Peter Smith, *Introduction to Formal Logic* (CUP, 2nd edition)

Exercises 3: Forms of argument

(a) *Which of the following patterns of inference are deductively reliable, meaning that all their instances are valid? (Here 'F', 'G' and 'H' hold the places for general terms.) If you suspect an inference pattern is unreliable, find an instance which has to be invalid because it has true premisses and a false conclusion.*

(1) *Some F are G; no G is H; so, some F are not H.*

Valid; those Fs which are G are not H.

(2) *Some F are G; some H are F; so, some G are H.*

Invalid: consider 'Some philosophers are men; some women are philosophers; so some men are women'.

(3) *All F are G; some F are H; so, some H are G.*

Valid: those F which are H are also G.

(4) *No F is G; some G are H; so, some H are not F.*

Valid: those G which are H are also not F.

(5) *No F is G; no H is G; so, some F are not H.*

Invalid: consider 'No brother is a woman; no woman is a man; so some brothers are not men.'

(6) *All F are G; no G is H; so, no H is F.*

Valid. Take anything which is H. By the second premiss it is not G. Which by the first premiss implies that it is not F either.

(b) *What of the following patterns of argument? Are these deductively reliable?*

(1) *All F are G; so, nothing that is not G is F.*

Valid!

(2) *All F are G; no G are H; some J are H; so, some J are not F.*

Valid: those J which are H are not G, and hence not F.

(3) *There is an odd number of F, there is an odd number of G; so there is an even number of things which are either F or G.*

Invalid. Maybe some things are both F and G).

(4) *All F are G; so, at least one thing is F and G.*

Disputable. Consider the store notice 'All shop-lifters will be reported to the police' ? couldn't that be true, and yet successfully deter all potential thieves, so no one is ever reported? What about Newton's first law of motion 'Objects subject to no net forces have constant velocity'? Can't this be true of a world where, as it happens, no object is subject to some net force?

(5) *m is F; n is F; so, there are at least two F.*

Invalid. Maybe m and n are the same person!

(6) *Any F is G; no G are H; so, any J is J.*

There is no way the premisses can be true and conclusion false, because there is no way the conclusion can be false. So the argument counts as deductively valid according to our classical definition, even though the premisses are irrelevant to the conclusion. Is this a problem for our definition? We return to this point in Chapter 6!

(c) *Arguments of the kinds illustrated in (a) are so-called (categorical)* syllogisms. *These syllogisms are formed from three propositions, each being of one of the following four forms, which have traditional labels:*

A: All X are Y
E: No X is Y
I: Some X are Y
O: Some X are not Y..

A syllogism then consists of two premisses and a conclusion, each having one of these forms. The two terms in the conclusion occur in separate premisses, and then there is a third or 'middle' term completing the pattern – as in our six schematic examples above. Two questions arising:

(1) *Which valid types of syllogism of this kind have a conclusion of the form A, 'All* S *are* P*'? (Use '*M *for the 'middle' term in a syllogism.)*

There's only one valid type of syllogism with this conclusion, with premisses of the form 'All S are M', 'All M are P' (the order of the premisses is irrelevant).

(2) *Which have a conclusion of the form O, 'Some* S *are not* P*'?*

This is trickier. First there are some unproblematic cases.

1. Some S are M. No M are P. Hence some S are not P.
2. Some S are M. No P are M. Hence some S are not P.
3. Some M are S. No M are P. Hence some S are not P.
4. Some M are S. No P are M. Hence some S are not P.

Since some S are M if and only some M are S, and likewise no M are P if and only if No P are M, these four types of syllogism do come to much the same! Then these too are unproblematic

5. Some S are not M. All P are M. Hence some S are not P.
6. All M are S. Some M are not P. Hence some S are not P.

But what about e.g. this sort of inference?

7. All S are M. No M are P. Hence some S are not P.

Whether you think this is acceptable will depend on whether you think the premiss 'All S are M' implies that there are some S. And as we saw above, that's disputable. Here's another contentious case:

8. All M are S. No M are P. Hence some S are not P.

Again, you verdict on this depends on whether you think 'All M are S' implies that there are some M. If it does, then these M will be S and not P.

(d) *Ancient Stoic logicians concentrated on a different family of arguments. Using 'A' and 'B' to stand in for whole propositions, and 'not-A' to stand in for the denial of what 'A' stands in for, they endorsed the following five basic forms of arguments. Indeed they held them to be so basic as to be 'indemonstrable':*

(1) *If A then B; A; so B.*

(2) *If A then B; not-B; so not-A.*

(3) *not-(A and B); A; so not-B.*

(4) *A or B; A; so not-B.*

(5) *A or B; not-A; so B.*

Which of these principles are acceptable, which – if any – are questionable? Give illustrations to support your verdicts!

The first three forms of argument are uncontentiously correct, as is the last. (For the second, just note that given if A then B, then if A were true, B would have to be true which is ruled out by the second premiss, so A can't be true, i.e. not-A.)

The fourth, however, is more problematic. The Stoics evidently took 'disjunctive propositions of the form *A or B* to mean, by default, *A or B but not both*. But 'or' is often meant inclusively. To my surprise the vote passes in a secret ballot. I say, 'How did that happen? It must be that Jack changed his mind or Jill changed hers.' I don't thereby rule out their both swapping sides.

What about these further forms of argument? Which are correct?

(6) *If A then B; not-A; so not-B.*

Invalid. If it is raining, the grass is wet. It is not raining. It doesn't follow the grass isn't wet – the sprinklers might be on!

(7) *If A then B; B; so A.*

Invalid. If it is raining, the grass is wet. The grass is wet. It doesn't follow that it is raining – again, the sprinklers might be on!

(8) *not-(A and B); so either not-A or not-B.*

Valid. If not both A and B are true, one of them must be false. It isn't the case that Jack *and* Jill are at the party (one of them is baby-sitting). So either Jack isn't at the party or Jill isn't at the party.

(9) *A or B; so not-(not-A and not-B).*

Valid. If one of A and B is true, that rules out their both being false. If either Jack or Jill is at the party (maybe both), then it isn't the case that both Jack isn't there and Jill isn't there!

(10) *not-not-A; so A.*

As we will see, on a standard or 'classical' conception of negation, this is valid. Or at least, it is if we e.g. ignore cases of vagueness (Bill is a borderline case – it isn't the case that he is (definitely) *not* bald; but he isn't (definitely) bald either). More about this principle in due course!

What about these general principles?

(11) *If the inference* A so B *is valid, and the inference* B so C *is valid, then the inference* A so C *is also valid.*

(12) *If the inference* A, B so C *is valid, then so is the inference* A, not-C so not-B.

Both these principles are correct. If C is true in any possible situation where B is true, and B is true in any possible situation where A is true, then C is true in any possible situation where A is true.

Suppose the inference A, B *so* C is valid. Then if we have A but can rule out C, then we are in a position to rule out B.