Outline

- Where next?
- Introducing PL trees
- Branching trees
Recap: The logician’s ‘divide and rule’ strategy

- Assessing real life arguments for (classical) validity in the general case involves two steps

1. Clarifying the content of the premisses and conclusions (sorting out ambiguities, making clear the reference of pronouns, completing unfinished propositions, etc. etc.)
2. Then evaluating the resulting clarified argument for validity.

- One powerful clarificatory technique is to render the vernacular argument into a suitable formal language which is expressly designed to be free of ambiguities, etc.

- Hence the standard logician’s assessment strategy
  1. Render vernacular arguments into a suitable formal language.
  2. Evaluate the formalized argument.
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Two questions

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  1. Can we test for tautological validity in more elegant ways than using a brute-force truth-table test?
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Two questions

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► We translate arguments into PL (or PLC if we include the conditionals) and then test for tautological validity by using a brute-force truth-table test.

► Two questions arising:
  1. Can we test for tautological validity in more elegant ways than using a brute-force truth-table test?
  2. How do we extend our technique to other kinds of arguments (e.g. those involving quantifiers)?
Two answers – 1

- Our answers to these two questions are not unrelated.
Two answers – 1

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- Consider an argument of the type $P \supset Q$, $P$, so $Q$. This is valid if, there’s no way of giving a ‘semantic value’ to $P$ and $Q$ which makes the premisses true and conclusion false. There are just four cases to look through.
Two answers – 1

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- Consider an argument of the type All $Fs$ are $Gs$, $n$ is an $F$; so $n$ is a $G$. This is valid if, there’s no way of giving a ‘semantic value’ to $F$, $G$ and $n$ which makes the premisses true and conclusion false.
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- What are truth-relevant semantic values for predicates like $F$, $G$ and names like $n$?
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- What are truth-relevant semantic values for predicates like \( F, \ G \) and names like \( n \)?
- Something like objects for \( n \), sets of objects (‘extensions’) for predicates like \( F \).
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- What are truth-relevant semantic values for predicates like $F$, $G$ and names like $n$?
- Something like objects for $n$, sets of objects (‘extensions’) for predicates like $F$.
- So there are indefinitely many different assignments of semantic values that we might make to $F$, $G$ and $n$: so we can’t do a brute-force search through all the possibilities.
Two answers – 2

- There’s another way of testing for tautological validity, namely the ‘tree method’ (a.k.a. method of ‘semantic tableaux’). This is always more elegant and very often much more efficient than brute force.
Where next?

Two answers – 2

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- And this ‘tree method’ can be carried over to the assessment of argument couched in a formal language QL for dealing with quantified arguments.
Two answers – 2

- There’s another way of testing for tautological validity, namely the ‘tree method’ (a.k.a. method of ‘semantic tableaux’). This is always more elegant and very often much more efficient than brute force.
- And this ‘tree method’ can be carried over to the assessment of argument couched in a formal language QL for dealing with quantified arguments.
- So first we introduce PL trees, then the language QL, then QL trees.
Introducing PL trees

■ Where next?

■ Introducing PL trees

■ Branching trees
‘Working backwards’ – 1

To take an utterly trivial example, consider the argument
\[ \neg P, \neg Q, \neg R, \neg S, \neg P' \text{ So } Q' \]

That is obviously not tautologically valid: and it would be daft to do a 64-line truth table to show this!
‘Working backwards’ – 1

To take an utterly trivial example, consider the argument
\[-P, \neg Q, \neg R, \neg S, \neg P' \text{ So } Q'\]
That is obviously not tautologically valid: and it would be daft to do a 64-line truth table to show this!

We can immediately see that there is a ‘counter-valuation’ (a ‘bad line’ on a truth-table) which makes the premisses true and conclusion false, without a brute-force search through the space of possibilities. To make the premisses true and conclusion false, i.e.
\[-P \Rightarrow T, \neg Q \Rightarrow T, \neg R \Rightarrow T, \neg S \Rightarrow T, \neg P' \Rightarrow T, Q' \Rightarrow F,\]
we just need
\[P \Rightarrow F, Q \Rightarrow F, R \Rightarrow F, S \Rightarrow F, P' \Rightarrow F, Q' \Rightarrow F\]
‘Working backwards’ – 2

- Another, only slightly less trivial example: consider the argument

\[(P \land \neg Q), (\neg Q \land R), (P \land S)\] So \(\neg P\)

Again, we don’t need to do a 32 line truth-table. For we can argue like this:
'Working backwards' – 2

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\[(P \land \neg Q), (\neg Q \land R), (P \land S)\] So \(\neg P'\)

Again, we don’t need to do a 32 line truth-table. For we can argue like this:

- The inference is invalid if there is a counter-valuation which makes

\[(P \land \neg Q) \Rightarrow T\]
\[(\neg Q \land R) \Rightarrow T\]
\[(P \land S) \Rightarrow T\]
\[\neg P' \Rightarrow F\]
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- And evidently there is such a valuation:

\[P \Rightarrow T, \; Q \Rightarrow F, \; R \Rightarrow T, \; S \Rightarrow T, \; P' \Rightarrow T\]
Introducing PL trees

‘Working backwards’ – 3

Consider the argument

\((P \land Q), (R \land S), (P' \land Q')\) So \(Q\)

The inference is invalid if there is a counter-valuation which makes \((P \land Q) \implies T\), \((R \land S) \implies T\), \((P' \land Q') \implies T\) and \(Q \implies F\). But we've already hit a contradiction, between \(Q \implies F\) and \(Q \implies T\). So there can't be a counter-valuation, so the original inference is valid.
‘Working backwards’ – 3

- Consider the argument

\[(P \land Q), (R \land S), (P' \land Q')\] So \(Q\)

- The inference is invalid if there is a counter-valuation which makes

\[
(P \land Q) \Rightarrow T \\
(R \land S) \Rightarrow T \\
(P' \land Q') \Rightarrow T \\
Q \Rightarrow F
\]
‘Working backwards’ – 3

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- The inference is invalid if there is a counter-valuation which makes
  
  \[
  \begin{align*}
  (P \land Q) & \Rightarrow T \\
  (R \land S) & \Rightarrow T \\
  (P' \land Q') & \Rightarrow T \\
  Q & \Rightarrow F
  \end{align*}
  \]

- The first of those assignments in turn requires
  
  \[
  \begin{align*}
  P & \Rightarrow T \\
  Q & \Rightarrow T
  \end{align*}
  \]
‘Working backwards’ – 3

Consider the argument

\((P \land Q), (R \land S), (P' \land Q')\) So \(Q\)

The inference is invalid if there is a counter-valuation which makes

\[
(P \land Q) \Rightarrow T \\
(R \land S) \Rightarrow T \\
(P' \land Q') \Rightarrow T \\
Q \Rightarrow F
\]

The first of those assignments in turn requires

\[
P \Rightarrow T \\
Q \Rightarrow T
\]

But we’ve already hit a contradiction, between \(Q \Rightarrow F\) and \(Q \Rightarrow T\). So there can’t be countervaluation, so the original inference is valid.
Introducing PL trees

‘Working backwards’ – 4

The basic idea again. Given an inference up for evaluation, assume that there is a countervaluation and see if we can work out this must look like if it exists.

1. If that assumption leads to contradiction, we know there can’t be a countervaluation and the inference is valid.

2. If we can work from that assumption to a countervaluation then the inference is invalid.

And this ‘working backwards’ approach – which doesn’t involve a brute-force search through the space of all valuations – can be applied to QL arguments as much as to PL arguments (though in the QL case it isn’t always guaranteed to deliver a verdict)
‘Working backwards’ – 4

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“But where do ‘trees’ come into it?” The story continues . . .
Branching trees

- Where next?

- Introducing PL trees

- Branching trees
Branching trees

Branching trees – 1

Consider the inference

\((P \lor Q), \text{ So } P.\)

This is evidently invalid. But how do we show this by the ‘working backwards’ method?
Branching trees

Branching trees – 1

Consider the inference

\((P \lor Q), \text{ So } P.\)

This is evidently invalid. But how do we show this by the ‘working backwards’ method? We start by supposing

\((P \lor Q) \Rightarrow T\)

\(P \Rightarrow F\)
Branching trees – 1

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This is evidently invalid. But how do we show this by the ‘working backwards’ method? We start by supposing

\[(P \lor Q) \Rightarrow T\]

\[P \Rightarrow F\]

We now need to consider branching alternatives:

- \(P \Rightarrow T\)
- \(Q \Rightarrow T\)
Branching trees

Branching trees – 1

Consider the inference

\[(P \lor Q), \text{ So } P.\]

This is evidently invalid. But how do we show this by the ‘working backwards’ method? We start by supposing

\[(P \lor Q) \Rightarrow T \]
\[P \Rightarrow F\]

We now need to consider branching alternatives:

\[
\begin{array}{c}
P \Rightarrow T \\ Q \Rightarrow T \\
\ast 
\end{array}
\]

where on the left branch we hit a contradiction: but the right branch gives us a coherent counter-valuation.
Consider next the inference

\[(P \lor Q), \neg P \text{ So } Q\]
Branching trees – 2

Consider next the inference

\((P \lor Q), \neg P \text{ So } Q\)

Using the ‘working backwards’ method we suppose

\((P \lor Q) \Rightarrow T\)
\(\neg P \Rightarrow T\)
\(Q \Rightarrow F\)
Branching trees – 2

Consider next the inference

\((P \lor Q), \neg P \quad \text{So} \quad Q\)

Using the ‘working backwards’ method we suppose

\[(P \lor Q) \Rightarrow T\]
\[\neg P \Rightarrow T\]
\[Q \Rightarrow F\]

We again need to consider branching alternatives:

\[
\begin{array}{c}
P \Rightarrow T \\
Q \Rightarrow T
\end{array}
\]
Branching trees – 2

Consider next the inference

\[(P \lor Q), \neg P \text{ So } Q\]

Using the ‘working backwards’ method we suppose

\[(P \lor Q) \Rightarrow T\]
\[\neg P \Rightarrow T\]
\[Q \Rightarrow F\]

We again need to consider branching alternatives:

\[
\begin{array}{c}
P \Rightarrow T \\
* \\
Q \Rightarrow T \\
*
\end{array}
\]

where we hit a contradiction on both branches. So the inference is valid.
Consider next the inference

\[ \neg P, (P \lor Q), (\neg Q \lor R) \text{ So } R \]
Branching trees – 3

Consider next the inference

\[ \neg P, (P \lor Q), (\neg Q \lor R) \text{ So } R \]

Here’s a corresponding tree:

\[ \neg P \Rightarrow T \]
\[ (P \lor Q) \Rightarrow T \]
\[ (\neg Q \lor R) \Rightarrow T \]
\[ R \Rightarrow F \]
Branching trees

Consider next the inference

\[ \neg P, (P \lor Q), (\neg Q \lor R) \; \text{So} \; R \]

Here's a corresponding tree:

\[
\begin{align*}
\neg P & \Rightarrow T \\
(P \lor Q) & \Rightarrow T \\
(\neg Q \lor R) & \Rightarrow T \\
R & \Rightarrow F \\
\end{align*}
\]

\[
\begin{align*}
P & \Rightarrow T \\
Q & \Rightarrow T \\
\end{align*}
\]
Branching trees

Branching trees – 3

Consider next the inference

\[ \neg P, (P \lor Q), (\neg Q \lor R) \text{ So } R \]

Here’s a corresponding tree:

\[-P \Rightarrow T\]

\[(P \lor Q) \Rightarrow T\]

\[\neg Q \lor R \Rightarrow T\]

\[R \Rightarrow F\]

\[P \Rightarrow T\]

\[Q \Rightarrow T\]

\[\ast\]
Branching trees

Branching trees – 3

Consider next the inference

\[ \neg P, (P \lor Q), (\neg Q \lor R) \text{ So } R \]

Here’s a corresponding tree:

\[
\begin{align*}
\neg P & \Rightarrow T \\
(P \lor Q) & \Rightarrow T \\
(\neg Q \lor R) & \Rightarrow T \\
R & \Rightarrow F \\
P & \Rightarrow T \\
\neg Q & \Rightarrow T \\
Q & \Rightarrow T \\
R & \Rightarrow T
\end{align*}
\]
Branching trees

Branching trees – 3

Consider next the inference

\( \neg P, (P \lor Q), (\neg Q \lor R) \) So \( R \)

Here’s a corresponding tree:

\[
\begin{aligned}
\neg P & \Rightarrow T \\
(P \lor Q) & \Rightarrow T \\
(\neg Q \lor R) & \Rightarrow T \\
R & \Rightarrow F \\
P & \Rightarrow T \\
Q & \Rightarrow T \\
\neg Q & \Rightarrow T \\
R & \Rightarrow T
\end{aligned}
\]
Branching trees

Branching trees – 4

\[ P, \neg(P \land \neg Q), (Q \lor R) \text{ So } R \]
Branching trees

Branching trees – 4

\[ P, \neg(P \land \neg Q), (Q \lor R) \text{ So } R \]

\[ P \Rightarrow T \]

\[ \neg(P \land \neg Q) \Rightarrow T \]

\[ (Q \lor R) \Rightarrow T \]

\[ R \Rightarrow F \]
Branching trees

Branching trees – 4

\[ P, \neg(P \land \neg Q), (Q \lor R) \text{ So } R \]

\[ P \Rightarrow T \]

\[ \neg(P \land \neg Q) \Rightarrow T \]

\[ (Q \lor R) \Rightarrow T \]

\[ R \Rightarrow F \]

\[ (P \land \neg Q) \Rightarrow F \]
$P, \neg(P \land \neg Q), (Q \lor R)$ So $R$

$P \Rightarrow T$

$\neg(P \land \neg Q) \Rightarrow T$

$(Q \lor R) \Rightarrow T$

$R \Rightarrow F$

$(P \land \neg Q) \Rightarrow F$

$P \Rightarrow F$  \hspace{2cm} $\neg Q \Rightarrow F$
Branching trees

Branching trees – 4

\[ P, \neg(P \land \neg Q), (Q \lor R) \text{ So } R \]

\[ P \Rightarrow T \]
\[ \neg(P \land \neg Q) \Rightarrow T \]
\[ (Q \lor R) \Rightarrow T \]
\[ R \Rightarrow F \]
\[ (P \land \neg Q) \Rightarrow F \]

\[ P \Rightarrow F \quad \neg Q \Rightarrow F \]

\*
Branching trees

Branching trees – 4

\[ P, \neg(P \land \neg Q), (Q \lor R) \text{ So } R \]

\[
\begin{align*}
P & \Rightarrow T \\

\neg(P \land \neg Q) & \Rightarrow T \\

(Q \lor R) & \Rightarrow T \\

R & \Rightarrow F \\

(P \land \neg Q) & \Rightarrow F \\

\end{align*}
\]

\[
\begin{array}{c}
P \Rightarrow F \\

\ast \\

Q \Rightarrow T \\

R \Rightarrow T \\

\neg Q \Rightarrow F \\

\end{array}
\]
Branching trees

Branching trees – 4

\[ P, \neg(P \land \neg Q), (Q \lor R) \text{ So } R \]

\[
\begin{align*}
P & \Rightarrow T \\
\neg(P \land \neg Q) & \Rightarrow T \\
(Q \lor R) & \Rightarrow T \\
R & \Rightarrow F \\
(P \land \neg Q) & \Rightarrow F
\end{align*}
\]

\[
\begin{array}{c}
P \Rightarrow F \\ \ast \end{array}
\quad
\begin{array}{c}
\neg Q \Rightarrow F \\ Q \Rightarrow T \\ R \Rightarrow T \quad \ast
\end{array}
\]
Summary so far

To operate the ‘working backwards’ method for seeing whether an inference is tautologically valid we . . .

1. Start by assuming the inference is invalid, i.e. there is a valuation which makes the premisses true and conclusion false.
Summary so far

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1. Start by assuming the inference is invalid, i.e. there is a valuation which makes the premisses true and conclusion false.

2. Then work out the implications of those assumptions: we may need to consider branching alternatives.
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1. Start by assuming the inference is invalid, i.e. there is a valuation which makes the premisses true and conclusion false.

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3. If a branch contains a contradictory assignment of values to wffs, close it off with an absurdity marker – this branch can’t represent a possible way of making the premisses true and conclusion false.
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To operate the ‘working backwards’ method for seeing whether an inference is tautologically valid we . . .

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3. If a branch contains a contradictory assignment of values to wffs, close it off with an absurdity marker – this branch can’t represent a possible way of making the premisses true and conclusion false.

4. If all branches eventually get closed off with an absurdity marker, then the assumption that the inference is invalid has lead to unavoidable absurdity and the inference is valid.
Summary so far

To operate the ‘working backwards’ method for seeing whether an inference is tautologically valid we . . .

1. Start by assuming the inference is invalid, i.e. there is a valuation which makes the premisses true and conclusion false.

2. Then work out the implications of those assumptions: we may need to consider branching alternatives.

3. If a branch contains a contradictory assignment of values to wffs, close it off with an absurdity marker – this branch can’t represent a possible way of making the premisses true and conclusion false.

4. If all branches eventually get closed off with an absurdity marker, then the assumption that the inference is invalid has lead to unavoidable absurdity and the inference is valid.

5. If a branch remains left open when we have unpacked the implications of every assignment of values to wffs other than atoms and their negations (so there is no more information to be used) then no contradiction has emerged, and the argument is indeed invalid.