

Take the following  $q$ -valuation –

The domain is {Romeo, Juliet, Benedick, Beatrice}

Constants are assigned references as follows:

‘m’  $\Rightarrow$  Romeo

‘n’  $\Rightarrow$  Juliet

Predicates are assigned extensions as follows:

‘F’  $\Rightarrow$  {Romeo, Benedick}

‘G’  $\Rightarrow$  {Juliet, Beatrice}

‘L’  $\Rightarrow$  {(Romeo, Juliet), (Juliet, Romeo), (Benedick, Beatrice),  
(Beatrice, Benedick), (Benedick, Benedick)}

Then what are the truth values of the following wffs?

1.  $\exists x Lmx$

True. ‘ $\exists x Lmx$ ’ is true if for some object  $o$  in the domain, the pair of objects (Romeo,  $o$ ) is in the extension of ‘L’, and that condition is satisfied.

2.  $\forall x Lxm$

False. ‘ $\forall x Lxm$ ’ is true if for every object  $o$  in the domain, the pair of objects (Romeo,  $o$ ) is in the extension of ‘L’, and that condition isn’t satisfied.

3.  $(\exists x Lmx \supset Lmn)$

True. ‘ $\exists x Lmx$ ’ is true – see (1) – and ‘Lmn’ is true, so material conditional is true.

4.  $\forall x (Fx \equiv \neg Gx)$

True. ‘ $\forall x (Fx \equiv \neg Gx)$ ’ is if every  $x$ -variant of our valuation makes ‘ $(Fx \equiv \neg Gx)$ ’ true; but an  $x$ -variant makes that true if, whatever object in the domain we assign to ‘ $x$ ’ if it is in the extension of ‘F’ it isn’t in the extension of ‘G’ and vice versa. And that condition holds.

5.  $\forall x (Gx \supset (Lxm \vee \neg Lmx))$

True. ‘ $\forall x (Gx \supset (Lxm \vee \neg Lmx))$ ’ is if every  $x$ -variant makes ‘ $(Gx \supset (Lxm \vee \neg Lmx))$ ’ true. The  $x$ -variants which assign Romeo or Benedick to ‘ $x$ ’ make ‘Gx’ false so the conditional true. The  $x$ -variant which assign Juliet to ‘ $x$ ’ makes ‘Gx’ true but also ‘Lxm’ true, so makes the conditional true. The  $x$ -variant which assign Beatrice to ‘ $x$ ’ makes ‘Gx’ true but also ‘ $\neg Lmx$ ’ true, so makes the conditional true. So every  $x$ -variant does indeed make ‘ $(Gx \supset (Lxm \vee \neg Lmx))$ ’ true.

6.  $\forall x (Gx \supset \exists y Lxy)$

True. ‘ $\forall x (Gx \supset \exists y Lxy)$ ’ is if every  $x$ -variant makes ‘ $(Gx \supset \exists y Lxy)$ ’ true. The  $x$ -variants which assign Romeo or Benedick to ‘ $x$ ’ make ‘Gx’ false so the conditional true. The  $x$ -variant which assign Juliet to ‘ $x$ ’ makes ‘Gx’ true; but also ‘ $\exists y Lxy$ ’ true, since there is further extension of our valuation to give a  $y$ -variant which makes ‘Lxy’ true (namely the  $y$ -variant that assigns Romeo to ‘ $y$ ’). Likewise the  $x$ -variant which assign Beatrice to ‘ $x$ ’ makes ‘Gx’ true; but also ‘ $\exists y Lxy$ ’ true, since there is further extension of our valuation to give a  $y$ -variant which makes ‘Lxy’ true (namely the  $y$ -variant that assigns Benedick to ‘ $y$ ’). So every  $x$ -variant does indeed make ‘ $(Gx \supset \exists y Lxy)$ ’ true.

7.  $\exists x (Fx \wedge \forall y (Gy \supset Lxy))$

False. The  $x$ -variants which assign Juliet or Beatrice to ‘ $x$ ’ make ‘Fx’ false so make the conjunction ‘ $(Fx \wedge \forall y (Gy \supset Lxy))$ ’ false. The  $x$ -variant which assigns Romeo to ‘ $x$ ’ makes the first conjunct true but makes the second conjunct false so ‘ $(Fx \wedge \forall y (Gy \supset Lxy))$ ’ is again false (for note, the second conjunct is true if further extension to give a  $y$ -variant makes

' $(Gy \supset Lxy)$ ' true, but the assignment of Beatrice to 'y' makes that conditional false). Similarly the x-variant which assigns Benedick to 'x' makes ' $(Fx \wedge \forall y(Gy \supset Lxy))$ ' false again. So, no x-variant of our original valuation makes ' $(Fx \wedge \forall y(Gy \supset Lxy))$ ' true.

Now take the following *q*-valuation –

The domain is  $\{4, 7, 8, 11, 12\}$

Constants are assigned references as follows:

'm'  $\Rightarrow$  7

'n'  $\Rightarrow$  12

Predicates are assigned extensions as follows:

'F'  $\Rightarrow$  the even numbers in the domain

'G'  $\Rightarrow$  the odd numbers in the domain

'L'  $\Rightarrow$  the set of pairs  $\langle m, n \rangle$  where *m* and *n* are in the domain and *m* is less than *n*

What are the truth values of the wffs (1) to (7) now?

- |    |  |       |
|----|--|-------|
| 1. | $\exists x Lmx$                                  | True  |
| 2. | $\forall x Lxm$                                  | False |
| 3. | $(\exists x Lmx \supset Lmn)$                    | True  |
| 4. | $\forall x(Fx \equiv \neg Gx)$                   | True  |
| 5. | $\forall x(Gx \supset (Lxm \vee \neg Lmx))$      | False |
| 6. | $\forall x(Gx \supset \exists y Lxy)$            | True  |
| 7. | $\exists x(Fx \wedge \forall y(Gy \supset Lxy))$ | False |