Outline

- Our next task
- Basic subject/predicate structure
- How not to add quantifiers
The intended content of arguments presented in the vernacular is often obscure; the individual premisses and/or conclusion often unclear or ambiguous.
Our next task

Divide and rule

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  1. Clarify at least the relevant logical structure of premisses and conclusion by regimenting the argument into an appropriate formalized language.
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- Our way – the standard way – of handling this double task, already illustrated in the case of basic propositional logic:
  1. Clarify at least the relevant logical structure of premisses and conclusion by regimenting the argument into an appropriate formalized language.
  2. Assess the argument as couched in the formalized language.
The formalized languages of logic are intended to be contentful languages in which real arguments can be presented. Using a formalized language isn't just playing a game with uninterpreted symbols.
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**Formalized languages**

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- So to specify a formal language, we give its **syntax** (rules for defining its wffs) and its **semantics** (rules that determine what the wffs mean).
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1. syntactic form determines semantic structure,
2. the rules determine unique interpretation for each wff.
Our next main task is to develop the formal language QL for representing propositions whose logically relevant structure includes not just the familiar connectives but quantifiers (like ‘all’, ‘each’, ‘some’, ‘none’, ‘only’, …).
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We’ll need to define the syntax of QL: which strings of symbols are well-formed formulae (wffs)?

We’ll need to define the semantics of QL: how are we to interpret the wffs?
Our next task

Arguments involving quantifiers

▶ Consider arguments like

- All good philosophers like logic
- Bertrand is a good philosopher
- Bertrand likes logic
- Every analytic philosopher admires some logician
- No logician is admired by Jean-Paul
- Jean-Paul is not an analytic philosopher
- Only Bob and Ted love Mary
- Mary kissed someone who loves her
- Mary kissed either Bob or Ted

▶ They depend for their validity on the sub-propositional structure of the premisses and conclusions.

▶ QL needs to have ways of representing sub-propositional structure.

Peter Smith: Formal Logic, Lecture 14
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Our next task

Basic subject/predicate structure

How not to add quantifiers
QL will need, for a start, two classes of expressions constants (or names) and predicates.

Constants/names serve to pick out particular people/things (Bertrand, Jean-Paul, Fido, Mount Everest, the martini glass on the table, a particular water atom, the number three, . . . , any individual thing).
Names, predicates – 1

- QL will need, for a start, two classes of expressions **constants** (or names) and **predicates**.

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- Predicates express properties and relations. Cf. the English
  - ‘. . . is blue’
  - ‘. . . is even’
  - ‘. . . loves . . . ’
  - ‘. . . is shorter than . . . ’
  - ‘. . . is between . . . and . . . ’
  - ‘. . . is to . . . as . . . is to . . . ’

Basic subject/predicate structure
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Names, predicates – 2

- QL doesn’t need to segment propositional clauses any finer than name/predicate structure. So we’ll use the simplest possible expressions for names and predicates.
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  - monadic: \( F, G, H, \ldots \)
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  - monadic: \( F, G, H, \ldots \)
  - dyadic: \( L, M, \ldots \)

NB: we shouldn’t really use open-ended lists, but we’ll be careless for the moment.

NB: QL predicates all have a fixed adicity (compare ordinary language multigrade predicates like ‘work well together’, ‘conspired to commit murder’).
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Atomic sentences – 1

- The most simple kind of sentence in QL – the atomic sentences – are formed by taking an \( n \)-place predicate and following it by \( n \) names.
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  - \( Lab, Lba, Laa \ldots \)
  - \( Rcab, Raaa, Rbab \ldots \)
Atomic sentences – 2

- The interpretation of an atomic sentence – a predicate followed by name(s) – is as you’d expect. The sentence says that the individuals named have the property/stand in the relation expressed by the predicate (order of names matters!).

  - Fa means Romeo is a boy
  - Lab means Romeo loves Juliet
  - Lba means Juliet loves Romeo
  - Rabc means Romeo prefers Juliet to Rosaline
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Suppose that:

- a names Romeo,
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Adding connectives – 1

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We'll now add to QL the now-familiar truth-functional propositional connectives.

Then how would we translate the following?

- Juliet is not a boy
- Romeo loves Juliet and she loves him
- Juliet and Rosaline are both girls
- Romeo loves either Rosaline or Juliet
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- Juliet is not a boy $\Rightarrow \neg Fb$
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NOT $G(b \land c)$
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  - NOT $G(b \land c)$
- Romeo loves either Rosaline or Juliet $\Rightarrow (Lab \lor Lac)$
  - NOT $La(b \lor c)$
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- Juliet is not a boy \( \Rightarrow \neg Fb \)
- Romeo loves Juliet and she loves him \( \Rightarrow (L_{ab} \land L_{ba}) \)
- Juliet and Rosaline are both girls \( \Rightarrow (G_{b} \land G_{c}) \)
  \( \neg G(b \land c) \)
- Romeo loves either Rosaline or Juliet \( \Rightarrow (L_{ab} \lor L_{ac}) \)
  \( \neg L_{a}(b \lor c) \)
- If Romeo prefers himself to Juliet, then she doesn’t love him
  \( \Rightarrow (Ra_{ab} \supset \neg L_{ba}) \)
How not to add quantifiers

- Our next task
- Basic subject/predicate structure
- How not to add quantifiers
How not to add quantifiers

Some vagaries of English quantification

▶ Compare

All students like logic
Every student likes logic
Any student likes logic
Each student likes logic

▶ These might seem equivalent, but they embed differently, e.g.

If all students like logic, I'll be surprised
If any student likes logic, I'll be surprised

▶ And compare

Not all students turned up to the class.
Not any students turned up to the class

▶ In QL we'll have just one style of universal quantifier.
How not to add quantifiers

Some vagaries of English quantification

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Some vagaries of English quantification

- Compare

\[
\begin{align*}
\text{All students like logic} \\
\text{Every student likes logic} \\
\text{Any student likes logic} \\
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\end{align*}
\]

- These might seem equivalent, but they embed differently, e.g.
  compare

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Some vagaries of English quantification

- Compare

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  Every student likes logic
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  If all students like logic, I’ll be surprised
  If any student likes logic, I’ll be surprised

- And compare

  Not all students turned up to the class.
  Not any students turned up to the class [?]
Some vagaries of English quantification

- Compare
  
  *All students like logic*
  *Every student likes logic*
  *Any student likes logic*
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- These might seem equivalent, but they embed differently, e.g. compare

  *If all students like logic, I’ll be surprised*
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- And compare

  *Not all students turned up to the class.*
  *Not any students turned up to the class [?]*

- In QL we’ll have just one style of universal quantifier.
Quantifiers modelled on English?

- To a fair extent, “everyone” can grammatically appear in English sentences in slots where (personal) names can appear. Thus compare

Juliet loves Romeo / Everyone loves Romeo

If Juliet does, Romeo does / If everyone does, Romeo does

Juliet loves Romeo / Everyone loves everyone

(But cf. “Oh Juliet, my beloved...”: we can’t have “Oh everyone, my beloved”. Or cf. “Someone other than Juliet loves Romeo”: we can’t have “Someone other than everyone loves Romeo”.)

Can we dismiss exceptions as linguistic quirks?
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1. syntax: if $\varphi(a)$ is grammatical, so is $\varphi(\mathcal{E})$. (So we get the syntactic interchangeability of name and quantifier which we often have in English.)
2. semantics: $\varphi(\mathcal{E})$ says of everyone what $\varphi(a)$ says of what $a$ names.
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- Will this work?
Hopeless for logic! –1

▶ To repeat, the suggestion is:

1. syntax: if \( \phi(a) \) is grammatical, so is \( \phi(E) \).

2. semantics: \( \phi(E) \) says of everyone what \( \phi(n) \) says of what \( n \) names.

▶ Suppose again \( a \) names Romeo, \( b \) names Juliet, \( L \) means \( x \) loves \( y \),

▶ Then the syntactic rule makes e.g. \( \neg L_E b \) ambiguous in terms of its constructional history.

1. Do we first “quantify into” \( L_E b \) to get \( L_E b \), and then negate the result to get \( \neg L_E b \)?

2. Or do we first negate \( L_E b \) to get \( \neg L_E b \), and then “quantify in” to get \( \neg L_E b \)?

▶ And the semantic rule generates a corresponding semantic ambiguity.
Hopeless for logic! –1

To repeat, the suggestion is:

1. syntax: if \( \varphi(a) \) is grammatical, so is \( \varphi(\mathcal{E}) \).
How not to add quantifiers

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Then the syntactic rule makes e.g. \( \neg L \mathcal{E} b \) ambiguous in terms of its constructional history.

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- To repeat, the suggestion is:
  1. syntax: if $\varphi(a)$ is grammatical, so is $\varphi(\mathcal{E})$.
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  2. Or do we first negate $Lab$ to get $\neg Lab$, and then "quantify in" to get $\neg L\mathcal{E}b$?

- And the semantic rule generates a corresponding semantic ambiguity.
How not to add quantifiers

Hopeless for logic! –2

- *a* names Romeo, *b* names Juliet, *L* means ① *loves* ②,

\[
\text{semantics: } \varphi(E) \text{ says of everyone what } \varphi(n) \text{ says of what } n \text{ names.}
\]

So, "quantifying into" *Lab* to get *LEb* gives us a proposition which means everyone loves Juliet. Negating that to get \(\neg \text{LEb}\) gives us a proposition which means that not everyone loves Juliet.

- But negating *Lab* to get \(\neg \text{Lab}\) gives us a proposition which says Romeo doesn't love Juliet. And then our semantic rule tells us that \(\neg \text{LEb}\) says of everyone what \(\neg \text{Lab}\) says of what *a* names. So \(\neg \text{LEb}\) says of everyone that he/she doesn't love Juliet – i.e. no-one loves Juliet.

- So our suggested device for quantifying introduces an ambiguity into the language, exactly what we don't want in a formalized language.
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- But negating Lab to get $\neg Lab$ gives us a proposition which says Romeo doesn’t love Juliet. And then our semantic rule tells us that $\neg LEb$ says of everyone what $\neg Lab$ says of what a names. So $\neg LEb$ says of everyone that he/she doesn’t love Juliet – i.e. no-one loves Juliet.
- So our suggested device for quantifying introduces an ambiguity into the language, exactly what we don’t want in a formalized language.
How not to add quantifiers

Similar ambiguities in English

Consider how **scope ambiguities** can arise when a quantifier is similarly combined with negation in English. E.g.

- Conversation 1:
  - 'Everyone seems to be here, so we can begin.'
  - 'No! Hold on. Everyone has not yet arrived. Jack is missing.'

- Conversation 2:
  - 'If anyone arrives early, the surprise will be spoilt.'
  - 'Don't worry! It's still dead quiet outside. Everyone has not yet arrived.'
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- Consider how **scope ambiguities** can arise when a quantifier is similarly combined with negation in English. E.g.

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