Lorem ipsum dolor sit amet, consectetur adipiscing elit. Sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat.
The ingredients of QL – 1

- Constants/names: $a, b, c, l, m, n, \ldots$ to name individual things.

- Predicates:
  - Monadic: $F, G, H, \ldots$ to express properties
  - Dyadic: $L, M, \ldots$ to express two-place relations
  - Triadic: $R, S, \ldots$ to express three-place relations
  - (other polyadic predicates ...)

- Atomic wffs: an $n$-place predicate followed by $n$ names (says the named individual(s) have the property/stand in the relation expressed by the predicate).

Now add the usual propositional connectives.
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- To express generality, we add variables: $x, y, z, \ldots$
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- \(\ldots \) and two quantifier-formers: \(\forall, \exists\)
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- \(\ldots\) and two quantifier-formers: \(\forall, \exists\)
- The basic syntactic rule: if \(\varphi(n)\) is a wff, so are \(\forall v \varphi(v)\) and \(\exists v \varphi(v)\). [Here, \(\varphi(n)\) represents any wff containing one or more occurrences of some name \(n\), and \(\varphi(v)\) is the result of replacing those occurrences of the name \(n\) by the variable \(v\).]
The ingredients of QL – 2

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- Semantics: $\forall v \varphi(v)$ says that everything [in some antecedently specified domain] satisfies the condition expressed by $\varphi$ – i.e. says that what $\varphi(n)$ claims about $n$ is in fact true of everything.
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- $\exists v \varphi(v)$ says that at least one thing [in the relevant domain] satisfies the condition expressed by $\varphi$. 
The point of the quantifier-variable idea is that it marks the scope of a quantifier. [Recall our troubles over e.g. “every experience might be delusory” where the relative scope of the quantifier and the modality is unclear.]
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Compare: $\neg\forall x Fx$ and $\forall x \neg Fx$. 
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Compare: \( \neg \forall x Fx \) and \( \forall x \neg Fx \).

Compare: \( \forall x \exists y Lxy \) and \( \exists y \forall x Lxy \).
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Compare: \( \neg \forall x Fx \) and \( \forall x \neg Fx \).

Compare: \( \forall x \exists y Lxy \) and \( \exists y \forall x Lxy \).

The quantifier/variable way of marking scope was discovered by Frege (though he used a more cumbersome notation).
Frege developed the quantifier/variable idea in his *Begriffsschrift* [“Concept script”] (1879).
QL in action

- QL so far

- QL in action

- Existential commitment
Examples – 1

‘m’ denotes Socrates
‘n’ denotes Plato
‘o’ denotes Aristotle

Domain is people

‘F’ means 1 is wise
‘G’ means 1 is a philosopher
‘K’ means 1 teaches 2
‘L’ means 1 loves 2
‘R’ means 1 prefers 2 to 3

Translate

- $\forall x Lxx$
- $\forall y (Lym \supset Rymn)$
- $\exists x (Kxm \land Lmx)$
- $\forall y ((Fy \land Gy) \supset Lyo)$
- $\forall x (Fx \supset \exists y (Gy \land Lxy))$
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Domain is people

\( \forall x Lxx \Rightarrow \) Everyone loves themself[!]

Peter Smith: Formal Logic, Lecture 16
Examples – 1

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Domain is people

‘F’ means 1 is wise  
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∀xLxx ⇒ Everyone loves themself[!]  
∀y(Lym ⊃ Rymn) ⇒ Anyone who loves Socrates prefers him to Plato.
QL in action

Examples – 1

‘\( m \)’ denotes Socrates
‘\( n \) denotes Plato
‘\( o \)’ denotes Aristotle

Domain is people

‘\( F \)’ means \( 1 \) is wise
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‘\( R \)’ means \( 1 \) prefers \( 2 \) to \( 3 \)

- \( \forall x Lxx \Rightarrow \) Everyone loves themself[!]
- \( \forall y (Lym \supset Rymn) \Rightarrow \) Anyone who loves Socrates prefers him to Plato.
- \( \exists x (Kxm \land Lmx) \Rightarrow \) Socrates loves at least one of his teachers.
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Domain is people

- $\forall x Lxx \Rightarrow$ Everyone loves themself[!]
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Examples – 1

‘\( m \)’ denotes Socrates
‘\( n \) denotes Plato
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Domain is people

‘\( F \)’ means \( \circ \) is wise
‘\( G \)’ means \( \circ \) is a philosopher
‘\( K \)’ means \( \circ \) teaches \( \diamond \)
‘\( L \)’ means \( \circ \) loves \( \diamond \)
‘\( R \)’ means \( \circ \) prefers \( \diamond \) to \( \heartsuit \)

\[\begin{align*}
\forall x Lxx & \Rightarrow \text{Everyone loves themself[!]}
\forall y (Lym \supset Rymn) & \Rightarrow \text{Anyone who loves Socrates prefers him to Plato.}
\exists x (Kxm \land Lmx) & \Rightarrow \text{Socrates loves at least one of his teachers.}
\forall y ((Fy \land Gy) \supset Lyo) & \Rightarrow \text{Every wise philosopher loves Aristotle.}
\forall x (Fx \supset \exists y (Gy \land Lxy)) & \Rightarrow \text{Anyone wise loves some philosopher.}
\end{align*}\]
Examples – 2

| ‘m’ denotes Socrates | ‘F’ means ① is wise |
| ‘n’ denotes Plato    | ‘G’ means ① is a philosopher |
| ‘o’ denotes Aristotle| ‘K’ means ① teaches ② |
|                     | ‘L’ means ① loves ② |
|                     | ‘R’ means ① prefers ② to ③ |

Domain is people

Translate

- If everyone loves Socrates, then Plato loves him.
- If anyone loves Aristotle, then Plato does.
- Aristotle loves anyone.
- Aristotle loves anyone who is wise.
Examples – 2

| ‘m’ denotes Socrates | ‘F’ means 1 is wise |
| ‘n’ denotes Plato    | ‘G’ means 1 is a philosopher |
| ‘o’ denotes Aristotle| ‘K’ means 1 teaches 2 |
|                     | ‘L’ means 1 loves 2 |
| Domain is people    | ‘R’ means 1 prefers 2 to 3 |

Translate

- If everyone loves Socrates, then Plato loves him
  \[ \Rightarrow (\forall x Lx m \supset Lnm) \]
Examples – 2

‘m’ denotes Socrates
‘n denotes Plato
‘o’ denotes Aristotle
Domain is people

‘F’ means ① is wise
‘G’ means ① is a philosopher
‘K’ means ① teaches ②
‘L’ means ① loves ②
‘R’ means ① prefers ② to ③

Translate

▶ If everyone loves Socrates, then Plato loves him
   ⇒ (∀xLxm ⊃ Lnm)

▶ If anyone loves Aristotle, then Plato does
   ⇒ (∃xLxo ⊃ Lnm)
Examples – 2

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
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<tr>
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</tr>
</tbody>
</table>

Translate

- If everyone loves Socrates, then Plato loves him
  \[ \Rightarrow (\forall x L_x m \supset L_{nm}) \]
- If anyone loves Aristotle, then Plato does
  \[ \Rightarrow (\exists x L_x o \supset L_{nm}) \]
- Aristotle loves anyone
  \[ \Rightarrow \forall x L_o x \]
### Examples – 2

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**Domain is people**

### Translate

- If everyone loves Socrates, then Plato loves him
  \[ \Rightarrow (\forall x Lx m \supset Lnm) \]

- If anyone loves Aristotle, then Plato does
  \[ \Rightarrow (\exists x Lxo \supset Lnm) \]

- Aristotle loves anyone
  \[ \Rightarrow \forall x Lo x \]

- Aristotle loves anyone who is wise
  \[ \Rightarrow \forall x (Fx \supset Lo x) \]
Comments

- NB behaviour of ‘anyone’: sometimes gets translated by ∀, sometimes by ∃. (Our formal language avoids the semantic variability of English.)
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  - Aristotle loves anyone $\Rightarrow \forall xLox$
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- NB our translation of the restricted quantifier:
  - Aristotle loves anyone $\Rightarrow \forall x Lox$
  - Aristotle loves anyone who is wise $\Rightarrow \forall x (Fx \supset Lox)$
Comments

- NB behaviour of ‘anyone’: sometimes gets translated by ∀, sometimes by ∃. (Our formal language avoids the semantic variability of English.)
- NB our translation of the restricted quantifier:
  - Aristotle loves anyone ⇒ ∀xLox
  - Aristotle loves anyone who is wise ⇒ ∀x(Fx ⊃ Lox)
- As well as restricting quantifications by relative clauses like ‘who is wise’. English also restricts quantifications by using kind terms, as in ‘All philosophers are wise’, ‘Some students love logic’ (or by both ‘All even numbers which are greater than two are the sum of two primes).
Translating restricted ‘all’ and ‘some’

All philosophers are wise ⇒ ∀x(Gx ⊃ Fx)
Some philosophers are wise ⇒ ∃x(Gx ∧ Fx)

(Translation loses plural implication.)
QL in action

Translating restricted ‘all’ and ‘some’

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Note the shift from ‘⊃’ to ‘∃’ as we move from restricted Universal to restricted Existential. Why is this?
Translating restricted ‘all’ and ‘some’

All philosophers are wise ⇒ ∀x(Gx ⊃ Fx)
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▶ (Translation loses plural implication.)
▶ Note the shift from ‘⊃’ to ‘∃’ as we move from restricted Universal to restricted Existential. Why is this?
▶ Remember the equivalence (P ⊃ ¬Q) ⇔ ¬(P ∧ Q).
Translating restricted ‘all’ and ‘some’

All philosophers are wise $\Rightarrow \forall x (Gx \supset Fx)$
Some philosophers are wise $\Rightarrow \exists x (Gx \land Fx)$

(Translation loses plural implication.)

Note the shift from ‘$\supset$’ to ‘$\exists$’ as we move from restricted Universal to restricted Existential. Why is this?

Remember the equivalence $(P \supset \neg Q) \iff \neg (P \land Q)$.

Some philosophers are wise

$\iff$ It isn’t the case that (all philosophers are unwise)
$\iff$ It isn’t the case that $\forall x (Gx \supset \neg Fx)$
$\iff$ It isn’t the case that $\forall x \neg (Gx \land Fx)$
$\iff$ $\neg \forall x \neg (Gx \land Fx)$
$\iff$ $\exists x (Gx \land Fx)$
Examples – 3

‘m’ denotes Socrates  
‘n’ denotes Plato  
‘o’ denotes Aristotle  
Domain is people

‘F’ means ① is wise  
‘G’ means ① is a philosopher  
‘L’ means ① loves ②

Translate

▶ Every philosopher loves Socrates
▶ Every philosopher loves someone
▶ Socrates loves someone wise
▶ Every philosopher loves someone wise
▶ Every philosopher loves someone who loves Socrates
▶ Every wise philosopher loves someone who loves Socrates
‘m’ denotes Socrates
‘n’ denotes Plato
‘o’ denotes Aristotle
Domain is people

‘F’ means ① is wise
‘G’ means ① is a philosopher
‘L’ means ① loves ②

Translate

► Every philosopher loves Socrates \(\Rightarrow \forall x (Gx \supset Lxm)\)
QL in action

‘m’ denotes Socrates
‘n’ denotes Plato
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Domain is people

‘F’ means ① is wise
‘G’ means ① is a philosopher
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Translate

▶ Every philosopher loves Socrates ⇒ ∀x(Gx ⊃ Lxm)
▶ Every philosopher loves someone ⇒ ∀x(Gx ⊃ ∃yLxy)
QL in action

‘m’ denotes Socrates
‘n’ denotes Plato
‘o’ denotes Aristotle
Domain is people

‘F’ means ① is wise
‘G’ means ① is a philosopher
‘L’ means ① loves ②

Translate

- Every philosopher loves Socrates ⇒ ∀x(Gx ⊃ Lxm)
- Every philosopher loves someone ⇒ ∀x(Gx ⊃ ∃yLxy)
- Socrates loves someone wise ⇒ ∃y(Fy ∧ Lmy)
‘m’ denotes Socrates
‘n’ denotes Plato
‘o’ denotes Aristotle
Domain is people

‘F’ means ① is wise
‘G’ means ① is a philosopher
‘L’ means ① loves ②

Translate

- Every philosopher loves Socrates ⇒ ∀x (Gx ⊃ Lxm)
- Every philosopher loves someone ⇒ ∀x (Gx ⊃ ∃y Lxy)
- Socrates loves someone wise ⇒ ∃y (Fy ∧ Lmy)
- Every wise philosopher loves someone who loves Socrates ⇒ ∀x (Fx ∧ Gx) ⊃ ∃y (Lym ∧ Lxy)
‘m’ denotes Socrates
‘n’ denotes Plato
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Domain is people

‘F’ means ① is wise
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Translate

- Every philosopher loves Socrates ⇒ ∀x(Gx ⊃ Lxm)
- Every philosopher loves someone ⇒ ∀x(Gx ⊃ ∃yLxy)
- Socrates loves someone wise ⇒ ∃y(Fy ∧ Lmy)
- Every philosopher loves someone wise
  ⇒ ∀x(Gx ⊃ ∃y(Fy ∧ Lxy))
- Every philosopher loves someone who loves Socrates
  ⇒ ∀x(Gx ⊃ ∃y(Lym ∧ Lxy))
QL in action

‘m’ denotes Socrates
‘n’ denotes Plato
‘o’ denotes Aristotle
Domain is people

‘F’ means 1 is wise
‘G’ means 1 is a philosopher
‘L’ means 1 loves 2

Translate

▶ Every philosopher loves Socrates ⇒ ∀x (Gx ⊃ Lxm)
▶ Every philosopher loves someone ⇒ ∀x (Gx ⊃ ∃yLxy)
▶ Socrates loves someone wise ⇒ ∃y (Fy ∧ Lmy)
▶ Every philosopher loves someone wise
  ⇒ ∀x (Gx ⊃ ∃y (Fy ∧ Lxy))
▶ Every philosopher loves someone who loves Socrates
  ⇒ ∀x (Gx ⊃ ∃y (Lym ∧ Lxy))
▶ Every wise philosopher loves someone who loves Socrates
  ⇒ ∀x ((Fx ∧ Gx) ⊃ ∃y (Lym ∧ Lxy))
Existential commitment

- QL so far
- QL in action
- Existential commitment
Existential commitment

Simple valuations

Domain is animals

‘$F$’ means $\exists$ is a unicorn

‘$G$’ means $\exists$ is white

What are the truth-values of the following?

- $\forall x Gx$
- $\exists x (Fx \land Gx)$
- $\forall x (Fx \supset Gx)$
Existential commitment

Simple valuations

Domain is animals

‘F’ means ① is a unicorn
‘G’ means ① is white

What are the truth-values of the following?

- \( \forall x Gx \) is False
- \( \exists x (Fx \land Gx) \)
- \( \forall x (Fx \supset Gx) \)
Existential commitment

Simple valuations

Domain is animals

‘$F$’ means $\Box$ is a unicorn
‘$G$’ means $\Box$ is white

What are the truth-values of the following?

- $\forall x Gx$ is False
- $\exists x (Fx \land Gx)$ is False
- $\forall x (Fx \supset Gx)$
Domain is animals

‘F’ means ⊢ is a unicorn
‘G’ means ⊢ is white

What are the truth-values of the following?

- \( \forall x G x \) is False
- \( \exists x (F x \land G x) \) is False
- \( \forall x (F x \supset G x) \) is True
Existential commitment

An invalid inference

Domain is animals

‘F’ means ① is a unicorn

‘G’ means ① is white

Since $\forall x (Fx \supset Gx)$ is true and $\exists x (Fx \land Gx)$ is false on this interpretation, that means $\forall x (Fx \supset Gx)$ doesn’t logically entail $\exists x (Fx \land Gx)$.
Existential commitment

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Since $\forall x (Fx \supset Gx)$ is true and $\exists x (Fx \land Gx)$ is false on this interpretation, that means $\forall x (Fx \supset Gx)$ doesn’t logically entail $\exists x (Fx \land Gx)$.

Traditionally(?) it is claimed that All Fs are Gs entails Some Fs are Gs – i.e. it is said that universal propositions have existential commitment.
Existential commitment

An invalid inference

Domain is animals

‘F’ means  \( \mathfrak{1} \) is a unicorn

‘G’ means  \( \mathfrak{1} \) is white

- Since  \( \forall x (Fx \supset Gx) \) is true and  \( \exists x (Fx \land Gx) \) is false on this interpretation, that means  \( \forall x (Fx \supset Gx) \) doesn’t logically entail  \( \exists x (Fx \land Gx) \).

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- But do they really? Consider
Existential commitment

An invalid inference

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  1. All objects subject to zero net force move in a straight line with constant velocity.
Existential commitment

An invalid inference

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‘F’ means 1 is a unicorn

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Since $\forall x (Fx \supset Gx)$ is true and $\exists x (Fx \land Gx)$ is false on this interpretation, that means $\forall x (Fx \supset Gx)$ doesn’t logically entail $\exists x (Fx \land Gx)$.

Traditionally(?) it is claimed that *All Fs are Gs* entails *Some Fs are Gs* – i.e. it is said that universal propositions have existential commitment.

But do they really? Consider

1. All objects subject to zero net force move in a straight line with constant velocity.
2. All trespassers will be prosecuted.
Existential commitment

An invalid inference

Domain is animals
‘F’ means ① is a unicorn
‘G’ means ① is white

- Since ∀x(Fx ⊃ Gx) is true and ∃x(Fx ∧ Gx) is false on this interpretation, that means ∀x(Fx ⊃ Gx) doesn’t logically entail ∃x(Fx ∧ Gx).
- Traditionally(?) it is claimed that All Fs are Gs entails Some Fs are Gs – i.e. it is said that universal propositions have existential commitment.
- But do they really? Consider
  1. All objects subject to zero net force move in a straight line with constant velocity.
  2. All trespassers will be prosecuted.
- Maybe All Fs is more natural when we think there are some Fs, Any F when we are neutral/doubtful about that.
Existential commitment

A valid inference(?)

- In QL, restricted universal quantifier don’t have existential import: what about unrestricted quantifications?
Existential commitment

A valid inference(?)

- In QL, restricted universal quantifier don’t have existential import: what about unrestricted quantifications?
- It is standardly assumed that, in any application, the domain of interpretation is non-empty.
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- Why this asymmetry of treatment between the cases? Nothing deep.
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- Keeping things single-sorted buys a lot of simplicity at the cost of some departure from the logical form of ordinary arguments. A price mostly worth paying.
And now read on . . .

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