A Suppose ‘m’ denotes Myfanwy, ‘n’ denotes Ninian, ‘o’ denotes Olwen, ‘Fx’ means \( x \) is a philosopher, ‘Gx’ means \( x \) speaks Welsh, ‘Lxy’ means \( x \) loves \( y \), and ‘Rxyz’ means that \( x \) is a child of \( y \) and \( z \). Take the domain of discourse to consist of human beings. Translate the following into QL:

1. Ninian is loved by Myfanwy and Olwen
   \( (Lmn \land Lon) \)

2. Neither Myfanwy nor Ninian love Olwen
   \( \neg (Lmo \lor Lno) \) \text{ or } \( \neg Lmo \land \neg Lno \)

3. Someone is a child of Myfanwy and Ninian
   \( \exists x Rxmn \)

4. No philosopher loves Olwen
   \( \neg \exists x (Fx \land Lxo) \) \text{ or } \( \forall x (Fx \supset \neg Lxo) \)

5. Myfanwy and Ninian love everyone
   \( \forall x ((Lmx \land Lnx) \lor (\forall x Lmx \land \forall x Lnx)) \)

6. Some philosophers speak Welsh
   \( \exists x (Fx \land Gx) \)

7. No Welsh-speaker who loves Myfanwy is a philosopher
   \( \neg \exists x ((Gx \land Lxm) \land Fx) \) \text{ or } \( \forall x ((Gx \land Lxm) \supset \neg Fx) \)

8. Some philosophers love both Myfanwy and Olwen
   \( \exists x (Fx \land (Lxm \land Lxo)) \)

9. Some philosophers love every Welsh speaker
   \( \exists x (Fx \land \forall y (Gy \supset Lxy)) \)

10. Everyone who loves Ninian is a philosopher who loves Myfanwy
    \( \forall x (Lxn \supset (Fx \land Lxm)) \)

11. Some philosopher is a child of Olwen and someone or other
    \( \exists x (Fx \land \exists y Rxoy) \)

12. Whoever is a child of Myfanwy and Ninian loves them both
    \( \forall x (Rxmn \supset (Lxm \land Lxn)) \)

13. Everyone speaks Welsh only if Olwen speaks Welsh
    \( (\forall x Gx \supset Go) \) \text{ [not } \forall x (Gx \supset Go), \text{ which isn’t equivalent]} \)

14. Myfanwy is a child of Ninian and of someone who loves Ninian
    \( \exists x (Rmxn \land Lxn) \)

    [oops, first printing has ‘Bethan’ etc. Sorry! Using obvious translation, that would be
    \( \exists x (Rbcx \land Lxc) \)]

15. Some philosophers love no Welsh speakers
    \( \exists x (Fx \land \neg \exists y (Gy \land Lxy)) \) \text{ or } \( \exists x (Fx \land \forall y (Gy \supset \neg Lxy)) \)

16. Every philosopher who speaks Welsh loves Olwen
    \( \forall x ((Fx \land Gx) \supset Lxo) \)

17. Every philosopher who speaks Welsh loves someone who loves Olwen
    \( \forall x ((Fx \land Gx) \supset \exists y (Lxy \land Lyo)) \)

18. If Ninian loves every Welsh speaker, then Ninian loves Myfanwy
    \( (\forall x (Gx \supset Lnx) \supset Lnm) \)

19. No Welsh speaker is loved by every philosopher
    \( \neg \exists x (Gx \land \forall y (Fy \supset Lxy)) \) \text{ or } \( \forall x (Gx \supset \exists y (Fy \land \neg Lxy)) \)

20. Every Welsh speaker who loves Ninian loves no one who loves Olwen
    \( \forall x ((Gx \land Lxm) \supset \neg \exists y (Lxy \land Lyo)) \)
21. Whoever loves Myfanwy, loves a philosopher only if the latter loves Myfanwy too
   \( \forall x(Lmx \supset \forall y((Lxy \land Fy) \supset Lym)) \)

22. Anyone whose parents are a philosopher and someone who loves a philosopher is a philosopher too.
   \( \forall x[(\exists y \exists z Rxyz \land \{ y \text{ is a philosopher and } z \text{ loves a philosopher}\}) \supset Fx] \)
   \( \forall x[(\exists y \exists z Rxyz \land (Fx \land \exists w(Lzw \land Fw))) \supset Fx] \)

23. Only if Ninian loves every Welsh-speaking philosopher does Myfanwy love him
   \( (Lmn \supset \forall x((Fx \land Gx) \supset Lnx)) \)

24. No philosophers love any Welsh-speaker who has no children
   \( \forall x(Fx \supset \neg \exists y(Lxy \land \{ Gx \land y \text{ has no children}\})) \)
   \( \forall x(Fx \supset \neg \exists y(Lxy \land \{ Gx \land \neg \exists z \exists w Rwzyw\})) \)

B

Take the domain of quantification to be the (positive whole) numbers, and let ‘\( n \)’ denote the number one, ‘\( Fx \)’ mean \( x \text{ is odd} \), ‘\( Gx \)’ mean \( x \text{ is even} \), ‘\( Hx \)’ mean \( x \text{ is prime} \), ‘\( Lxy \)’ mean \( x \text{ is greater than } y \), ‘\( Rxyz \)’ mean that \( x \text{ is the sum of } y \text{ and } z \). Then translate the following from QL into natural English:

1. \( \neg \exists x(Fx \land \neg Gx) \)
   \( \Rightarrow \) No odd number is not even [\text{which is false! to get a truth, which is what}]
   \( \text{I'd intended, delete the second negation in both (1) and its translation!} \)

2. \( \forall x \forall y \exists z Rxyz \)
   \( \Rightarrow \) Every pair of numbers has a sum

3. \( \forall x \exists y Lxy \)
   \( \Rightarrow \) For any number, there’s a larger one

4. \( \forall x \forall y((Fx \land Ryxn) \supset Gy) \)
   \( \Rightarrow \) If a number is one more than an odd number, then it is even.

5. \( \forall x \forall y((Gx \land Rxyn) \supset Fy) \)
   \( \Rightarrow \) If a number is one less than an even number, then it is odd.

6. \( \forall x \exists y((Gx \land Fy) \land Rxyy) \)
   \( \Rightarrow \) Any even number is equal to twice some odd number \text{(more literally: any even number is equal to some odd number added to itself – false of course!)}

7. \( \forall x \forall y(\exists z(Rzxn \land Rzyn) \supset (Gx \supset Gy)) \)
   \( \Rightarrow \) If two numbers differ by two, then if one is even, so is the other.

8. \( \forall x \forall y \forall z(((Fx \land Fy) \land Rzxy) \supset Gz) \)
   \( \Rightarrow \) The sum of two odd numbers is even.

9. \( \forall x(Gx \supset \exists y(\exists z(Hx \land Hz) \land Rxyz)) \)
   \( \Rightarrow \) Every even number is the sum of two primes. \text{[Goldbach’s conjecture]}

10. \( \forall w \exists x \exists y(((Hx \land Hy) \land (Lwx \land Lyw)) \land \exists z(Rzxn \land Rzyn)) \)
    \( \Rightarrow \) Take any number, then there is a pair of primes larger than it which differ by two. \text{[The twin primes conjecture]}

C

Which of the following pairs are equivalent, and why?

1. \( \forall x(Fx \supset Gx); (\exists xFx \supset \exists xGx) \)
   \( \text{Interpret ‘} F \text{’ as } \text{man, ‘} G \text{’ as } \text{woman, and take the domain to be } \text{people. Then ‘} \forall x(Fx \supset Gx) \text{’ is false; but ‘} \exists xFx \text{’ and ‘} \exists xGx \text{’ are both false so ‘} (\exists xFx \supset \exists xGx) \text{’ is true. So these wffs are not equivalent.} \)

2. \( \exists x(Fx \supset Gx); (\exists xFx \supset \exists xGx) \)
   \( \text{Interpret ‘} F \text{’ as } \text{horse, ‘} G \text{’ as } \text{unicorn, and take the domain to be } \text{living creatures. Then ‘} \exists xFx \text{’ is true, and ‘} \exists xGx \text{’ is false so ‘} (\exists xFx \supset \exists xGx) \text{’ is false. Suppose ‘} a \text{’ denotes a dog in the domain; then ‘} Fa \text{’ is false, as is ‘} Ga \text{’, so ‘} (Fa \supset Ga) \text{’ is true, so ‘} \exists x(Fx \supset Gx) \text{’ is true. So these wffs are not equivalent.} \)

3. \( \exists x(Fx \supset Gx); (\forall xFx \supset \exists xGx) \)
   \( \text{Equivalent: for consider this chain } \exists x(Fx \supset Gx) = \neg \forall x(\neg (Fx \supset Gx)) = \neg \forall x(Fx \land \neg Gx) = \neg (\forall xFx \land \forall x \neg Gx) = (\forall xFx \supset \neg \forall x \neg Gx) = (\forall xFx \supset \exists xGx) - \text{which relies on the equivalence of} \)
\( \forall x (Ax \land Bx) \) and \( \forall x Ax \land \forall x Bx \).

4. \( \forall x (Fx \supset Gx); (\exists x Fx \supset \forall x Gx) \)

Take the domain to be living things, interpret ‘F’ as man, ‘G’ as human. Then \( \forall x (Fx \supset Gx) \) is true and \( (\exists x Fx \supset \forall x Gx) \) false, so the wffs are not equivalent.

Q: The claim that, e.g., that a wff of the form \( (A \lor \exists x Fx) \) is equivalent to one of the form \( \exists x (A \lor Fx) \) depends on our stipulation that the domain of quantification isn’t empty. Why?

A: Because in an empty domain, \( \exists x C \) is always false; so if \( A \) is true, \( (A \lor \exists x Fx) \) is true but \( \exists x (A \lor Fx) \) is false; so the wffs aren’t equivalent.

Q: Which other equivalences we stated in §24.3 above also depend on that stipulation?

A: Similarly, if \( A \) is false, \( (A \supset \exists x Fx) \) is true and \( \exists x (A \supset Fx) \) false, so those are no equivalent in empty domains. The other equivalences stated remain correct.