Answers to Exercises 29

A Use QL trees to evaluate the entailment claims (1) to (10) in Exercises 28A.

1. \( \forall x (Fx \supset Gx) \vdash \forall x (Gx \supset Fx) \)

The tree doesn't close and there are no more rules to apply. We can read off the open branch a valuation which makes the premises true and conclusion false, and (recall) the trick is to pick a valuation which makes the 'primitives' on the branch, i.e. the atoms and negated atoms, all true, and which puts into the domain just enough objects to give references to every constant on the branch. We want a valuation with just the object named by 'a' in the domain and which makes '¬Fa' and 'Ga' both true. So, put just the number 0 in the domain as its sole member, and let \( F \) have as extension the empty set, and 'G' has the extension \{0\}. Then, as desired, '\( \forall x (Fx \supset Gx) \)' is true and '\( \forall x (Gx \supset Fx) \)' is false.

2. \( \forall x (Fx \supset Gx) \vdash \forall x (\neg Gx \supset \neg Fx) \)

The inference is valid and the q-validity claim is true.

3. \( \forall x \exists y Lxy \vdash \forall y \exists x Lxy \)

The inference is valid and the q-validity claim is true.
4. \( \forall x ((Fx \land Gx) \supset Hx) \vdash \forall x (Fx \supset (Gx \supset Hx)) \)

\[
\begin{align*}
(1) & \quad \forall x ((Fx \land Gx) \supset Hx) \\
(2) & \quad \neg \forall x (Fx \supset (Gx \supset Hx)) \checkmark \\
(3) & \quad \exists x \neg (Fx \supset (Gx \supset Hx)) \checkmark \quad \text{From 2} \\
(4) & \quad \neg (Fa \supset (Ga \supset Ha)) \checkmark \quad \text{From 3} \\
(5) & \quad Fa \quad \text{From 4} \\
(6) & \quad \neg (Ga \supset Ha) \checkmark \quad \text{From 4} \\
(7) & \quad Ga \quad \text{From 6} \\
(8) & \quad \neg Ha \quad \text{From 6} \\
(9) & \quad ((Fa \land Ga) \supset Ha) \checkmark \quad \text{From 1} \\
(10) & \quad \neg (Fa \land Ga) \checkmark \\
(11) & \quad \neg Fa \quad \neg Ga \\
\quad \checkmark \quad \checkmark \\
\end{align*}
\]

The inference is valid and the \( q \)-validity claim is again true.

5. \( (\forall x Fx \lor \forall x Gx) \vdash \forall x (Fx \lor Gx) \)

\[
\begin{align*}
(1) & \quad (\forall x Fx \lor \forall x Gx) \checkmark \\
(2) & \quad \neg \forall x (Fx \lor Gx) \checkmark \\
(3) & \quad \exists x \neg (Fx \lor Gx) \checkmark \quad \text{From 2} \\
(4) & \quad \neg (Fa \lor Ga) \checkmark \quad \text{From 3} \\
(5) & \quad \neg Fa \quad \text{From 4} \\
(6) & \quad \neg Ga \quad \text{From 4} \\
(7) & \quad \forall x Fx \quad \forall x Gx \quad \text{From 1} \\
(8) & \quad Fa \quad Ga \quad \text{From 7} \\
\quad \checkmark \quad \checkmark \\
\end{align*}
\]

The inference is valid and the \( q \)-validity claim is again true.

6. \( \forall x (Fx \supset Gx), \forall x (\neg Gx \supset Hx) \vdash \forall x (Fx \supset \neg Hx) \)

\[
\begin{align*}
(1) & \quad \forall x (Fx \supset Gx) \\
(2) & \quad \forall x (\neg Gx \supset Hx) \\
(3) & \quad \neg \forall x (Fx \supset \neg Hx) \checkmark \\
(4) & \quad \exists x \neg (Fx \lor \neg Hx) \checkmark \\
(5) & \quad \neg (Fa \lor \neg Ha) \checkmark \\
(6) & \quad Fa \\
(7) & \quad \neg Ha \checkmark \\
(8) & \quad Ha \\
(9) & \quad (Fa \lor Ga) \checkmark \quad \text{From 1} \\
(10) & \quad (\neg Ga \lor Ha) \checkmark \quad \text{From 2} \\
(11) & \quad \neg Fa \quad Ga \\
\quad \checkmark \quad \checkmark \\
(12) & \quad \neg \neg Ga \checkmark \quad Ha \\
(13) & \quad Ga \\
\end{align*}
\]

The tree doesn’t close and there are no more rules to apply. We can read off each open branch a valuation which makes the premises true and conclusion false – in fact the same valuation, as each branch contains the same primitives, ‘Fa’, ‘Ga’ and ‘Ha’. We want a valuation with just the
object named by a in the domain and which makes each of those primitives true. So, put just the number 0 in the domain as its sole member, and let ‘F’, ‘G’ and ‘H’ have the extension {0}. Then, as desired, ‘∀x(Fx ⊃ Gx)’ and ‘∀x(¬Gx ⊃ Hx)’ are true and ‘∀x(Fx ⊃ ¬Hx)’ is false.

7. \[ \exists x(Fx \land Gx), \forall x(¬Hx \circ ¬Gx) \vdash \exists x(Fx \land Hx) \]

(1) \[ \exists x(Fx \land Gx) \]
(2) \[ ∀x(¬Hx \circ ¬Gx) \]
(3) \[ ¬∃x(Fx \land Hx) \quad \checkmark \]
(4) \[ ∀x(¬(Fx \land Hx)) \quad \checkmark \]
(5) \[ (Fa \land Ga) \quad \checkmark \quad \text{From 1} \]
(6) \[ Fa \]
(7) \[ Ga \]
(8) \[ (¬Ha \circ ¬Ga) \quad \checkmark \quad \text{From 2} \]
(9) \[ ¬(Fa \land Ha) \quad \checkmark \quad \text{From 4} \]

\[ (10) \quad ¬Ha \quad ¬Ga \]
\[ (11) \quad ¬Fa \quad ¬Ha \]
\[ * \quad * \quad * \]

The inference is valid and the q-validity claim is again true.

8. \[ ∀x∃y(Fy \circ Gx) \vdash ∀y∃x(Gx \circ Fy) \]

(1) \[ ∀x∃y(Fy \circ Gx) \]
(2) \[ ¬∀y∃x(Gx \circ Fy) \quad \checkmark \]
(3) \[ ∃y¬∃x(Gx \circ Fy) \quad \checkmark \]
(4) \[¬∃x(Gx \circ Fa) \quad \checkmark \]
(5) \[ ∀x¬(Gx \circ Fa) \quad \text{From 1} \]

The only name in play is a so let’s now instantiate both universal quantifiers with this name

(6) \[ ∃y(Fy \circ Ga) \quad \checkmark \]
(7) \[ ¬(Ga \circ Fa) \]
(8) \[ Ga \]
(9) \[¬Fa \]

Now, the tree hasn’t finished, and indeed the tree will never close if we carry on applying every rule we can. For then we’d instantiate (6) to get

\[ (10) \quad (Fb \circ Ga) \quad \checkmark \]

And instantiating both our universal quantifiers with the new name b we’d get

\[ (11) \quad ∃y(Fy \circ Gb) \quad \checkmark \]
\[ (12) \quad ¬(Gb \circ Fb) \]

And now we’ve got another existential quantifier to instantiate, which introduces another name, and off we go down an infinite tree.

But go back and look at (8) and (9). In fact a minimal valuation that makes these two primitives true makes (6) true. So consider the valuation with just the number 0 in the domain as its sole member, and let ‘F’ have as extension the empty set, and ‘G’ has the extension {0}. Then, this makes the premiss of the argument true and conclusion false.
9. \( \forall x \forall y (Lxy \supset Lyx) \vdash \forall x Lxx \)

(1) \( \forall x \forall y (Lxy \supset Lyx) \) 
(2) \( \neg \forall x Lxx \) \( \checkmark \) 
(3) \( \exists x \neg Lxx \) \( \checkmark \) From 2 
(4) \( \neg \exists x \neg Lxx \) \( \checkmark \) From 3 
(5) \( \forall y (Lay \supset Lya) \) From 1 
(6) \( (Laa \supset Laa) \) \( \checkmark \) From 1 

(10) \( \neg Laa \) Laa * From 9 

There are no more moves to make. So consider the valuation with just the number 0 in the domain as its sole member, and let ‘L’ have as extension the empty set. Then that makes the premises true and conclusion false.

10. \( \forall x (\exists y Lxy \supset \forall z Lzx) \vdash \forall x \forall y (Lxy \supset Lyx) \)

(1) \( \forall x (\exists y Lxy \supset \forall z Lzx) \) 
(2) \( \neg \forall x \forall y (Lxy \supset Lyx) \) \( \checkmark \) 
(3) \( \exists x \neg \forall y (Lxy \supset Lyx) \) \( \checkmark \) From 2 
(4) \( \exists y \neg (Lay \supset Lya) \) \( \checkmark \) From 3 
(5) \( \neg (Lab \supset Lba) \) \( \checkmark \) From 4 
(6) Lab 
(7) \( \neg Lba \) 
(6) \( (\exists y Lay \supset \forall z Lza) \) From 1 

(10) \( \neg \exists y Lay \) VzLza From 9 
(11) \( \forall y \neg Lay \) Lba From 10 
(12) \( \neg Lab \) * 

The inference is valid and the q-validity claim is again true.

B Using trees, show the following arguments are valid:

1. Some philosophers admire Jacques. No one who admires Jacques is a good logician. So some philosophers are not good logicians.

\( \exists x (Fx \land Gx) , \forall x (Gx \supset \neg Hx) \vdash \exists x (Fx \land \neg Hx) \)

(1) \( \exists x (Fx \land Gx) \) \( \checkmark \) 
(2) \( \forall x (Gx \supset \neg Hx) \) 
(3) \( \neg \exists x (Fx \land \neg Hx) \) \( \checkmark \) Negated conclusion 
(4) \( \forall x \neg (Fx \land \neg Hx) \) From 3 
(5) \( (Fa \land Ga) \) \( \checkmark \) Instantiating 1 
(6) \( (Ga \supset \neg Ha) \) \( \checkmark \) From 2 
(7) \( \neg (Fa \land \neg Ha) \) \( \checkmark \) From 4 
(8) Fa 
(9) Ga \( \checkmark \) 

(11) \( \neg Ga \) \( \neg Ha \) * 

(12) \( \neg Fa \) \( \neg \neg Ha \) * *
2. *Some philosophy students admire all logicians; no philosophy student admires any rotten lecturer; hence, no logician is a rotten lecturer.*

\[ \exists x (F_x \land \forall y (G_y \supset R_{yx})), \neg \exists x (F_x \land \exists y (H_y \land R_{yx})) \vdash \neg \exists x (G_x \land H_x) \]

Other translations of the ‘no’ propositions are possible. For example, we could have translated the second premiss as ‘\( \forall x (F_x \supset \neg \exists y (H_y \land R_{xy})) \)’ or ‘\( \forall x (F_x \supset \forall y (H_y \supset \neg R_{xy})) \)’. The conclusion can be translated ‘\( \forall x (G_x \supset \neg H_x) \)’. The tree will go similarly with each combination of translations:

\[
\begin{align*}
(1) & \quad \exists x (F_x \land \forall y (G_y \supset R_{yx})) \\
(2) & \quad \neg \exists x (F_x \land \exists y (H_y \land R_{yx})) \quad \checkmark \\
(3) & \quad \neg \neg \exists x (G_x \land H_x) \quad \checkmark \\
(4) & \quad \exists x (G_x \land H_x) \\
(5) & \quad \forall x \neg (F_x \land \exists y (H_y \land R_{yx})) \quad \checkmark
\end{align*}
\]

We now have two existentials to instantiate: we should start with (1) — as the other involves predicates buried inside the wffs (1) and (5).

\[
\begin{align*}
(6) & \quad (F_a \land \forall y (G_y \supset R_{ya})) \quad \checkmark \\
\text{From 1}
\end{align*}
\]

And now we immediately use the new name to instantiate the universal quantifier to get

\[
\begin{align*}
(7) & \quad \neg (F_a \land \exists y (H_y \land R_{ya})) \quad \checkmark \\
(8) & \quad F_a \\
(9) & \quad \forall y (G_y \supset R_{ya}) \quad \text{Unpacking 6}
\end{align*}
\]

\[
\begin{align*}
(10) & \quad \neg F_a \quad \neg \exists y (H_y \land R_{ya}) \quad \checkmark \\
(11) & \quad \ast \quad \forall y \neg (H_y \land R_{ya}) \quad \text{Pushing in the negation sign}
\end{align*}
\]

At this point, we have three universals and an unchecked existential in play: so we now instantiate the existential and unpack the result ...

\[
\begin{align*}
(12) & \quad (G_b \land H_b) \quad \checkmark \\
(13) & \quad G_b \\
(14) & \quad H_b \\
\end{align*}
\]

We now instantiate the two universals we haven’t so far used and the rest is plain sailing:

\[
\begin{align*}
(15) & \quad (G_b \supset R_{ab}) \quad \checkmark \\
(16) & \quad \neg (H_b \land R_{ab}) \quad \checkmark \\
\end{align*}
\]

\[
\begin{align*}
(17) & \quad \neg G_b \quad \neg R_{ab} \\
\ast & \quad \ast \\
(18) & \quad \neg H_b \quad \neg R_{ab} \\
\ast & \quad \ast
\end{align*}
\]

3. *There’s a town to which all roads lead. So all roads lead to a town.*

\[ \exists x (F_x \land \forall y (G_y \supset R_{yx})), \quad \forall x (G_x \supset \exists y (F_y \land R_{xy})) \]

where ‘Rab’ expresses a leads to b.

\[
\begin{align*}
(1) & \quad \exists x (F_x \land \forall y (G_y \supset R_{yx})) \\
(2) & \quad \neg \forall x (G_x \supset \exists y (F_y \land R_{xy})) \quad \checkmark \\
(3) & \quad \exists x \neg (G_x \supset \exists y (F_y \land R_{xy})) \\
\end{align*}
\]

We’ll instantiate the first wff and unpack the result to get ...

\[
\begin{align*}
(4) & \quad (F_a \land \forall y (G_y \supset R_{ya})) \quad \checkmark \\
(5) & \quad F_a \\
(6) & \quad \forall y (G_y \supset R_{ya}) \\
\end{align*}
\]

Now we’ll instantiate the other existential wff and unpack the result to get ...

\[
\begin{align*}
(7) & \quad \neg (G_b \supset \exists y (F_y \land R_{by}))
\end{align*}
\]
Answers to Exercises 29

4. Some good philosophers admire Frank; all wise people admire any good philosopher; Frank is wise; hence there is someone who both admires and is admired by Frank.

\[ \exists x (Fx \land Rxn), \forall x (Gx \supset \forall y (Fy \supset Rxy)), Gn \therefore \exists x (Rxn \land Rnx) \]

‘F’ means good philosopher, ‘n’ denotes Frank, etc.,

1. \[ \exists x (Fx \land Rxn) \]
2. \[ \forall x (Gx \supset \forall y (Fy \supset Rxy)) \]
3. \[ Gn \]
4. \[ \neg \exists x (Rxn \land Rnx) \]
5. \[ \forall x \neg (Rxn \land Rnx) \]

The obvious first move is to instantiate (2) to get ‘Gn’ as the antecedent to combine with (3) …

6. \[ (Gn \supset \forall y (Fy \supset Rny)) \]

We now have the initial existential wff at (1) plus two universals at (5) and (7) which we haven’t yet made use of. So we now proceed in the obvious way:

8. \[ (Fa \land Ran) \]
9. \[ \neg (Ran \land Rna) \]
10. \[ (Fa \supset Rna) \]

And now everything quickly closes:

11. \[ Fa \]
12. \[ Ran \]

13. \[ \neg Ran \]
14. \[ \neg Fa \]

\[ \neg Ran \]

\[ \neg Fa \]

\[ RnA \]
5. Any true philosopher admires some logician. Some students admire only existentialists. No existentialists are logicians. Therefore not all students are true philosophers.

$$\forall x(Fx \supset \exists y(Gy \land Rxy)), \exists x(Hx \land \forall y(Rxy \supset Ey)), \forall x(Ex \supset \neg Gx) \therefore \neg \forall x(Hx \supset Fx)$$

(1) $$\forall x(Fx \supset \exists y(Gy \land Rxy))$$
(2) $$\exists x(Hx \land \forall y(Rxy \supset Ey))$$
(3) $$\forall x(Ex \supset \neg Gx)$$
(4) $$\neg \forall x(Hx \supset Fx) \quad \checkmark$$ Negated conclusion
(5) $$\forall x(Hx \supset Fx)$$

The first move must be to instantiate the existential quantifier (2) to give

(6) $$(Ha \land \forall y(Ray \supset Ey))$$
(7) $$Ha$$
(8) $$\forall y(Ray \supset Ey)$$

The obvious next move is to instantiate (5) in order to use the antecedent ‘Ha’:

(9) $$(Ha \supset Fa)$$
(10) $$\neg Ha \quad Fa \quad \star$$

And we now instantiate (1) in order to use the antecedent ‘Fa’:

(11) $$(Fa \supset \exists y(Gy \land Ray))$$
(12) $$\neg Fa \quad \exists y(Gy \land Ray) \quad \star$$

We now have an uninstantiated existential wff at (12), and two as-yet-unused universally quantified wffs at (3) and (8). So let’s proceed in the obvious way to get

(13) $$(Gb \land Rab)$$
(14) $$(Eb \supset \neg Gb)$$
(15) $$(Rab \supset Eb)$$

The last steps are trivial!

(16) $$Gb$$
(17) $$Rab$$
(18) $$\neg Eb \quad \neg Gb \quad \star$$
(19) $$\neg Rab \quad Eb \quad \star$$
6. *Everyone loves a lover; hence if someone is a lover, everyone loves everyone!*

For the translation, see E, p. 278: *someone is a lover* is equivalent to *there is someone who is such that there is someone that they love*, so

\[ \forall x \exists y (\exists z (Lyz \supset Lxy)) \supset (\exists x \exists y (Lxy \supset \forall x \forall y Lxy)) \]

(1) \[ \forall x \forall y (\exists z (Lyz \supset Lxy)) \]
(2) \[ \neg (\exists y z (Lxy \supset \forall x \forall y Lxy)) \quad \checkmark \quad \text{Negated conclusion} \]
(3) \[ \exists y z Lxy \]
(4) \[ \neg \forall y Lxy \quad \checkmark \]
(5) \[ \exists x \neg \forall y Lxy \]

We’ve now got a lot of existentials to instantiate!

(6) \[ \exists y Lax \quad \checkmark \quad \text{From } 3, \text{ now checked off} \]
(7) \[ \exists y Lax \quad \text{From } 6 \]
(8) \[ \neg \forall y Lcy \quad \checkmark \quad \text{From } 5, \text{ now checked off} \]
(9) \[ \exists y \neg Lcy \quad \checkmark \quad \text{From } 8 \]
(10) \[ \neg Lcd \quad \text{From } 9 \]

Everything other than primitive wffs is now checked off, except (1), so we now need to use that. Let’s first instantiate to get ‘Lcd’ as the consequent, to conflict with (10) …

(11) \[ \forall y (\exists z (Lyz \supset Lcy)) \]
(12) \[ (\exists z Ld z \supset Lcd) \]

(13) \[ \neg \exists z Ld z \quad \text{Lcd} \]
(14) \[ \forall z \neg Ld z \quad \star \]

But now what? Well, we want eventually to make use of (7), so we’ll aim to eventually get an occurrence of ‘\(\neg Lb\)’ to contradict (7). But how are we going to get that? Presumably by using (1) again. But the consequent of instantiations of (1) don’t involve negations: so our needed wff will come – if at all – via the antecedent of that instantiation. Which means that the ‘y’ variable will need to be instantiated with ‘a’. But now, if we also instantiate the ‘x’ variable in (1) with ‘d’ we’ll get an occurrence of ‘Lda’ as the consequent, which will contradict (14). So, let’s try that line …

(15) \[ \forall y (\exists z (Lyz \supset Ldy)) \]
(16) \[ (\exists z L dz \supset Lda) \]

(17) \[ \neg \exists z Laz \quad \text{Lda} \]
(18) \[ \forall z \neg Laz \quad \neg Lda \quad \text{From } 17 \mid \text{From } 14 \]
(19) \[ \neg Lb \quad \star \]

And we are done!

Note that the argument is intuitively valid. Assume everyone loves a lover. Then, supposing someone is a lover, everyone loves him (because everyone loves a lover)! So everyone is a lover. So everyone loves everyone (again because everyone loves a lover)! This double invocation of the premiss in the informal argument is matched by the double invocation in our formal tree-argument.
7. If anyone speaks to anyone, then someone introduces them; no one introduces anyone to anyone unless they know them both; everyone speaks to Frank; therefore everyone is introduced to Frank by someone who knows him. [Use ‘Rxyz’ to render ‘x introduces y to z’.

For the translation, use ‘Sxy’ for x speaks to y, and ‘Kxy’ for x knows y. Translation requires a bit of thought. (a) The first premiss is plainly intended to involve a universal generalization (that any pair of people, x, y, if x talks to y, then they’ve been introduced). (b) Being introduced is (strictly speaking) a matter of someone (i) introducing the first to the second and (ii) the second to the first, though it doesn’t in fact matter for the validity of this argument if you forget about (ii).

∀x∀y(Sxy ⊃ ∃z(Rzxy ∧ Rzyx)), ∀x∀y∀z(Rxyz ⊃ (Kxy ∧ Kxz)), ∀xSxn

∴ ∀x∃y(Ryxn ∧ Kyn)

(1) ∀x∀y(Sxy ⊃ ∃z(Rzxy ∧ Rzyx))
(2) ∀x∀y∀z(Rxyz ⊃ (Kxy ∧ Kxz))
(3) ∀xSxn
(4) ¬∀x∃y(Ryxn ∧ Kxn) √ Negated conclusion
(5) ∃x¬∃y(Ryxn ∧ Kxn)

The first move has to be to instantiate the existential quantifier (5) to give

(6) ¬∃y(Ryan ∧ Kyn)

(7) ∀y¬(Ryan ∧ Kyn)

We now have two names in play, ‘n’ and ‘a’: that’s not enough to make use of the triply quantified (2), so forget that for the moment. But if we instantiate (3) with ‘a’ to get ‘San’, and (1) with both names we’ll get an occurrence of ‘San’ as the antecedent of a conditional, thus ...

(8) San
(9) ∀y(Say ⊃ ∃z(Rzay ∧ Rzya))
(9) (San ⊃ ∃z(Rzan ∧ Rzna))

(10) ¬San ∃z(Rzan ∧ Rzna) *

Obviously, we now instantiate our new existential wff to get:

(11) {Rban ∧ Rbna}
(11) Rban
(12) Rbna

We’ve now got two universals that we haven’t yet made use of, at (2) and (7). Take the simpler one first and instantiate with ‘b’ (of course! — to give us an occurrence of ‘Rban’ in the scope of a negation, to contradict (11)):

(13) ¬(Rban ∧ Kbn)

(14) ¬Rban ¬Kbn *

We now at last use (2): to get something contradicting ‘¬Kbn’, we must instantiate ‘x’ by ‘b’:

(13) ∀y∀z(Ryz ⊃ (Kby ∧ Kbz))

Now it should be obvious how to continue ...

(14) ∀z(Rbaz ⊃ (Kba ∧ Kbz))
(15) (Rban ⊃ (Kba ∧ Kbn))

(16) ¬Rban (Kba ∧ Kbn) *
(17) Kba
(18) Kbn *

(19)
8. Any elephant weighs more than any horse. Some horse weighs more than any donkey. If a first thing weighs more than a second thing, and the second thing weighs more than a third, then the first weighs more than the third. Hence any elephant weighs more than any donkey.

\[ \forall x \forall y ((Fx \land Gy) \supset Rxy), \exists x (Gx \land \forall y (Hy \supset Rxy)), \forall x \forall y \forall z ((Rxy \land Ryz) \supset Rxz) \]

\[ \therefore \forall x \forall y ((Fx \land Hy) \supset Rxy) \]

(1) \[ \forall x \forall y ((Fx \land Gy) \supset Rxy) \]
(2) \[ \exists x (Gx \land \forall y (Hy \supset Rxy)) \]
(3) \[ \forall x \forall y \forall z ((Rxy \land Ryz) \supset Rxz) \]
(4) \[ \neg \forall x \forall y ((Fx \land Hy) \supset Rxy) \quad \checkmark \quad \text{Negated conclusion} \]
(5) \[ \exists x \neg \forall y ((Fx \land Hy) \supset Rxy) \]

The first move has to be to instantiate our two existential quantifiers to give

(6) \[ (Ga \land \forall y (Hy \supset Ray)) \]
(7) \[ \neg \forall y ((Fb \land Hy) \supset Rby) \quad \checkmark \]
(8) \[ \exists x \neg ((Fb \land Hy) \supset Rby) \quad \checkmark \]
(9) \[ \neg (Fb \land Hc) \supset Rbc \]

where we've just instantiated the new existential at (8) too. Let's just now unpack (6) and (9) to give

(10) \[ Ga \]
(11) \[ \forall y (Hy \supset Ray) \]
(12) \[ (Fb \land Hc) \quad \checkmark \]
(13) \[ \neg Rbc \]
(14) \[ Fb \]
(15) \[ Hc \]

We now have three names in play, and three universals at (1), (3) and (11) to instantiate. Obviously we should chose instantiations which neatly tie in with the primitives at (10), (13), (14), (15), thus …

(16) \[ \forall y ((Fb \land Gy) \supset Rby) \]
(17) \[ ((Fb \land Ga) \supset Rba) \]
(18) \[ (Hc \supset Rac) \]
(19) \[ \forall y \forall z ((Rby \land Ryz) \supset Rbz) \]
(20) \[ \forall z ((Rba \land Raz) \supset Rbz) \]
(21) \[ ((Rba \land Rac) \supset Rbc) \]

(22) \[ \neg Hc \quad \text{Rac} \]
(23) \[ (Fb \land Ga) \quad \text{Rba} \]

(24) \[ \neg Fb \quad \neg Ga \]

\[ (Rba \land Rac) \quad \text{Rbc} \quad \checkmark \]

\[ \neg Rba \quad \neg Rac \quad \checkmark \]

\[ \neg Rba \quad \neg Rac \quad \checkmark \]

\[ \neg Rba \quad \neg Rac \quad \checkmark \]
C  Redo the first three examples of §29.2 as signed trees (as in §28.2).

A  \[\exists x Fx, \forall x \forall y(Fy \supset \neg Lxy) \vdash \exists x \forall y \neg Lxy\]

We suppose that there is a valuation \(q\) such that

(1) \[\exists x Fx \Rightarrow q T\]
(2) \[\forall x \forall y(Fy \supset \neg Lxy) \Rightarrow q T\]
(3) \[\neg \exists x \forall y \neg Lxy \Rightarrow q T\]

So from (3) we get

(4) \[\forall x \neg \forall y \neg Lxy \Rightarrow q T\]

(1) tells us that there then there must be an extension \(q^*\) of \(q\) to cover the new name 'a', such that

(5) \[Fa \Rightarrow q^* T\]

So from (4) — since the extended valuation doesn't change what's in the domain or anything's properties, but just dubs something with a new name —we know

(6) \[\neg \forall y \neg Lya \Rightarrow q^* T\]
(7) \[\exists y \neg \neg Lya \Rightarrow q^* T\]

(1) tells us that there then there must be a further extension \(q^{**}\) of \(q\) to cover the new name 'b', such that

(8) \[\neg \neg Lba \Rightarrow q^{**} T\]

Whence (why??) ...

(9) \[\forall y(Fy \supset \neg Lby) \Rightarrow q^{**} T\]
(10) \[(Fa \supset \neg Lba) \Rightarrow q^{**} T\]

(11) \[\neg Fa \Rightarrow q^{**} T \quad \neg Lba \Rightarrow q^{**} T\]

\[\ast \quad \ast\]

B  \[\forall x \exists y(Fx \land Lxy), \forall x \forall y(Lxy \supset Mxy) \vdash \forall x \exists y(Fx \land Mxy)\]

We suppose that there is a valuation \(q\) such that

(1) \[\forall x \exists y(Fx \land Lxy) \Rightarrow q T\]
(2) \[\forall x \forall y(Lxy \supset Mxy) \Rightarrow q T\]
(3) \[\neg \forall x \exists y(Fx \land Mxy) \Rightarrow q T\]

So from (3) we get

(4) \[\exists x \neg \exists y(Fx \land Mxy) \Rightarrow q T\]

(4) tells us that there then there must be an extension \(q^*\) of \(q\) to cover the new name 'a', such that

(5) \[\neg \exists y(Fa \land May) \Rightarrow q^* T\]

whence ...

(6) \[\forall y \neg (Fa \land May) \Rightarrow q^* T\]
(7) \[\exists y(Fa \land Lay) ) \Rightarrow q^* T\]
(8) \[\forall y(Lay \supset May) \Rightarrow q^* T\]

(7) tells us that there then there must be a further extension \(q^{**}\) of \(q\) to cover the new name 'b', such that

(9) \[(Fa \land Lab) \Rightarrow q^{**} T\]

whence ...

(10) \[\neg (Fa \land Mab) \Rightarrow q^{**} T\]
(11) \[(Lab \supset Mab) \Rightarrow q^{**} T]\n(12) \[Fa \Rightarrow q^{**} T\]
Similarly for C.