Revisit summer ... go to the Fitzwilliam Museum!
Propositional connectives, and the assumption of bivalence

Propositions and worlds

Complex propositions

Two kinds of symbols?
Introducing the language $\textbf{PL}$

- We are going to be examining how we can systematically evaluate arguments which depend for their validity or invalidity on the distribution of ‘$\text{and}$’, ‘$\text{or}$’, and ‘$\text{not}$’ in premisses and conclusions.
Introducing the language **PL**

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- This isn’t because learning how to deal with such simple arguments is intrinsically exciting. It isn’t! But we can introduce various key ideas and techniques in this ‘baby’ case without having to tangle with distracting complexities.
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- Vernacular ‘and’, ‘or’, and ‘not’ are subject to various (lexical) semantic complexities and ambiguities and their use can create (structural) scope ambiguities.
Propositional connectives, and the assumption of bivalence

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- Vernacular ‘and’, ‘or’, and ‘not’ are subject to various (lexical) semantic complexities and ambiguities and their use can create (structural) scope ambiguities.
- So following the ‘divide and rule’ strategy, we first need to characterize a formal language **PL** for regimenting arguments clearly and without ambiguities. Then we discuss how to assess arguments once regimented.
We introduced the three basic connectives

‘∧’ and ‘∨’ are called propositional connectives, for obvious reasons. ‘¬’ is also treated as an honorary connective.

\[
\begin{array}{c|c|c|c}
\phi & \psi & (\phi \land \psi) & (\phi \lor \psi) \\
\hline
T & T & T & T \\
T & F & F & T \\
F & T & F & T \\
F & F & F & F \\
\end{array}
\]

\[
\begin{array}{c|c}
\phi & \neg \phi \\
\hline
T & F \\
F & T \\
\end{array}
\]
Propositions and worlds

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We are going to be assuming that any proposition $\phi$ that we are concerned with is either true or false . . . i.e. has one of the two 'truth-values', true and false . . . is bivalent.

So we are going to set aside cases where there are arguably 'truth-value gaps':

1. vague propositions (it is neither true nor false that the borderline bald man is bald).
2. liar propositions ("This proposition is not true" is neither true nor false).
3. propositions involving denotationless names ("Mr Sviatolak Brintangle is married" is neither true nor false if there is no such person as Mr B.)
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Propositions and possible worlds

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So given a proposition $\phi$ let’s write $\Phi$ for the corresponding set containing all possible worlds where $\phi$ is true.
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Conversely, given a set of possible worlds, ways that things could be, there is the proposition that the actual world is one of those.

So given a proposition \( \phi \) let’s write \( \Phi \) for the corresponding set containing all possible worlds where \( \phi \) is true.

Then \( \phi \) is equivalent to the proposition that the actual world is in \( \Phi \), i.e. the actual world is one of the worlds where \( \phi \) is true, i.e. \( \phi \) is true.
Suppose $\phi$ and $\psi$ are propositions, and that $\Phi$ and $\Psi$ are the corresponding sets of possible worlds where $\phi$ and $\psi$ respectively are true.
The connectives again

- Suppose $\phi$ and $\psi$ are propositions, and that $\Phi$ and $\Psi$ are the corresponding sets of possible worlds where $\phi$ and $\psi$ respectively are true.
- The proposition $(\phi \land \psi)$ corresponds to $(\Phi \cap \Psi)$, the intersection of $\Phi$ and $\Psi$. 
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- The proposition $(\phi \lor \psi)$ corresponds to $(\Phi \cup \Psi)$, the union of $\Phi$ and $\Psi$. 
Suppose \( \phi \) and \( \psi \) are propositions, and that \( \Phi \) and \( \Psi \) are the corresponding sets of possible worlds where \( \phi \) and \( \psi \) respectively are true.

The proposition \( (\phi \land \psi) \) corresponds to \( (\Phi \cap \Psi) \), the intersection of \( \Phi \) and \( \Psi \).

The proposition \( (\phi \lor \psi) \) corresponds to \( (\Phi \cup \Psi) \), the union of \( \Phi \) and \( \Psi \).

The proposition \( \neg \phi \) corresponds to \( \Phi \), the complement of \( \Phi \).
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- The proposition $\neg \phi$ corresponds to $\overline{\Phi}$, the complement of $\Phi$.
- (This happy alignment of notation is not an accident!)
Complex propositions

- Propositional connectives, and the assumption of bivalence
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Complex propositions

Using more than one connective

- We can build up complex propositions by using more than one connective, as in:

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(P \land \neg Q) \\
\neg (P \land \neg Q) \\
(R \lor \neg (P \land \neg Q)) \\
\neg (R \lor \neg (P \land \neg Q))
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Using more than one connective

▶ We can build up complex propositions by using more than one connective, as in:

\[(P \land \neg Q)\]
\[\neg (P \land \neg Q)\]
\[(R \lor \neg (P \land \neg Q))\]
\[\neg (R \lor \neg (P \land \neg Q))\]

▶ NB, to reiterate:

1. Every occurrence of the connectives ‘\(\land\)’ and ‘\(\lor\)’ always comes with an accompanying pair of brackets, which make the scope of the connectives entirely clear.
2. The connective ‘\(\neg\)’ never introduces extra brackets – the rule (in rough terms) is that ‘a negation governs what immediately follows it’.
Complex propositions

Interpreting examples

Suppose:

\[ P = \text{Felix is on the mat.} \]
\[ Q = \text{Fido is on the mat.} \]

Then:

1. \( (P \land \neg Q) \) = Felix is on the mat and Fido isn’t.
Interpreting examples

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1. \((P \land \neg Q) = \text{Felix is on the mat and Fido isn’t.}\)
2. \(\neg(P \lor Q) = \text{It isn’t the case that either Felix or Fido are on the mat}\)
   \[= \text{Neither Felix nor Fido are on the mat.}\]
Complex propositions

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2. \( \neg (P \lor Q) \) = It isn’t the case that either Felix or Fido are on the mat
   = Neither Felix nor Fido are on the mat.
3. \( (\neg P \lor Q) \) = Either Felix isn’t on the mat or Fido is.
Complex propositions

Calculating truth values

Suppose:

\[ P \] is false.
\[ Q \] is true.

Then:

\[ \neg P \] is true.
\[ \neg Q \] is false.

NB, to calculate the truth-values of the complex ('molecular') propositions we only need to know the truth-values of the simple ('atomic') ones. You don't need to remember what they mean!
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NB, to calculate the truth-values of the complex (‘molecular’) propositions we only need to know the truth-values of the simple (‘atomic’) ones. You don’t need to remember what they mean!
Some more examples

<table>
<thead>
<tr>
<th>φ</th>
<th>ψ</th>
<th>(φ ∧ ψ)</th>
<th>(φ ∨ ψ)</th>
<th>φ</th>
<th>¬φ</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
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Suppose that $P$ is True, $Q$ is False, $R$ is True.
Then, what are the truth-values of

1. $(Q ∧ R)$
2. $((Q ∧ R) ∨ P)$
3. $(Q ∧ (R ∨ P))$
4. $¬(P ∧ ¬Q)$
Some more examples

$P$ is $T$, $Q$ is $F$, $R$ is $T$.

1. $(Q \land R)$ is $F$. 

Note the importance of bracketing here: $Q \land R \lor P$ would be scope-ambiguous (and have different truth-values depending on how we disambiguate it): hence the need to insist on the brackets with the connectives $\land$ and $\lor$.

4. $\neg (P \land \neg Q)$ is $F$ – because $(P \land \neg Q)$ is $T$. 

Peter Smith: Formal Logic, Lecture 5
Some more examples

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Peter Smith: Formal Logic, Lecture 5
Two kinds of symbols?

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Two kinds of symbols?

An aside on $P, Q$ versus $\varphi, \psi$

- Why have we been using two kinds of symbols?

Roughly: $P, Q, \text{etc.}$ are shorthand for particular propositions (maybe we don't specify which propositions or we can forget which, but they are – now informally, soon formally – apt to convey particular messages).

Roughly: $\varphi, \psi$ are devices for generalization about propositions, as in: whatever propositions $\varphi, \psi$ are in play, the result of writing '(' followed by $\varphi$ followed by $\lor$ followed by ')'$ is true just when one of $\varphi, \psi$ is true.

More on this in due course (IFL, Ch. 10).
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