Faculty of Philosophy

Formal Logic

Lecture 4

Peter Smith
Outline

- Divide and rule
- ‘And’, ‘or’ and ‘not’
- The three basic propositional connectives
  - Conjunction
  - Disjunction
  - Negation
- Bivalent propositions and sets of possible worlds
- Complex propositions
To evaluate arguments we often first need to do interpretative work to fill out the exact message being conveyed by premisses and conclusions.
Issues of interpretation – 1

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- E.g. we’ll need to work out what is being said, in context, by ‘Mary went to the bank’ (lexical ambiguity), ‘She kissed him’ (referential ambiguity), “Mary had a little lamb’.
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E.g. we’ll need to work out what is being said, in context, by ‘Mary went to the bank’ (lexical ambiguity), ‘She kissed him’ (referential ambiguity), “Mary had a little lamb’.

Even if words are clear, we may have to deal with structural ambiguities: e.g. ‘Every experience might be delusory’.
Issues of interpretation – 2

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- But notice the kind of interpretative step isn’t the sort of thing that we can readily give any sort of systematic theory for.

- The possibility of a systematic logical theory kicks in at the next stage, where we evaluate inferences between sufficiently clearly regimented propositions.

- But natural languages like English aren’t ideally suited for doing the regimentation. (Consider e.g. the vagaries of quantifiers in English.)
How to cope

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That’s why logicians construct artificial languages of logic for perspicuously representing the logically relevant content of premisses and conclusions.

Then the overall evaluation of arguments can proceed in two steps:

1. We regiment an ordinary-language argument into some appropriate symbolic language.
2. Assess the regimented argument in its regimented symbolic form.
Implementing the divide and rule strategy

We will illustrate this strategy first with a toy example, involving arguments whose relevant logical materials are just ‘and’, ‘or’, and ‘not’.
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- We will illustrate this strategy first with a toy example, involving arguments whose relevant logical materials are just ‘and’, ‘or’, and ‘not’.
- This will introduce us to the language called PL (the language of propositional logic). It will be easy to introduce a range of key logical ideas in the context of this toy language – so that's our excuse for spending a lot of time on it at the outset.
‘And’, ‘or’ and ‘not’

- Divide and rule
- ‘And’, ‘or’ and ‘not’
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We start with some very simple examples of inferences (valid and invalid) that owe their validity/invalidity to the distribution of ‘and’, ‘or’ and ‘not’ in the premisses and conclusions.
‘And', ‘or' and ‘not’

‘Disjunctive syllogism’

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1. Either we are out of petrol or the carburettor is blocked.  
   We are not out of petrol.  
   So, the carburettor is blocked.
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1. Either we are out of petrol or the carburettor is blocked.
   We are not out of petrol.
   So, the carburettor is blocked.

2. Either Jill is in the library or she is in the bar.
   She is not in the library.
   So Jill is in the bar.

These inferences are both valid, as is any of the type

Either \( P \) or \( Q \)
Not-\( P \)
So: \( Q \)
‘And’, ‘or’ and ‘not’

An evidently invalid argument

1. It’s not the case that Alice is clever and Bertie is clever too. Hence Alice is not clever.
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‘And’, ‘or’ and ‘not’

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1. It’s not the case that Alice is clever and Bertie is clever too. Hence Alice is not clever.

2. It’s not true that Brown and Blair both supported the policy. Therefore Brown didn’t support the policy.

These inferences are both invalid, as is any that relies on an inference of the type

\[ \text{Not-} (P \text{ and } Q) \]

\[ \text{So: Not-} P \]
A further example

1. It’s not the case that Alice is clever and also that Bertie is clever too.
   Alice is clever
   So Bertie is not clever.

2. It’s not true that Brown and Blair both supported the policy
   Brown supported the policy
   So Blair did not support it.

These inferences are this time both valid, as is any of the type

\[ \text{Not-(P and Q)} \]
\[ P \]
\[ So: \text{Not-Q} \]
Scope matters!

Note the use of bracketing to make it clear what the ‘not’ applies to (what its scope is). So compare

1. Not-(P and Q)
2. (Not-P) and Q

If

P = Obama will order an attack
Q = There will be devastation

then

Not-(P and Q) = it isn’t true that Obama will order an attack and there will be devastation;
(Not-P) and Q = Obama will not order an attack, but there will still be devastation.

The scope of the ‘not’ matters.
Introducing the language PL

Our next task, then, is to consider how we can systematically evaluate arguments as in our examples, which depend for their validity or invalidity on the distribution of ‘and’, ‘or’, and ‘not’ in premisses and conclusions.
‘And’, ‘or’ and ‘not’

Introducing the language **PL**

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- As we’ll see, vernacular ‘and’, ‘or’, and ‘not’ are subject to various semantic complexities and ambiguities. And also they can be put together into sentences in ways that create so-called **scope ambiguities**.
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- For example, consider the party invitation ‘You can bring your partner or come alone and have a good time’. (How does that ‘group together’, how should we ‘bracket’ it?)
Introducing the language \textbf{PL}

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- As we’ll see, vernacular ‘and’, ‘or’, and ‘not’ are subject to various semantic complexities and ambiguities. And also they can be put together into sentences in ways that create so-called \textit{scope ambiguities}.
- For example, consider the party invitation ‘You can bring your partner or come alone and have a good time’. (How does that ‘group together’, how should we ‘bracket’ it?)
- So following the ‘divide and rule’ strategy, we first need to characterize a formal language \textbf{PL} for regimenting arguments clearly and without ambiguities (whether word ambiguities or structural scope ambiguities).
The three basic propositional connectives

- Divide and rule
- ‘And’, ‘or’ and ‘not’

The three basic propositional connectives
- Conjunction
- Disjunction
- Negation

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The three basic propositional connectives

Bare Conjunction – 1

- We are going to isolate the core sense of ‘and’ (maybe some uses of English ‘and’ involve more than the core sense). The core sense is **bare conjunction**. The bare conjunction of two propositions is true exactly when both propositions are true together (no more, no less).
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We will use ‘∧’ to signify bare conjunction. So

\[(\phi \land \psi) \text{ is true just when both } \phi \text{ and } \psi \text{ are true}\]
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- Note that

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(P \land Q) \\
(Q \land P)
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therefore say exactly the same – both hold exactly when each of the constituent propositions \(P\) and \(Q\) hold.
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- NB ‘∧’ always comes with a pair of brackets.
The three basic propositional connectives

Bare Conjunction – 2

- Contrast

Mary became pregnant and she got married.
Mary got married and she became pregnant.

with

\((\text{Mary became pregnant } \land \text{ she got married})\).
\((\text{Mary got married } \land \text{ she became pregnant})\).

So does ordinary language ‘and’ mean more than ‘\(\land\)’?
The three basic propositional connectives

Bare Conjunction – 2

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Mary got married and she became pregnant.

with

(Mary became pregnant ∧ she got married).
(Mary got married ∧ she became pregnant).

So does ordinary language ‘and’ mean more than ‘∧’?
- Perhaps not. For compare

Mary became pregnant. She got married.
Mary got married. She became pregnant.

Standard narrative conventions still generate implications of temporal order, so we don’t need to say that ‘and’ is responsible for such implications.
The three basic propositional connectives

The truth table for conjunction – 1

- \((\phi \land \psi)\) is true just when \(\phi\) and \(\psi\) are both true.
The three basic propositional connectives

The truth table for conjunction – 1

- \((\phi \land \psi)\) is true just when \(\phi\) and \(\psi\) are both true.
- Given two propositions, \(\phi\) and \(\psi\), there are four ways that the world way may turn out:

<table>
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<tr>
<th>(\phi)</th>
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<td>True</td>
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The three basic propositional connectives

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- We are assuming \(\phi\) and \(\psi\) are determinate propositions (not vague or subject to other failings – more on this later).
- And each way the world can turn out fixes whether \((\phi \land \psi)\) is true or false – as follows . . .
The three basic propositional connectives

### The truth table for conjunction – 2

- $(\phi \land \psi)$ is true just when $\phi$ and $\psi$ are both true.
- Given two propositions, $\phi$ and $\psi$, there are four ways that the world way may turn out, and they fix whether $(\phi \land \psi)$ is true or false:

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- And we can now naturally abbreviate this – as follows . . .
The three basic propositional connectives

The truth table for conjunction – 2

- \((\phi \land \psi)\) is true just when \(\phi\) and \(\psi\) are both true.
- Given two propositions, \(\phi\) and \(\psi\), there are four ways that the world way may turn out, and they fix whether \((\phi \land \psi)\) is true or false:

\[
\begin{array}{ccc}
\phi & \psi & (\phi \land \psi) \\
T & T & T \\
T & F & F \\
F & T & F \\
F & F & F \\
\end{array}
\]

- This is the truth-table for ‘\(\land\)’.
The three basic propositional connectives

The truth table for conjunction – 3

Note that the statement

\[(\phi \land \psi)\] is true just when \(\phi\) and \(\psi\) are both true.

and the table

<table>
<thead>
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<th>(\phi)</th>
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<th>((\phi \land \psi))</th>
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are two ways of presenting the same basic characterization of bare conjunction.
The three basic propositional connectives

Disjunction

Compare

1. Osborne will either increase taxes or reduce government spending.
2. Either the peace initiative will be succeed or the war will carry on at least another year.

(1) is naturally read **INCLUSIVELY** – i.e. read ‘P or Q or both’. (2) is naturally read **EXCLUSIVELY**.

Inclusive ‘or’ is standardly symbolized by ‘\(\lor\)’ [for Latin ‘vel’].

\[(\phi \lor \psi)\] is true just when at least one of \(\phi\) and \(\psi\) is true.

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\begin{array}{|c|c|}
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The three basic propositional connectives

Negation

- The strict NEGATION of a proposition \( \phi \) is true exactly when proposition \( \phi \) is false.
The three basic propositional connectives

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- ‘It is not the case that \( \phi \)’ almost always expresses the negation of plain \( \phi \).
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Negation

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- ‘It is not the case that \( \phi \)’ almost always expresses the negation of plain \( \phi \).
- Inserted ‘not’ often expresses negation more simply – e.g. compare

  (a) Jill is married.
  (b) Jill is not married.
  
  (c) Thatcher was a KGB spy.
  (d) Thatcher was not a KGB spy.
Negation

- But inserted ‘not’ doesn’t always produce the negation of a proposition. Consider

  (a) Some Welshmen are rugby fanatics.
Negation

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- The only place you can insert ‘not’ preserving grammaticality is:

  (b) Some Welshmen are not rugby fanatics.

But (b) is not the negation of (a). For on the natural readings, both are true (while the negation of the true (a) has to be a falsehood).
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But (b) is not the negation of (a). For on the natural readings, both are true (while the negation of the true (a) has to be a falsehood).

- Two ways of expressing the negation of (a):

  (c) It is not the case that some Welshmen are rugby fanatics.
  (d) No Welshman is a rugby fanatic.
The three basic propositional connectives

Negation

- We will use the sign ‘¬’ for negation.
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- ‘¬’ always works as ‘it is not the case that’ usually works: so ‘¬φ’ expresses the strict negation of φ (in other words, ‘¬φ’ is the strict contradictory of φ).
The three basic propositional connectives

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► In a truth-table

<table>
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- Unlike ‘∧’ and ‘∨’, the negation-sign ‘¬’ does not need brackets.
The three basic propositional connectives

‘∧’ and ‘∨’ are called propositional connectives, for obvious reasons. ‘¬’ is also treated as an honorary connective.

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