Peter Smith, *Introduction to Formal Logic* (CUP, 2nd edition)

Exercises 35: Q-valuations

Thanks to "spamegg" (https://github.com/spamegg1/) for draft solutions!

(a) Take a QL language, with a couple of proper names, a couple of unary predicates, and a binary predicate, and consider the following q-valuation:

The domain is {Romeo, Juliet, Benedick, Beatrice}
m: Romeo
n: Juliet
F: {Romeo, Benedick}
G: {Juliet, Beatrice}
L: {⟨Romeo, Juliet⟩, ⟨Juliet, Romeo⟩, ⟨Benedick, Beatrice⟩, ⟨Beatrice, Benedick⟩, ⟨Benedick, Benedick⟩}.

Then what are the truth values of the following wffs?

(1) $\exists$xLmx
(2) $\forall$xLxm
(3) ($\exists$xLmx $\to$ Lmn)
(4) $\forall$x(Fx $\to$ $\neg$Gx)
(5) $\forall$x(Gx $\to$ (Lxm $\lor$ $\neg$Lmx))
(6) $\forall$x(Gx $\to$ $\exists$yLxy)
(7) $\exists$x(Fx $\land$ $\forall$y(Gy $\to$ Lxy))

(b) Now take the following q-valuation:

The domain is {4, 7, 8, 11, 12}
m: 7
n: 12
F: the set of even numbers in the domain
G: the set of odd numbers in the domain
L: the set of pairs $\langle m, n\rangle$ where m and n are in the domain and $m < n$.

What are the truth values of the wffs (1) to (7) now?

(c) Take the language QL_3 of Exercises 30(b) whose non-logical vocabulary comprises the name n, the one-place predicates F, G, H, the two-place L, and three-place predicate R. Consider the following q-valuation (the natural one suggested by the given glossary for the language):

The domain is the set of natural numbers (the integers from zero up)
n: one
F: the set of odd numbers
G: the set of even numbers
H: the set of prime numbers
L: the set of pairs $\langle m, n\rangle$ such that $m < n$
R: the set of triples $\langle l, m, n\rangle$ such that $l = m + n$.

Carefully work out the values of the wffs (1), (2), (4) and (5) from Exercises 30(b), i.e. the wffs

(1) $\forall$x$\forall$y$\exists$zRzxy
(2) $\exists$y$\forall$xLxy
(4) $\forall$x(Hx $\to$ $\exists$y(Lxy $\land$ Hy))
(5) $\forall$x$\forall$y((Fx $\land$ Ryxn) $\to$ $\neg$Fy)