Answers to Exercises 33

A More Welsh affairs! Using the same translation manual as §33.2, translate the following into **QL**$^=$:

1. Angharad and Mrs Jones are one and the same.

   $a = m$

2. At least one Welsh speaker loves Bryn.

   $\exists x(Fx \wedge Lxb)$

   (Remember: 'at most one' needs the identity symbol to translate it: 'at least one' doesn't – it is the familiar existential quantifier!)

3. Either Angharad or Bryn is Mrs Jones.

   $(a = m \vee b = m)$

   (Remember: '$(a \vee b) = m$' is garbage: propositional connectives have to connect propositional clauses – see §22.2)

4. Someone other than Bryn loves Angharad.

   $\exists x(\neg x = b \wedge Lxa)$

   (Remember: the negation sign negates '$x = b$', the whole clause, not just the variable!)

5. Mrs Jones loves everyone but Angharad.

   $\forall x(\neg x = a \supset Lmx)$

6. Some girls only love Bryn.

   $\exists x((Gx \wedge Lxb) \wedge \forall y(Lxy \supset y = b))$

   ('Some girls only love Bryn' surely implies that some girls do love Bryn: hence the first clause: and the second clause says that these girls love only him. Equivalently, we could have written '$\exists x((Gx \wedge Lxb) \wedge \forall y(\neg y = b \supset \neg Lxy))$'.)

7. Some girls only love people who love them.

   $\exists x\forall y(Gxb \wedge \forall y(Lxy \supset Lyx))$

   (The natural reading of 'Some girls only love people who love them' doesn't imply that these girls love all the people who love them — and maybe not any of them! Anyway, the point about this example, is that 'only' is not always translated by using identity. As of course the next example makes plain too!)

8. Only if she loves Bryn is Angharad loved by him.

   $(Lba \supset Lab)$

9. Exactly one girl who loves Bryn speaks Welsh.

   Exactly one *J* is *K translates as* $\exists x((Jx \wedge Kx) \wedge \forall y((Jy \wedge Ky) \supset y = x))$

   Hence in this example:

   $\exists x(([Gx \wedge Lxb] \wedge Fx) \wedge \forall y(([Gy \wedge Lyb] \wedge Fy) \supset y = x))$

10. If Angharad isn't Mrs Jones, then at least two people love Bryn.

    $(\neg a = m \supset \exists x\exists y((Lxb \wedge Lyb) \wedge \neg x = y))$

B Take the domain of quantification to be the (positive whole) numbers, and let 'm' denote the number zero, 'n' denote the number one, let 'Mxy' mean that *x immediately follows y in the number series* (equivalently, *x is an immediate successor of y*), and 'Rxyz' mean that *x equals y plus z*. Then translate the following from **QL⁼** into natural English:

1. ∀x∀y∀z((Mxz ∧ Myz) ⊃ x = y)
   Any two successors of a number are equal – *or* a number has at most one immediate successor.

2. ∀y∃xMxy
   Every number has an immediate successor.

3. ¬∃zMmz
   Zero has no predecessors.

4. ∀x∀y∀z((Mxy ∧ y = z) ⊃ Mxz)
   Equal numbers have equal sucessors.

5. ∀x∀y(Rxym = y)
   Oops, typo (as noted on the website)! Sorry! This is garbage, and what was intended was ∀x∀y(Rxym ⊃ x = y), *i.e. a number plus zero equals itself!*

6. ∀x∀y(Rxyn ⊃ Mxy)
   If a number is onee more than another, it is its successor.

7. ∀x∀y∀z(Rxyz = Rxzy)
   Oops, another typo (as noted on the website)! The identity sign should be an equivalence sign, and what was intended was ∀x∀y∀z(Rxyz ≡ Rxzy), *i.e. it doesn't matter what order you add numbers, you get the same result.*

8. ∃x(Rxxx ∧ ∀y(Ryyy ⊃ y = x))
   There is a unique number which when added to itself yields itself. (*Zero!*)

9. ∀x∀y∃z(Rzxy ∧ $\forall v_0(Rv_0xy \supset v_0 = z)$)
   Any two numbers have a unique sum.

10. ∀z∃x((Mxz ∧ ∀y(Myz ⊃ y = x)) ∧ Rznx)
    This is a wff, which says every number has a unique successor, and is one greater than its successor – a falsehood slipped in to keep you alert!

11. ∃x∃y({[((Mxm ∨ Mxn) ∧ (Mym ∨ Myn))] ∧ ¬x = y}
    ∧ ∀z{(Mzm ∨ Mzn) ⊃ (z = x ∨ z = y)})

    There are exactly two numbers which are successors of zero or one.