\)
5. Some wise people aren’t philosophers
   \(\exists x (Gx \land \neg Fx)\)
6. No one taught Socrates
   \(\neg\exists x Mxm\) or \(\forall x \neg Mxm\)
7. If Socrates taught Plato, then someone taught Plato
   \((Mmn \supset \exists x Mxn)\)
8. Whoever Socrates taught is wise
   \(\forall x (Mmx \supset Gx)\)
9. Any philosopher who was taught by Plato taught Aristotle
   i.e. Everyone x is such that, if x is a philosopher and x was taught by Plato, then x taught
   Aristotle, so:
   \(\forall x ((Fx \land Mnx) \supset Mxo)\)
10. No wise philosopher was taught by Aristotle
    \(\neg\exists x ((Fx \land Gx) \land Mox)\) or \(\forall x ((Fx \land Gx) \supset \neg Mox)\)

B  

Which of the following pairs of wffs are equivalent (i.e. imply each other), and why? When they aren’t equivalent, give interpretations to illustrate the non-equivalence.

1. \(\exists x \forall y \exists z Ryzx; \exists z \forall y \exists x Ryzx\)
   Swapping ‘x’ and ‘z’ in the first (which of course doesn’t change what it means) turns it into the second, so these are equivalent.

2. \(\exists x \forall y \exists z Ryzx; \exists z \forall x \exists y Ryzx\)
   Swapping ‘x’ and ‘z’ in the first (which of course doesn’t change what it means) turns it into
   \(\exists z \forall x \exists y Ryzx\)
   Now swap ‘x’ and ‘y’ to get
   \(\exists z \forall x \exists y Rxy\)
   The initial block of quantifiers is the same as in the original second wff. But the following expressions are now ‘Rxyz’ and ‘Rxy’ are plainly not equivalent – so the originals are not equivalent.

3. \((\forall x Fx \supset Fn); (\forall z Fz \supset Fn)\)
   Plainly equivalent.
4. \( (\forall xFx \supset \forall xFx); (\forall zFz \supset \forall yFy) \)

Also plainly equivalent, since each of ‘\( \forall xFx \)’, ‘\( \forall yFy \)’ and ‘\( \forall zFz \)’ are equivalent to each other.

5. \( \exists x \exists y Lxy; \exists y \exists x Lxy \)

These are equivalent – see §24.3 for more explanation.

6. \( \forall x \forall y Lxy; \forall y \forall x Lxy \)

These too are equivalent – see §24.3 for more explanation.

7. \( \forall x(Fx \land Gx); (\forall xFx \land \forall xGx) \)

These are equivalent. If everything is \( F \) and \( G \), then everything is \( F \) and everything is \( G \); and equally, if everything is \( F \) and everything is \( G \), then everything is \( F \) and \( G \).

8. \( \forall x(Fx \lor Gx); (\forall xFx \lor \forall xGx) \)

Not equivalent. Compare ‘everyone is male or female’ with ‘everyone is male or everyone is female’.

9. \( \exists x(Fx \land Gx); (\exists xFx \land \exists xGx) \)

Not equivalent. Compare ‘someone is male and female’ with ‘someone is male and someone is female’.

10. \( \exists x(Fx \lor Gx); (\exists xFx \lor \exists xGx) \)

Equivalent. If something is \( F \) or \( G \), then either something is \( F \) or something is \( G \); and vice versa. And if something is \( F \) or something is \( G \), then something is \( F \)-or-\( G \) or something is \( G \)-or-\( F \); so something is \( F \)-or-\( G \).

C We can render ‘Plato and Aristotle are philosophers’ by e.g. ‘(\( Fm \land Fn \))’. Why can’t we render ‘Plato and Aristotle are classmates’ by something like ‘(\( Gm \land Gn \))’? Consider other cases of predicates \( F \) where we can’t render something of the form ‘Plato and Aristotle are \( F \)’ by something of the type ‘(\( Fm \land Fn \))’. What can be learnt from such cases about the expressive limitations of QL?

(\( Gm \land Gn \)) entails \( Gm \). But Plato and Aristotle are classmates does not entail Plato is a classmate (which hardly makes sense). Likewise Socrates and Plato and Aristotle surrounded the escaped goat doesn’t entail Socrates surrounded the escaped goat (it takes more than one to do that!).

Let’s say that a predicate \( F \) is distributive if it sustains the inference ‘if \( m \) and \( n \) is \( F \), then \( m \) is \( F \). Then not all English predicates are distributive; but QL can only represent distributive predicates – so that’s an expressive limitation.