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Interlude: Formal and informal logic
1 What is logic?

The business of logic is the systematic evaluation of arguments for internal cogency. And the kind of internal cogency that will especially concern us is deductive validity.

But these brief headlines leave everything to be explained. What do we mean here by ‘argument’? What do we mean by evaluation for ‘internal cogency’? What do we mean, more particularly, by ‘deductive validity’? And what kinds of ‘systematic’ evaluation of arguments are possible? This introductory chapter makes a start on answering these questions.

1.1 What is an argument?

By ‘argument’ we mean a chain of reasoning, short or long, in support of a certain conclusion. So we must distinguish arguments from mere disagreements and disputes. The children who shout at each other ‘You did’, ‘I didn’t’, ‘Oh yes, you did’, ‘Oh no, I didn’t’, are certainly disagreeing: but they are not arguing in our sense, i.e. they are not yet giving any reasons in support of one claim or the other.

Reason-giving arguments are the very stuff of all serious enquiry, whether it is philosophy or physics, economics or experimental psychology. But of course, episodes of reasoning equally feature in everyday, street-level, enquiry into what explains our team’s losing streak, the likely winner of next month’s election, or the best place to train as a lawyer. We quite generally want our opinions to be true; which means that we should aim to have good reasons backing up our opinions, so raising the chances of getting things right. That in turn means that we have an interest in being skilful reasoners, using arguments which really do support their conclusions.

1.2 Kinds of evaluation

The business of logic, then, is the evaluation of stretches of reasoning. Let’s take a very simple one-step argument. Suppose you hold

(1) All philosophers are eccentric.

I then introduce you to Jack, who I tell you is a philosopher. So you come to believe

(2) Jack is a philosopher.
Putting these two thoughts together, you infer

(3) Jack is eccentric.

This little bit of reasoning, call it argument A, can now be evaluated along two quite independent dimensions.

• First, we can ask whether A’s premisses (1) and (2) are true: are the ‘inputs’ to your inference correct? (1) is in fact very disputable. Perhaps I have made a mistake, and (2) is false as well.

• Second, we can ask about the quality of the inferential step from the premisses (1) and (2) to the conclusion (3). In this particular case, the inferential step is surely absolutely compelling. We have agreed that it may be open to question whether (1) and (2) are both true. However, if (1) and (2) are granted to be true (granted ‘for the sake of argument’, as we say), then (3) has got to be true too. There’s just no way that (1) and (2) could be true and yet (3) false. To assert that Jack is a philosopher and that all philosophers are eccentric, but go on to deny that Jack is eccentric, would be implicitly to contradict yourself.

Generalizing, it is one thing to consider whether an argument starts from true premisses; it is another thing entirely to consider whether it moves on by reliable inferential steps.

To be sure, we normally want our arguments to pass muster on both counts. We normally want both to start from true premisses and to reason by steps which will take us on to further truths. But it is important to emphasize that these are distinct aims.

The premisses (and conclusions) of arguments can be about all sorts of topics: their truth is usually no business of the logician. If we are arguing about historical matters, then it is the historian who is the expert about the truth of our premisses; if we are arguing about some matter of physics, then the physicist is the one who might know whether our premisses are true; and so on. The specific concern of logic, by contrast, is not the truth of initial premisses but the way we argue from a given starting point. It is in this sense that logic is concerned with the ‘internal cogency’ of our reasoning.

1.3 Deduction vs. induction

The one-step argument A is a splendid bit of reasoning; if the premisses are true, then the conclusion is guaranteed to be true too. Here’s a similar case:

B

(1) Either Jill is in the library or she is in the coffee bar.
(2) Jill isn’t in the library.
So (3) Jill is in the coffee bar.

Who knows whether the premisses are true or not? But we can immediately see that necessarily, if the premisses are true, then the conclusion will be true too: the inferential move is completely watertight. If premisses B(1) and B(2) are both true, then B(3) cannot conceivably fail to be true.
§1.3 Deduction vs. induction

Now consider the following contrasting case. Here you are, sitting in your favourite coffee shop. Unrealistic philosophical scepticism apart, you are thoroughly confident that the cup of coffee you are drinking isn’t going to kill you – for if you weren’t really confident, you wouldn’t be calmly sipping as you read this, would you? What justifies your confidence?

Well, you believe something along the following lines:

C

1. Cups of coffee from GreatBeanz that looked and tasted just fine haven’t killed anyone in the past.
2. This present cup of GreatBeanz coffee looks and tastes just fine.

These premisses sustain your cheerful belief that

3. This present cup of GreatBeanz coffee won’t kill you.

And yes, the inference that moves from the premisses C(1) and C(2) to the conclusion C(3) is, in the circumstances, surely perfectly reasonable: the facts recorded in C(1) and C(2) do give you excellent grounds for believing that C(3) is true. However – and here is the crucial contrast with the earlier ‘Jack’ and ‘Jill’ examples – it isn’t the case that the truth of C(1) and C(2) absolutely guarantees C(3) to be true too.

Perhaps someone has slipped a slow-acting tasteless poison into the coffee, just to make the logical point that facts about how things have generally been in the past don’t guarantee that the trend will continue in the future.

Fortunately for you, C(3) no doubt is true. The tasteless poison is a fantasy. Still, it is a coherent fantasy. It illustrates the point that your grounds C(1) and C(2) for the conclusion that the coffee is safe to drink are strictly speaking quite compatible with the falsity of that conclusion. Someone who agrees to C(1) and C(2) and yet goes on to assert the opposite of C(3) might be saying something highly improbable, but they won’t actually be contradicting themselves. We can make sense of the idea of C(1) and C(2) being true and yet C(3) false.

In summary then, there is a crucial difference between the ‘Jack’ and ‘Jill’ examples on the one hand, and the ‘coffee’ example on the other. In the ‘Jack’ and ‘Jill’ cases, the premisses absolutely guarantee the conclusion. There is no conceivable way that A(1) and A(2) could be true and yet A(3) false: likewise if B(1) and B(2) are true then B(3) has to be true too. Not so with the ‘coffee’ case: it is conceivable that C(1) and C(2) are true while C(3) is false. What’s happened in the past is a very good guide to what will happen next (what else can we rely on?): but reasoning from past to future isn’t absolutely watertight.

We need some terminology to mark this fundamental difference. So we will say an inference step from premisses to a conclusion is deductively valid if it is absolutely watertight, as in arguments A and B. Equivalently, when an inference step is deductively valid, we’ll say that its premisses deductively entail the conclusion.

Constrast the coffee argument C. This argument involves reasoning from past cases to a new case in a way which leaves room for error, however unlikely. This kind of extrapolation from the past to the future, or more generally from some sample cases to further cases, is standardly called inductive. The inference in C might be an inductively strong, but it is not deductively valid.
4 What is logic?

We’ll return to give a sharper definition of deductive validity at the beginning of the next chapter. But for now, we should stress that the deductive/inductive distinction is not the distinction between good and bad reasoning. The ‘coffee’ argument is a perfectly decent one. It involves the sort of generally reliable reasoning to which we rightly trust our lives, day in, day out. The conclusion is highly likely to be true given the premises. It is just that the inference here doesn’t completely guarantee that the conclusion is true, even assuming that the given premises are true.

Non-conclusive, probabilistic, inductive inferences are a very important and decidedly difficult topic. But they are not our topic. We will be concentrating on arguments which aim to use deductively valid inferences, where the premises are supposed to strictly entail the conclusion. And our principal task in this book will be to find various techniques for establishing whether a putatively valid inferential move really is deductively valid.

1.4 Just a few more examples

The ‘Jack’ and ‘Jill’ arguments are examples where the inferential moves are obviously deductively valid. Compare this next mini-argument:

\[ D \]

(1) All Republican voters support capital punishment.
(2) Jo supports capital punishment.
So (3) Jo is a Republican voter.

The inference step here is equally obviously invalid. Even if D(1) and D(2) are true, D(3) doesn’t follow. Maybe lots of people in addition to Republican voters support capital punishment, and Jo is one of them.

How about the following argument?

\[ E \]

(1) Most Irish are Catholics.
(2) Most Catholics oppose abortion.
So (3) At least some Irish oppose abortion.

Leave aside the question of whether the premises are in fact correct (that’s not a matter for logicians: it needs sociological investigation to determine the distribution of religious affiliation among the Irish, and to find out what proportion of Catholics support their church’s official teaching about abortion). What we can ask here – from our armchairs, so to speak – is whether the inferential move is valid: if the premises are true, then must the conclusion be true too?

Well, whatever the facts of the case, it is at least conceivable that the Irish are a tiny minority of the world total of Catholics. And it could also be that nearly all the other (non-Irish) Catholics oppose abortion, and hence most Catholics do, even though none of the Irish oppose abortion. But then E(1) and E(2) would be true, yet E(3) false. So the truth of the premises doesn’t absolutely guarantee the truth of the conclusion. Hence the inference step here can’t be deductively valid.

Here’s another trite argument: is the inference step valid this time?

\[ F \]

(1) Some philosophy students admire all logicians.
(2) No philosophy student admires any rotten lecturer.
So (3) No logician is a rotten lecturer.

With a little thought you should arrive at the right answer here too (we will return to this example in Chapter 3). Still, at the moment, faced with such examples, all you can do is to cast around hopefully, trying to work out somehow or other whether the truth of the premises would guarantee the truth of the conclusion. It would evidently be good to be able to proceed more methodically and to have some general techniques for evaluating arguments for deductive validity.

Indeed, ideally, we would like techniques that work mechanically, that can be applied to settle questions of validity as mechanically as we can settle arithmetical questions by calculation. We will have to wait to see how far this is possible. First, however, we should say a little more about what makes any kind of more systematic approach possible (whether mechanical or not).

1.5 The systematic evaluation of arguments

Here again is our first sample mini-argument with its deductively valid inference step:

\[ \begin{align*}
A & \quad (1) \text{ All philosophers are eccentric.} \\
 & \quad (2) \text{ Jack is a philosopher.} \\
 & \quad \text{So (3) Jack is eccentric.}
\end{align*} \]

Now compare it with the following arguments:

\[ \begin{align*}
A' & \quad (1) \text{ All robins have red breasts.} \\
 & \quad (2) \text{ Tweety is a robin.} \\
 & \quad \text{So (3) Tweety has a red breast.}
\end{align*} \]

\[ \begin{align*}
A'' & \quad (1) \text{ All logicians are cool.} \\
 & \quad (2) \text{ Russell is a logician.} \\
 & \quad \text{So (3) Russell is cool.}
\end{align*} \]

\[ \begin{align*}
A''' & \quad (1) \text{ All post-modernists write bosh.} \\
 & \quad (2) \text{ Derrida is a post-modernist.} \\
 & \quad \text{So (3) Derrida writes bosh.}
\end{align*} \]

We can obviously keep going on and on, churning out arguments to the same pattern, all involving equally valid inference steps.

It is no accident that these arguments share the property of being internally cogent. Comparing these examples makes it clear that the deductive validity of the inference step in the original argument A hasn’t anything especially to do with philosophers or with the notion of being eccentric. Likewise the validity of the inference move in argument A’ hasn’t anything especially to do with robins. Rather, the inferences all rely in the same way on the meaning of ‘all’. There’s a general principle here:
Given a pair of propositions, one saying that all things of a certain kind have a given property, the other saying that a particular individual is of the former kind, we can validly infer the conclusion that the individual in question has the latter property.

However, this wordy version is not the most perspicuous way of presenting the inferential principle at work in the A-family. How much easier it is to say instead something like this:

Any inference step of the following type

\[
\begin{align*}
\text{All } F & \text{ are } G \\
n & \text{ is } F \\
\text{So: } & \text{ } n \text{ is } G
\end{align*}
\]

is deductively valid.

Here the italic letters ‘n’, ‘F’, ‘G’ are being used to exhibit the skeletal pattern of an argument. We can think of ‘n’ as holding the place for a name, while ‘F’ and ‘G’ indicate predicates (using the term loosely for now to mean, roughly, expressions which attribute properties or pick out kinds of thing). It doesn’t matter how we flesh out this skeletal pattern or schema: any sensible way of substituting for the schematic variables (as we’ll call the likes of ‘n’, ‘F’, ‘G’) will evidently yield another argument with a valid inference step.

We said at the outset that logic aims to be a systematic study of validity. We can now see at least how to get some generality into the story. Noting that the same form or pattern of inference can feature in many different particular arguments, we can focus on these patterns and hence deal with whole families of arguments at once. Some forms of inference like the one above are deductively reliable – every instance is a valid inference step. Others aren’t. Consider the pattern of inference

\[
\begin{align*}
\text{Most } F & \text{ are } G \\
\text{Most } G & \text{ are } H \\
\text{So: } & \text{ At least some } F \text{ are } H.
\end{align*}
\]

This is the type of inference involved in the ‘Irish’ argument, and we now know that it isn’t trustworthy.

There will be more, much more, on the idea of schematic patterns of inference in later chapters; but first, a quick summary.

### 1.6 Summary

- We can evaluate a piece of reasoning in two distinct ways. We can ask whether the premises are actually true. And we can ask whether the truth of the premises actually supports the truth of the conclusion. Logic is concerned with the second dimension of evaluation.
§1.6 Summary

• We are setting aside inductive arguments (and other kinds of non-conclusive reasoning). We will be concentrating on arguments involving inferences that purport to be deductively valid. In other words, we are going to be concerned with the study of inferences that aim to strictly guarantee their conclusions, assuming the truth of their premisses.

• Arguments typically come in families whose members share good or bad types of inferential move; by looking at such general patterns of inference we can hope to make logic more systematic.

Exercises 1

By ‘conclusion’ we of course do not mean what concludes a passage of reasoning in the sense of what is stated at the end. We mean what the reasoning aims to establish – and that might in fact be stated at the outset. Likewise, ‘premiss’ does not mean (contrary to what the Concise Oxford Dictionary says!) ‘a previous statement from which another is inferred’. Reasons supporting a certain conclusion, i.e. the inputs to an inference, might well be given after that target conclusion has been stated. And the move from supporting reasons to conclusions can be signalled by inference-markers other than ‘So’.

Indicate the premisses, inference-markers, and conclusions of the following one-step arguments. Which of these arguments do you suppose involve deductively valid reasoning? Why? (We haven’t developed any techniques for you to use yet: just think, improvise, and answer the best you can!)

1. Whoever works hard at logic does well. Accordingly, if Russell works hard at logic, he does well.
2. Most politicians are corrupt. After all, most ordinary people are corrupt – and politicians are ordinary people.
3. It will snow tonight. For the snow clouds show up clearly on the weather satellite, heading this way.
4. Anyone who is well prepared for the exam, even if she doesn’t get an A grade, will at least get a B. Jane is well prepared, so she will get at least a B grade.
5. John is taller than Mary; and Jane is shorter than Mary. So John is taller than Jane.
6. At eleven, Fred is always either in the library or in the coffee bar. And assuming he’s in the coffee bar, he’s drinking an espresso. Fred was not in the library when I looked at eleven. So he was drinking an espresso then.
7. The Democrats will win the election. For the polls put them 20 points ahead, and no party has ever overturned even a lead of 10 points with only a week to go to polling day.
8. Jekyll isn’t the same person as Hyde. The reason is that no murderers are sane – but Hyde is a murderer, and Jekyll is certainly sane.
9. No experienced person is incompetent. Jenkins is always blundering. No competent person is always blundering. Therefore Jenkins is inexperienced.
10. Many politicians take bribes. Most politicians have extra-marital affairs. So many people who take bribes have extra-marital affairs.

11. (Lewis Carroll) Babies cannot manage crocodiles. Because babies are illogical. But illogical persons are despised. And nobody is despised who can manage a crocodile.

12. (Lewis Carroll again) No interesting poems are unpopular among people of real taste. No modern poetry is free from affectation. All your poems are on the subject of soap bubbles. No affected poetry is popular among people of real taste. Only a modern poem would be on the subject of soap bubbles. Therefore all your poems are uninteresting.

13. ‘If we found by chance a watch or other piece of intricate mechanism we should infer that it had been made by someone. But all around us we do find intricate pieces of natural mechanism, and the processes of the universe are seen to move together in complex relations; we should therefore infer that these too have a maker.’

14. ‘I can doubt that the physical world exists. I can even doubt whether my body really exists. I cannot doubt that I myself exist. So I am not my body.’
2 Validity and soundness

Our first chapter introduced the idea of an inferential move being deductively valid. This chapter explores the notion of validity a little more, though still in a preliminary way. We also emphasize the special centrality of deductive reasoning in serious enquiry.

2.1 Validity of inference steps again

We said – as a first shot – that an inference step is deductively valid just if it is completely watertight (given that its premisses are true, then its conclusion is absolutely guaranteed to be true as well).

What exactly does this mean? Let’s try to pin down the intended idea of a valid entailment more carefully. Here is a standard kind of definition:

An inference step is *deductively valid* if and only if there is no possible situation in which its inputs, i.e. its premisses, would be true and its conclusion false.

Here and later, take the plural ‘premisses’ to cover the one-premiss case too.

But this definition still leaves something to be explained: it is only as clear as the notion of a ‘possible situation’. For a start, then, it needs to be stressed that ‘possible’ here is meant in the widest sense – a sense related to those notions of being conceivable, or involving no contradiction, that we met before.

To help fix ideas, consider the following argument:

A Jo jumped out of a twentieth floor window (without parachute, safety net, etc.) and fell unimpeded onto a concrete pavement. So of course she was injured!

And let’s grant that there is no situation that can really obtain in the actual world, with the laws of physics as they are, in which the premiss would be true and the conclusion false. In the world as it is, falling unimpeded onto concrete from twenty floors up will always produce serious (surely fatal) injury. Does that make the inference from A’s premiss to its conclusion deductively valid?

No. Although it isn’t, let’s agree, *physically* possible to jump in those circumstances without being injured, it remains a possible situation in a weaker sense. We can coherently, without self-contradiction, conceive of a situation in which the laws of nature are different or are miraculously suspended, and someone jumping from twenty floors
up will float delicately down like a feather. In this sense, there is (as we might say) a *logically possible* situation in which the A’s premiss would be true and conclusion false, and that’s enough for the inference to be deductively invalid.

This very weak notion of logical possibility – any internally consistent fantasy counts as picturing a possible situation in this sense – goes with a correspondingly strong notion of logical *im*possibility. If something is logically impossible, like a plane figure being circular and square at the same time, then it is ruled out in any situation at all, however outlandish; the very idea is inconsistent, incoherent, logically absurd. For a deductively valid inference, it is impossible in this very strong sense for its premisses to be true and the conclusion false. So another way of putting this is to say that a deductively valid inference is one that is *necessarily truth-preserving*, in the strongest sense of ‘necessarily’.

It will be useful to introduce a further definition:

| Propositions are logically **consistent** just if there is a logically possible situation in which they are all true together. |

Hence, some given propositions are *inconsistent* just if there is no logically possible situation which they are all true together.

In this sense, if there is no possible situation in which a bunch of premisses would be true and a certain conclusion false, then the premisses and the *denial* of the conclusion must together be inconsistent. So another equivalent way of characterizing the notion of validity is this: an inference is deductively valid if the premisses taken together with the denial of the conclusion are inconsistent.

Now, to be honest, we have been going round a rather tight circle of interconnected ideas here. We have defined deductive validity in terms of what is possible; we then explained the relevant notion of possibility in terms of what is coherently conceivable; but coherence in the relevant sense is a matter of a story about the conceived situation not involving self-contradiction, i.e. we can’t validly deduce a contradiction from it. However, we have hopefully said enough at least to give an initial sense of the intended web of ideas, enabling us to get started on our investigations over the next few chapters. Later, we will be able to give very much shaper definitions of validity for various key classes of argument (though these definitions will still be very much in the spirit of our preliminary elucidations).

### 2.2 The invalidity principle

To assess whether an inference is deductively valid, it is typically not enough to consider what is the case in the actual world; we also need to consider alternative possible situations, alternative ways things might have been – or, as some say, alternative possible worlds. Take, for example, the following argument:

| B | (1) No Welshman is a great poet. |
|   | (2) Shakespeare is a Welshman. |
|   | So (3) Shakespeare is not a great poet. |
The propositions here are all false in the actual world. But that doesn’t settle the inferential status of argument A. In fact, the inference step here is a valid one – there is no possible situation in which the premisses would be true and the conclusion false. In other words, any possible world which did make the premisses true (Shakespeare being brought up some miles to the west, and none of the Welsh, now including Shakespeare, going in for verse), would also make the conclusion true.

Here are two more inferences stamped out from the same mould:

C
(1) No father is female.
(2) Bill Clinton is a father.
So (3) Bill Clinton is not female.

D
(1) No one whose middle name is ‘William’ is a Democrat.
(2) George W. Bush’s middle name is ‘William’.
So (3) George W. Bush is not a Democrat.

These two arguments involve valid inferences too – in case C taking us from true premisses to a true conclusion, in case D taking us from false premisses to a true conclusion.

We might wonder about the last case: how can a truth be validly inferred from two falsehoods? But note that someone who believes D(3) on the basis of the false premisses D(1) and D(2) would be arriving at a true belief merely by luck. Their reasons for believing D(3) would have nothing to do with why D(3) in fact happens to be true.

There can of course be mixed cases too, where inferences have some true and some false premisses. So allowing for these, we have in summary:

- A deductively valid inference can have actually true premisses and a true conclusion, (some or all) actually false premisses and a false conclusion, or (some or all) false premisses and a true conclusion.
- A deductively invalid inference can evidently have any combination of actually true and/or false premisses and a true or false conclusion.
- The only combination ruled out by the definition of validity is a valid argument’s having all true premisses and a false conclusion. Validity is about the necessary preservation of truth – so a deductively valid inference cannot take us from actually true premisses to a false conclusion.

That last point is worth emphasizing, and indeed we might usefully dignify an equivalent version with a label:

*The invalidity principle* An inference with actually true premisses and an actually false conclusion must be deductively invalid.

We’ll see in §4.1 how this invalidity principle gets to do important work when combined with the observation that arguments come in families sharing the same kind of inference step.
2.3 Inferences and arguments

So far, we have spoken of inferential steps in arguments as being deductively valid or invalid (from now on, let’s take ‘deductively’ to be understood).

But it is more common simply to describe arguments as being valid or invalid. In this usage, we say that a one-step argument, like the toy examples we’ve been looking at, is valid just if the inference step from its premisses to its conclusion is valid.

This is absolutely standard shorthand. But it can mislead beginners. After all, saying that an argument is ‘valid’ can sound like an all-in endorsement. So let’s emphasize the point: to say that an argument is valid in our sense is only to commend the cogency of the inferential move between premisses and conclusion. A valid argument, i.e. one that is internally cogent, can still have premisses that are quite hopelessly false. It is useful, then, to have a term for arguments that do deserve all-in endorsement – arguments which both start from truths and proceed by deductively cogent inference steps. The usual term is ‘sound’. So:

A (one-step) argument is valid just if the inference step from the premisses to the conclusion is valid.

A (one-step) argument is sound just if it has all true premisses and the inference step from those premisses to the conclusion is valid.

A few authors of older texts use ‘sound’ to mean what we mean by ‘valid’. But everyone agrees there is a key distinction to be made between mere deductive cogency and the all-in virtue of having true premisses and making a cogent inference; there’s just an irritating divergence over how this agreed distinction should be labelled. Our usage of the labels follows the modern convention.

We typically want arguments that aim at deductive success to be sound in our double-barrelled sense. But not always! Sometimes we do want to argue, and argue absolutely compellingly, from premisses we believe aren’t all true (perhaps they are even inconsistent). For example, we might aim to deduce some obviously false consequence of the premisses, in the hope that a disputation will agree that this consequence has to be rejected and so come to acknowledge that the premisses aren’t all true.

Note three immediate consequences of our definition of soundness:

• any sound argument has a true conclusion;
• no pair of sound arguments can have conclusions which are inconsistent with each other;
• no sound argument has inconsistent premisses.

Why so? First, a sound argument starts from actually true premisses and involves a necessarily truth-preserving inference move – so it must end up with an actually true conclusion. Second, since a pair of sound arguments will have a pair of actually true conclusions, that means that the conclusions are true together. If they actually are true together, then of course they can be true together. And if they can be true together then (by definition) the conclusions are consistent with each other. Finally, since a
§2.4 What’s the use of deduction?

bunch of inconsistent premisses cannot all be true together, an argument starting from
those premisses cannot fulfil the first half of the definition of soundness. (If we replace
‘sound’ by simply ‘valid’ in those three claims, we get falsehoods instead. More about
valid arguments with inconsistent premisses in due course.)

What about multi-step arguments where there are intermediate steps between the
initial premisses and the final conclusion (and after all, real-life arguments very often
have more than one inference step)? When should we say that they are deductively
cogent? An obvious first shot is to say that multi-step arguments are valid when each
inference step along the way is valid (and are sound when the initial premisses are true
too). That indeed is part of the story. But we’ll see shortly that things are a bit more
complicated than that, so let’s hang fire for the moment on further developing an account
of what makes for a deductively compelling multi-step argument.

The propositions that occur in arguments as premisses and conclusions are assessed
for truth/falsity. Inference moves between them are assessed for validity/invalidity.
These dimensions of assessment, as we have stressed, are fundamentally different:
so we should keep the distinction carefully marked. So, despite the common misuse
of the terms, from now on never say that a premiss or conclusion or other proposition
is ‘valid’ when you mean it is true, and never say that an argument is ‘true’ when
you mean that it is valid (or sound).

2.4 What’s the use of deduction?

Valid inferences, meaning deductively valid inferences, are not the only acceptable
inferences. Concluding that Jo is injured from the premiss she fell twenty stories onto
concrete is of course perfectly reasonable. Inductive reasoning like this is very often
strong enough to trust your life to: the premisses may render the conclusion a racing
certainty. But such reasoning isn’t deductively valid.

Now consider a more complex kind of inference: take the situation of the detective,
call him Sherlock. Sherlock assembles a series of clues and then solves the crime by an
inference to the best explanation (an old term for this is abductive reasoning). In other
words, the detective arrives at an hypothesis that neatly accommodates all the strange
events and bizarre happenings. In the ideally satisfying detective story, this hypothesis
strikes us (once revealed) as obviously giving the right explanation – why didn’t we think
of it? Why is the bed bolted to the floor so it can’t be moved? Why is there a useless bell
rope hanging by the bed? Why is the top of the rope fixed near a ventilator grille leading
through into the next room? What is that strange music heard at the dead of night? All the
pieces fall into place when Sherlock infers a dastardly plot to kill the sleeping heiress in
her unmovable bed by means of a poisonous snake, trained to descend the rope through
the ventilator grille in response to the snake-charmer’s music. But although this is
indeed an impressive ‘deduction’ in one everyday sense of the term, it is not deductively
valid reasoning in the logician’s sense. We may have a set of clues, and the detective’s
hypothesis $H$ may be the only plausible explanation we can find: but in the typical
case it won’t be a contradiction to suppose that, despite the way all the evidence stacks up, hypothesis $H$ is actually false. That won’t be a logically inconsistent supposition, only perhaps a very unlikely one. Hence, the detective’s plausible ‘deductions’ are not (normally) valid deductions in the logician’s sense.  

This being so, we might begin to wonder: if our inductive reasoning about the future on the basis of the past is not deductive, and if inference to the best explanation is not deductive either, just how interesting is the idea of deductively valid reasoning? To make the question even more worrisome, consider that paradigm of systematic rationality, scientific reasoning. We gather data, and try to find the best theory that fits; rather like the detective, we aim to come up with the best explanation of the actually observed data. But a useful theory goes well beyond merely summarizing the data. In fact, it is precisely because the theory goes beyond what is strictly given in the data that the theory is useful for making novel predictions. Yet since the excess content isn’t guaranteed by the data, the theory cannot be validly deduced from observation statements. So again it might be asked: if deductive inference doesn’t feature even in the construction of scientific theories, why is it particularly important or interesting, except perhaps to mathematicians?

But that’s too quick! It is true that we can’t simply deduce a theory from the data it is based on. However, it doesn’t at all follow that deductive reasoning plays no essential part in scientific reasoning.

Here’s a picture of what goes on in science. Inspired by patterns in the data, or by models of the underlying processes, or by analogies with other phenomena, etc., we conjecture that a certain theory is true. Then we use the conjectured theory (together with assumptions about ‘initial conditions’, etc.) to deduce a range of testable predictions. The first stage, the conjectural stage, may involve flair and imagination, rather than brute logic, as we form our hypotheses e.g. about underlying processes (though our conjectures will usually not be mere guesses – there can be abductive reasoning, in a broad sense, involved in the process of hypothesis-formation.) At the second stage, having made our conjectures, we need to infer testable consequences; and now this does involve deductive logic. For we need to examine what else must be true if the theory is assumed true: we want to know what our hypothesized theory entails. Then, once we have deduced testable predictions, we can seek to check them out. Often our predictions turn out to be false. We have to reject the theory – or else we have to revise it as best we can to accommodate the new data, and then go on to deduce more testable consequences. The process is typically one of repeatedly improving and revising our hypotheses, deducing consequences which we can test, and then refining the hypotheses again in the light of test results.

This so-called hypotheticodeductive model of science (which highlights the role of deductive reasoning from theory to predictions) no doubt needs a lot of development and amplification and refinement. But with science thus conceived, we can see why deduction is absolutely central to the enterprise after all.

And what goes for science, narrowly understood, goes for rational enquiry more generally: deductive reasoning may not be the whole story, but it is an ineliminable core. That’s why logic, which teaches us how to appraise passages of reasoning for deductive validity, matters.
2.5 A word about premisses and conclusions

We should perhaps pause to mention an inconclusive philosophical debate, if only to note that for our purposes we can happily ignore the issue.

So consider: what sort of things are the premisses and conclusions of everyday arguments? Things that can be true or false – propositions, to use a standard term. But what kind of things are propositions? Here are two of the possible views.

According to the first view, in a rough version, propositions, the bearers of truth or falsity, are declarative sentences (sentences like ‘Jack kicks the ball’ as opposed to interrogatives like ‘Does Jack kick the ball?’ or imperatives like ‘Jack, kick the ball!’). But it would be odd to identify premisses and conclusions with sentences in the sense of the actual printed inscriptions, particular physical arrays of blobs of ink on the page. For we surely want to say that e.g. the same premiss can appear in the many different printed copies of this book. So, more plausibly, the view is that propositions are declarative sentence-types, which can have printed instances appearing in many copies, and can have spoken instances too.

Take, however, an ambiguous sentence like the proverbial wartime headline ‘Eighth Army push bottles up Germans’; are the Germans being bottled up by a push, or are the Eighth Army pushing bottles? The sentence can say two different things, one of which may be true and the other false: so, same sentence, different messages conveyed, different possible ingredients of arguments. In other cases, it seems natural to say that different declarative sentences can say the same thing. Don’t ‘Snow is white’ and ‘La neige est blanche’ say the same, express the same truth? Staying within a language, how about ‘Jack loves Jill’ and ‘Jill is loved by Jack’: different sentences, same message. Or consider ‘he loves her’, occurring in varying contexts: now we are back to cases where the same declarative sentence can express different messages, some true, some false.

These points suggest a second view: we should think of propositions, the bearers of truth and falsity, not as declarative sentences but rather as the messages that sentences express. After all, can’t the same argument, with the same premisses and conclusion, be presented using different sentences so long as the messages expressed stay the same? Can’t the same argument appear in this book and in its French translation?

(Terminology varies. Some would talk of ‘thoughts’ as what are expressed – meaning possible thought-contents as opposed to acts of thinking. Some use the word ‘proposition’ not in the non-committal way we do but specifically to refer to such contents.)

Defenders of the first view will respond that we have so far only been told what messages (thought-contents) are not: they are not sentences, but their positive nature is left mysterious, and it is quite unclear what theory to offer. So maybe we should try to rescue the less puzzling view that the bearers of truth are sentences by refining it. More carefully, then, the bearers of truth are fully interpreted declarative sentences (i.e. are disambiguated sentences parsed as having a certain meaning, with context supplying the references of pronouns etc.). And then we could insist that the premisses and conclusions of an argument in this book and the corresponding premisses and conclusions in the French translation are not strictly speaking the same – rather, they are more or less smooth translations of each other, and translatability comes in degrees. But how is this revised proposal to be spelt out?
Here, we will just have to leave the issue hanging in the air. Having flagged up that there are at least two different possible philosophical views here about the status of premises and conclusions in everyday arguments – sentences vs messages expressed – we aren’t going to try to settle the question of which is right. Which may sound rather irresponsible; for how can we leave unresolved such a basic question about everyday arguments as what they are made of? However, for our purposes in this book we rather fortunately won’t need to adjudicate this tricky issue in ‘philosophical logic’. Why? Because – spoiler alert! – our key technique for assessing everyday arguments will involve rendering them first into disciplined formal languages; so we will be concentrating on arguments couched in these formal languages; and such formal arguments can unproblematically be taken to consist of formal sentences in an entirely straightforward way.

2.6 Summary

• Although inductive arguments from past to future, and inferences to the best explanation, are not deductive, the hypothetico-deductive picture shows how there can still be a central role for deductive inference in scientific and other enquiry.

• Our preferred definition: an inference step is valid if and only if there is no possible situation in which its premises are true and the conclusion false.

• The relevant notion of a possible situation is (roughly) the notion of a coherently conceivable situation.

• A one-step argument is valid if and only if its inference step is valid. An argument which is valid and has true premises is said to be sound.

Exercises 2

Which of the following claims are true and which are false? Explain why the true claims hold good, and give counterexamples to the false claims.

1. If someone produces an invalid argument, their premises and conclusion must together be logically inconsistent.

2. If an argument has false premises and a true conclusion, then the truth of the conclusion can’t really be owed to the premises: so the argument cannot really be valid.

3. Any inference with actually true premises and a true conclusion must be truth-preserving and so valid.

4. You can make a valid inference invalid by adding extra premisses.

5. You can make a sound inference unsound by adding extra premisses.

6. You can make an invalid inference valid by adding extra premisses.

7. If some propositions are consistent with each other, then adding a further true proposition can’t make them inconsistent.
8. If a set of propositions is inconsistent, then if we remove some proposition $P$, we can validly infer that $P$ is false from the remaining propositions in the set.
3 Patterns of inference

We noted in the first chapter how arguments come in families which share the same type of inference step which we can represent using an abstract schema. We now continue to explore this key idea.

3.1 More patterns of inference

Consider again the argument:

A
(1) No Welshman is a great poet.
(2) Shakespeare is a Welshman.
So (3) Shakespeare is not a great poet.

This is valid as we noted, and likewise for the parallel ‘Clinton’ and ‘Bush’ arguments from §2.2. The following are valid too:

A′
(1) No three-year old understands quantum mechanics.
(2) Daisy is three years old.
So (3) Daisy does not understand quantum mechanics.

A″
(1) No elephant ever forgets.
(2) Jumbo is an elephant.
So (3) Jumbo never forgets.

We can improvise endless variations on the same theme. And plainly, the inference steps in these arguments aren’t validated by anything especially to do with poets, presidents, three-year-olds or elephants. Rather, they are all valid for the same reason, namely the meaning of ‘no’ and ‘not’ and the way that these logical concepts distribute in the premisses and conclusion (the same way in each argument).

In fact, we have the following:

Any inference step of the following type

\[ \text{No } F \text{ is } G \]
\[ n \text{ is } F \]
\[ \text{So: } n \text{ is not } G \]

is valid.
More patterns of inference 19

As before (§1.5), ‘F’ and ‘G’ here stand in for predicates – expressions that attribute properties like being an elephant or understanding quantum mechanics – and ‘n’ holds the place for a name. Fill in the schematic variables with appropriate expressions, smooth the grammar as necessary, and we’ll get a valid argument.

We can put the underlying principle or rule of inference like this:

Given a pair of propositions, one saying that nothing of some given kind has a certain property, the other saying that a certain individual is of the given kind, we can validly infer the conclusion that the individual in question lacks the property in question.

But it is surely a lot more vivid and much more immediately understandable to use the symbolic shorthand. And let’s be clear, there is nothing essentially mathematical involved in our use of schematic variables like ‘F’ and ‘n’. We are simply exploiting the fact that it is easier to talk about a pattern of inference by displaying the pattern using these variables as place-holders, instead of trying to describe it in cumbersome words.

Here’s another argument which we have also met before (§1.4):

B
(1) Some philosophy students admire all logicians.
(2) No philosophy student admires any rotten lecturer.
So (3) No logician is a rotten lecturer.

Do the premisses here deductively entail the conclusion?

Consider any situation where the premisses are true. Then by B(1) there will be some philosophy students (as it might be, Jack and Jill) who admire all logicians. We know from B(2) that Jack and Jill (since they are philosophy students) don’t admire rotten lecturers. That is to say, people admired by Jack and Jill aren’t rotten lecturers. So in particular, logicians – who are all admired by Jack and Jill – aren’t rotten lecturers. Which establishes B(3), and shows the inference step is indeed valid.

What about this next argument?

B’
(1) Some opera fans buy tickets for every new production of Wagner’s Siegfried.
(2) No opera fan buys tickets for any merely frivolous entertainment.
So (3) No new production of Wagner’s Siegfried is a merely frivolous entertainment.

This too is valid; and a moment’s reflection shows that it essentially involves the same type of valid inference step as before. Again, we can display the general principle in play by using schematic variables:

Any inference step of the following type

Some F are R to every G
No F is R to any H
So: No G is H

is valid.
Here we are using ‘is/are R to’ to stand in for a two-place relational expression – which means, roughly, an expression with (as it were) two slots in it waiting to be filled by subject terms, which denotes a relation. For example the predicate ‘... loves ...’ has two empty places waiting to be filled up by names like ‘Romeo’ and ‘Juliet’ (or by more complex terms like ‘Someone in this room’ and ‘every philosopher’), and it expresses the relation of loving. Likewise, ‘... is married to ...’ also expresses a two-place relation, as does ‘... is taller than ...’ and (to return to our examples) ‘... admires ...’ and ‘... buys tickets for ...’. So, with this notion of a relation in play, we can see that the arguments B and B’ are valid for the same reason, by virtue of sharing the same indicated pattern of inference.

And just try describing the shared pattern of inference without using the symbols: it can be done, to be sure, but at what a cost in perspicuity!

In summary, then, we have now used schemas to display three different patterns of valid inference moves. We initially met the form of inference

\[
\begin{align*}
\text{All } F \text{ are } G \\
n \text{ is } F \\
\text{So: } n \text{ is } G
\end{align*}
\]

And we have just noted the inference patterns

\[
\begin{align*}
\text{No } F \text{ is } G \\
n \text{ is } F \\
\text{So: } n \text{ is not } G
\end{align*}
\]

\[
\begin{align*}
\text{Some } F \text{ are } R \text{ to every } G \\
\text{No } F \text{ is } R \text{ to any } H \\
\text{So: } \text{No } G \text{ is } H
\end{align*}
\]

Any inference following one of these three patterns will be valid: the way that ‘all’, ‘every’ and ‘any’, ‘some’, ‘no’ and ‘not’ distribute between the premisses and conclusion means that inferences filling out these schematic patterns are necessarily truth-preserving.

Here, for the moment, are just three more examples of deductively reliable patterns of inference (which take varying numbers of inputs):

\[
\begin{align*}
\text{No } F \text{ is } G \\
\text{So: } \text{No } G \text{ is } F
\end{align*}
\]

\[
\begin{align*}
\text{All } F \text{ are } H \\
\text{No } G \text{ is } H \\
\text{So: } \text{No } F \text{ is } G
\end{align*}
\]

\[
\begin{align*}
\text{All } F \text{ are either } G \text{ or } H \\
\text{All } G \text{ are } K \\
\text{All } H \text{ are } K \\
\text{So: } \text{All } F \text{ are } K
\end{align*}
\]
§3.2 Four simple points about the use of schemas

(Construct some instances on one-step arguments involving these inference patterns, and convince yourself that your examples are indeed valid, in virtue of the meanings of the logical words ‘all’, ‘no’ and ‘either . . . or . . . ’.)

Now, assessing a particular argument for validity very often goes with seeing a certain principle or rule as being relied on in the argument – a principle we can display using a schematic pattern. And evaluating this shareable pattern for general reliability will then, in one fell swoop, simultaneously give a verdict on a whole range of arguments making the same sort of inferential move. As we said before, this observation underlies the whole business of logic as a systematic study.

3.2 Four simple points about the use of schemas

We have informally used schemas to display the shareable patterns of reasoning in some arguments couched in everyday language. Quite soon, we will be using schemas to display patterns of reasoning in arguments framed in artificial formal languages (languages that logicians love, for reasons which will become clear). Here are four quick and easy initial points that apply to the informal and more formal uses of schemas alike.

(a) Take again the schema

\[
\begin{align*}
\text{All } F & \text{ are } G \\
\text{n is } F & \\
\text{So: } n & \text{ is } G
\end{align*}
\]

Does the following argument count as an instance of the displayed pattern?

C

(1) All men are mortal.
(2) Tweety is a robin.
So (3) Bill Clinton is female.

Of course not! True, when taken in isolation, C(1) has the form \( \text{All } F \text{ are } G \) (i.e. this premiss attributes a certain property to everything of a given kind); taken in isolation C(2) is a simple subject/predicate proposition, an instance of the form \( n \text{ is } F \); and likewise taken in isolation C(3) has the form \( n \text{ is } G \). But when we describe an inference as exhibiting the displayed pattern we are indicating that the same predicate \( F \) is involved in the two premisses. Likewise, the name \( n \) and predicate \( G \) that are in the premisses recur in the conclusion. Indeed, the whole point of the schematic variables here is to represent patterns of recurrence in the premisses and conclusion. So in moving back from the abstract schema to a particular instance of it, we must preserve the patterns of recurrence by being consistent in how we interpret the ‘\( F \)’s and ‘\( G \)’s and ‘\( n \)’s.

(b) What about the following argument? Does this count as an instance of the same displayed inference pattern?

D

(1) All men are men.
(2) Socrates is a man.
So (3) Socrates is a man.
Instead of filling in the schema at random, we have this time at least been consistent, substituting for both occurrences of ‘F’ in the same way, and likewise for both occurrences of ‘G’; however, we happen to have substituted the same predicate each time.

We will allow this as a special case. If ‘F’’s and ‘G’’s get the same substitution, we still get a valid argument – obviously D can’t have true premises and a false conclusion!

Of course, argument D is no use at all as a means for persuading someone of the conclusion – since to make use of the argument you’d already need to accept that very proposition as a premiss. However, there’s a very important distinction to make here: being a sound argument with a reliably truth-preserving inference step is one thing, being persuasive is something else. And once we firmly make that distinction, counting D as a limiting case of a deductively virtuous argument is acceptable. After all, an inference that covers no ground has no chance to go wrong.

(c) And what about the following argument? How does this stand with respect to our displayed inference schema?

\[ \text{(1) Socrates is a man.} \]
\[ \text{(2) All men are mortal.} \]
\[ \text{So (3) Socrates is mortal.} \]

Here the premiss are ‘in the wrong order’. But so what? We said an inference is valid just when there is no possible situation in which all the premisses are true and the conclusion false. Assessments of validity for inference steps can therefore be quite blind to the order in which the premisses are stated. Hence for our purposes – concentrating, as logic does, on assessments of validity – the order in which the premisses for an inference step are presented is quite irrelevant. So we will take E as involving the same type of inference step as before.

(d) Finally, what is the relation between our favourite schema and the following three?

\[ \text{All } H \text{ are } G \]
\[ \text{m is } H \]
\[ \text{So: } m \text{ is } K \]
\[ \text{All } \Phi \text{ are } \Psi \]
\[ \text{α is } \Phi \]
\[ \text{So: } α \text{ is } \Psi \]
\[ \text{All } \imath \text{ are } \circ \]
\[ \text{⋆ is } \imath \]
\[ \text{So: } ⋆ \text{ is } \circ \]

Plainly, all four schemas are just alternative possible ways of representing the same kind of inferential move. The choice of ‘F’’s and ‘G’’s as against ‘H’’s and ‘K’’s, or Greek letters, or some other place-holders for predicates, doesn’t matter at all; similarly for the place-holders for names. What we are trying to reveal is a pattern of recurrence; the ‘F’’s or ‘Φ’’s or whatever are just different ways of marking the places in the common pattern.

3.3 Arguments can instantiate many patterns

Consider the ‘Socrates’ argument E again. This has two premisses and a conclusion distinct from both of them. Therefore it is an instance of the wildly unreliable general pattern of inference

\[ \text{(i) A, B, so C} \]
where ‘$A$’ etc. are being used as schematic variables for whole propositions, so they can
stand in for complete premisses and conclusions. Exposing a bit more structure, $E$ also
instantiates the equally unreliable pattern of inference

$$(ii)\quad \text{All } F \text{ are } G, \text{ } m \text{ is } H, \text{ so } n \text{ is } K$$
since it has a general premiss, and then a simple name/predicate premiss and conclusion.

Next, now filling in enough details about the structure of the inference to bring out
an inferentially relevant pattern of recurrence, the ‘Socrates’ argument also instantiates,
as we said before, the thoroughly reliable pattern

$$(iii)\quad \text{All } F \text{ are } G, \text{ } n \text{ is } F, \text{ so } n \text{ is } G.$$  
But we can go further. Leaving less merely schematic, $E$ is also an instance of e.g. the
perfectly reliable types of inference

$$(iv)\quad \text{All } F \text{ are } G, \text{ } \text{Socrates is } F, \text{ so } \text{Socrates is } G$$  
$$(iv')\quad \text{All men are } G, \text{ } n \text{ is a man, so } n \text{ is } G.$$  
We could even, going to the extreme, take the inference to be the one and only example
of the reliable type

$$(v)\quad \text{All men are mortal, Socrates is a man, so Socrates is mortal}$$
which is (so to speak) a pattern with all the details filled in!

In sum, the argument $E$ exemplifies a number of different patterns of inference, at
different levels of generality. Which all goes to illustrate an important moral:

| There is no such thing as the unique pattern of inference that a given argument can be seen as instantiating. |

A valid inference will typically be an instance of a number of general, shareable, forms
of reliable inference; but the inference will also be an instance of other, ‘too general’,
unreliable forms of inference. Conversely, a generally unreliable pattern of inference
can have specific instances that happen to be valid (being valid for some other reason
than exemplifying the unreliable pattern).

Of course, none of this is to deny that the pattern of inference (iii) has a special place
in the story about $E$. For (iii) is the most general, most widely shareable, reliable pattern
of inference that the argument exemplifies. In other words, the schema $\text{All } F \text{ are } G; \text{ } n \text{ is } F; \text{ so, } n \text{ is } G$ abstracts just enough of the structure of $E$ – but no more than we need – to
enable us to see that the argument is indeed valid.

### 3.4 Topic neutrality and logically valid arguments

Consider next the following three arguments: are they deductively valid?

<table>
<thead>
<tr>
<th>F</th>
<th>(1) Jill is a mother.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>So (2) Jill is female.</td>
</tr>
</tbody>
</table>
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G  (1) Jack has a first cousin.
    So (2) Jack’s parents are not both only children.

H  (1) Tom is taller than Dick.
    (2) Dick is taller than Harry.
    So (3) Tom is taller than Harry.

In any possible situation in which Jill is a mother, she must be a female parent, and hence female. In a situation in which Jack has a first cousin (where a first cousin is a child of one of his aunts or uncles) he must have an aunt or uncle, so his parents cannot both have been only children. Necessarily, if Tom, Dick and Harry are lined up in descending order of height, then the first must be taller than the last. So these arguments all involve deductively valid inference steps, according to our definition of validity in §2.1.

Now, in each of these cases, we can again abstract from some of the details, and find a relevant more general schema all of whose instances are valid. Thus, the first two arguments are not made valid by something peculiar to Jill or Jack; rather, what matters are the concepts of a mother or of a first cousin. So any inference of these kinds is valid:

\[ n \text{ is a mother} \]
\[ \text{So: } n \text{ is female} \]

\[ n \text{ has a first cousin} \]
\[ \text{So: } n \text{'s parents are not both only children} \]

Likewise, any argument with which suitably fills in the following schema is valid:

\[ k \text{ is taller than } l \]
\[ l \text{ is taller than } m \]
\[ \text{So: } k \text{ is taller than } m \]

Here we have further cases of reliable patterns of inference; but in an obvious sense, these are less abstract than the kinds of patterns we previously looked at.

In our earlier examples, we could display relevant inferential principles by using, in addition to the schematic variables, only very general vocabulary like ‘all’ or ‘none’ or ‘any’ or ‘or’ or ‘not’. That vocabulary can usefully be called topic-neutral, for it is vocabulary that we might equally well use in discussing any topic, be it history, pure mathematics, philosophy or whatever. By contrast, our new examples of valid inference patterns involve concepts which belong to more specific areas of interest, concerning familial relations, or physical size.

Still, they are none the worse for that; they are perfectly good kinds of valid inferences by our informal definition in §2.1.

In what follows, however, we are not going to be really concerned with arguments whose validity depends on features of special-interest concepts like motherhood or tallness. Our focus is going to be on the kinds of argument whose load-bearing patterns of inference can be laid out using only certain topic-neutral logical notions, and so can occur when we are reasoning about any subject matter at all.

There is a tradition of saying something like this:
§3.5 A word about forms of argument

An inference step is *logically valid* if and only if it is both valid (i.e. is necessarily truth-preserving) and the pattern of recurrence between premisses and conclusion of topic-neutral logical notions is sufficient to make it valid.

We can exhibit such a pattern of recurrence using a *purely logical schema*, i.e. one involving only variables and logical vocabulary like ‘if’, ‘or’, ‘not’, ‘some’, ‘all’, ‘no’, etc. So, according to the tradition, a logically valid inference is an instance of a purely logical schema all of whose instances are necessarily truth-preserving. In this common usage, the inferences in arguments **F**, **G** and **H**, though deductively valid in our original sense, are not *logically* valid.

There is a snag though. We started making a list of of purely logical notions, but how do we continue it, where do we stop? It just isn’t obvious what is to count as purely topic-neutral logical vocabulary (and hence not obvious what counts as a purely logical schema). This makes the supposed *general* notion of logical validity rather too indeterminate to be very useful as things stand.

However, we *can* go on to define a number of *particular* notions, special cases of logical validity, by explaining ways in which validity can depend on various carefully circumscribed packages of uncontroversially logical topic-neutral vocabulary. These narrower ideas of particular kinds of logical validity are in fact going to loom very large in our explorations in this book. More on this starting in Chapter [ref].

### 3.5 A word about forms of argument

In discussing forms of argument with their patterns of inference, we have so far been relaxed and informal – but hopefully clear enough for our introductory purposes. But when we start probing, we run up against an obvious question. At the end of the last chapter, we raised the following philosophical issue: what are the premisses and conclusions of everyday arguments, sentences or messages (thought-contents, or whatever)?

We can now raise a related question: when we talk about patterns of inference in everyday arguments, do we mean patterns at the surface level of the sentences used to state the argument, or do we mean patterns at the level of messages expressed?

To be honest, we have rather hedged our bets up to now. When we described forms of inference laboriously in words (in §1.5 and §3.1), we talked about what premisses and conclusions *say* – which looks to be at the level of messages expressed. When we described forms of inference using schemas, and talked about systematically substituting names and predicates for schematic letters, that chimes more with thinking of patterns of inference as patterns to be found at sentence level.

Let’s ask: do the following four arguments **I** share the same pattern of inference, so we should think of them all as instances of a single schematic form?

- All dogs have four legs.
- Fido *is* a dog.
- So, Fido *has* four legs.

- Every dog has four legs.
- Fido *is* a dog.
- So, Fido *has* four legs.
Any dog has four legs.  
Each dog has four legs.  
Fido is a dog.  
Fido is a dog.  
So, Fido has four legs.  
So, Fido has four legs.

Thinking at the level of sentences, we might claim that these arguments strictly speaking exemplify different forms of inference, because they involve the distinct words ‘all’, ‘every’, ‘any’ and ‘each’. However it is very tempting to respond that the differences here are only superficial and the underlying logical structure is really the same. The respective first premisses of the arguments I, which say that all things/everything/anything/each thing of a certain general kind have/has some property, are just stylistically different ways of expressing the very same thought-content; and then the rest of arguments are the same. So it is tempting to think that while the various I-versions may differ in surface sentential form, they in some sense share the same underlying logical form – thinking at the level of the messages expressed by the various sentences, the inferences are all instances of a single logical form, which we can represent using a single schema, e.g. by recycling the familiar All F are G, n is F, so n is G.

So which is it to be when thinking about the forms of everyday arguments? Sentential forms or forms of messages expressed? The second line is tempting, but do we really understand what it comes to? Fortunately, for our current rough-and-ready purposes, we just don’t need to pursue the issue: introductory uses of the notion of patterns of inference aren’t sensitively dependent on such decisions. And – another spoiler alert! – when we start to consider arguments regimented into formal languages, inferential forms can be understood quite unproblematically as patterns at the sentential level.

3.6 Summary

• Forms or patterns of inference can be conveniently represented using schematic variables, whose use is governed by the obvious convention that a given variable represents the same name or predicate whenever it appears within an argument schema.

• There is strictly no such thing as the unique form exemplified by an argument. Valid arguments will typically be instances of reliable forms; but they can also be instances of other, more general, unreliable forms.

• Our main concern henceforth will be with rather general, ‘topic-neutral’, patterns of inference, which are deductively valid in virtue of the way that various key logical concepts (like ‘all’, ‘no’ and ‘most’) are distributed in a patterned way between the premisses and conclusion. Such arguments are said to be logically valid (though it is not easy to see how to sharply demarcate this class of cases).

Exercises 3

(a) Which of the following types of inference step are valid (i.e. are such that all their instances are valid)? If you suspect an inference-type is invalid, find an instance which
§3.6 Summary

obviously fails because it has plainly true premisses and a false conclusion.

1. Some F are G; no G is H; so, some F are not H.
2. Some F are G; some F are H; so, some G are H.
3. All F are G; some F are H; so, some H are G.
4. No F is G; some G are H; so, some H are not F.
5. No F is G; no G is H; so, some F are not H.

By the way, arguments of these kinds, with two premisses and a conclusion, with each proposition being of one of the kinds ‘All ... are ...’, ‘No ... is ...’ or ‘Some ... are/are not ...’, and three predicates filling in the gaps (the two predicates in the conclusion occurring in separate premisses), are the traditional syllogisms first discussed by Aristotle.

(b) What of the following patterns of argument? Are these valid?

1. All F are G; so, nothing that is not G is F.
2. All F are G; no G are H; some J are H; so, some J are not F.
3. There is an odd number of F, there is an odd number of G; so there is an even number of things which are either F or G.
4. All F are G; so, at least one thing is F and G.
5. m is F; n is F; so, there are at least two F.
6. All F are G; no G are H; so, all H are H.

(c) Consider the argument ‘Dogs have four legs. Fido is a dog. So, Fido has four legs.’ Is it of the same inferential type as the F-family of arguments in §3.5?

(d) Consider the argument ‘Everyone is female. So, any siblings are sisters.’ Is this valid? Does it exemplify any general but still reliable form of inference? What, if anything, does this tell us about the slogan ‘Valid arguments are valid in virtue of their form’?
4 The counterexample method

In this chapter we exploit the fact that arguments typically come in families which share an inferential structure (as outlined in Chapter 3 – we can for our purposes in this chapter continue with a rough-and-ready understanding of this fact). We put this together with the trivial invalidity principle (which we met Chapter 2) to give a method for showing that various invalid arguments are indeed invalid.

4.1 The method illustrated, in slow motion

(a) Recall the following argument from §1.4:

A

1. Most Irish are Catholics.
2. Most Catholics oppose abortion.
So (3) At least some Irish oppose abortion.

We quickly persuaded ourselves that the inference is invalid by imagining a world in which the premisses would be true and conclusion false. But let’s now proceed more slowly, keeping ourselves grounded in the actual world.

If the inference in the ‘Irish’ argument is to be reliable, what could make it so? It can’t be anything other than the meaning of ‘most’ and ‘at least some’, and the way that these are distributed in the premisses and conclusion. In other needs, we need the pattern of inference

Most F are G
Most G are H
So: At least some F are H.

to be generally reliable. Is it? Are inferences of this shape always valid?

Well, consider this instance:

A’

1. Most chess grandmasters are men.
2. Most men are no good at chess.
So (3) At least some chess grandmasters are no good at chess.

The premisses of this argument are actually true. Chess is still (at least at the upper levels of play) a predominantly male activity, and is one that few people are any good at. But the conclusion is crazily false. So here we have true premisses and a plainly false conclusion. Hence by the invalidity principle (§2.2), argument A’ is invalid.
That shows the displayed inference pattern is not generally reliable. So that settles it: the original ‘Irish’ argument A isn’t valid.

(b) We need to be clear about what is going on here.

We can’t just say ‘The displayed inference pattern is unreliable, so any instance – such as A – is invalid’. That’s too simple. As we stressed in §3.3, a generally unreliable pattern of inference can have special instances that happen to be valid for some other reason.

Still, when an unreliable pattern does happen to have a valid instance, there must be something else about the instance (i.e. something other than exhibiting the unreliable pattern) which makes it valid. So, faced with an instance of an unreliable pattern, if there are no other redeeming features of the argument, then we can conclude that the argument isn’t valid.

That’s the case with A. We have shown that the displayed inference pattern is unreliable. There’s nothing else that the ‘Irish’ argument can be depending on for its validity. In other words there’s nothing to make it any better than the invalid argument A’. They really are parallel cases. So, as we said, A is invalid.

(c) Here’s another quick example. Consider this argument:

\begin{enumerate}
\item Some philosophers are great logicians.
\item Some philosophers are German.
\item Some great logicians are German.
\end{enumerate}

This has true premisses and a true conclusion; so we can’t yet apply the invalidity principle. So is it valid? The argument is evidently relying on the following pattern of inference (what else?):

\begin{align*}
\text{Some } F &\text{ are } G \\
\text{Some } F &\text{ are } H \\
\text{So: } &\text{ Some } G \text{ are } H.
\end{align*}

If this pattern is unreliable, then B is not a valid argument. But take e.g. this instance:

\begin{enumerate}
\item Some humans are adults.
\item Some humans are babies.
\item Some adults are babies.
\end{enumerate}

True premisses, false conclusion: so by the invalidity principle, the inference in B’ is invalid. Hence the last displayed inference pattern is reliable, and any argument like B that is relying just on this kind of inference move is invalid.

4.2 ‘But you might as well argue …’

Here is a summary description of the two-stage method we have just used to settle the status of the ‘Irish’ and ‘German’ arguments.
The counterexample method

Stage 1  Locate some form of inference that a given argument is relying on (i.e. a pattern of inference that needs to be reliable if the inference in the given argument is indeed to be necessarily truth-preserving).

Stage 2  Show that this form of inference is not a reliable one by finding a counterexample, i.e. find another argument with this pattern of inference which is uncontroversially invalid, e.g. because it has actually true premises and a false conclusion so we can apply the invalidity principle.

And there is absolutely nothing mysterious or difficult or novel about this counterexample method; we use it all the time in the everyday evaluation of arguments, when we use the ‘But you might as well argue . . . ’ gambit. Someone says that at least some Irish must be against abortion because most catholics are, and most Irish are catholics. You reply: ‘But you might as well argue that some chess masters are hopeless at chess, because most men are and most chess masters are men.’ Similarly, some gossip says that Mrs Jones must be an alcoholic, because she has been seen going to the Cheapo Booze Emporium and everyone knows that that is where all the local alcoholics hang out. You reply, ‘But you might as well argue that Hillary Clinton is a Republican Senator, because she’s been seen going into the Senate, and everyone knows that that’s where all the Republican Senators hang out’.

Note that in our general statement of the method, we do not require a counterexample to be generated by the invalidity principle, i.e. to have actually true premises and an actually false conclusion. Telling counterexamples don’t have to be constructed from ‘real life’ situations. A merely imaginable but uncontroversially coherent counterexample will do just as well. Why? Because that’s still enough to show that the inferential pattern in question is unreliable: it doesn’t necessarily preserve truth in all possible situations. However, using ‘real life’ counterexamples does often have the great advantage that you don’t get into any disputes about what is coherently imaginable.

4.3 The counterexample method applied again

(a) Consider the following quotation (in fact, the opening words of Aristotle’s Nicomachean Ethics):

Every art and every enquiry, and similarly every action and pursuit, is thought to aim at some good; and for this reason the good has rightly been declared to be that at which all things aim.

At first sight, there is an argument here with the initial premiss we can sum up as

\[ C \quad (1) \text{ Every practice aims at some good.} \]

And then a conclusion is drawn (note the inference marker ‘for this reason’)

\[ S \quad (2) \text{ There is some good (‘the good’) at which all practices aim.} \]
But this argument has a very embarrassing similarity to the following one:

\[ C' \]

(1) Every assassin’s bullet is aimed at some victim.

So (2) There is some victim at whom every assassin’s bullet is aimed.

And that is obviously bogus. Every assassin’s bullet has its target (let’s suppose), without there being a single target shared by them all. Likewise, every practice may aim at some valuable end or other without there being a single good which encompasses them all.

Drawing out some logical structure, Aristotle’s argument appears to rely on the inference form

\[
\text{Every } F \text{ is } R \text{ to some } G \\
\text{So: There is some } G \text{ such that every } F \text{ is } R \text{ to it.}
\]

and then the ‘assassin’ argument is a counterexample (an inference of the same pattern, which is evidently invalid).

(b) Did Aristotle really use the terrible argument \( C \)? Or can he wriggle off the hook here?

We have ripped the quotation from Aristotle out of all context, so there is room for scholars to debate what the intended argument really is. We can’t enter into such debates here. Still, anyone who wants to defend Aristotle must at least meet the challenge of saying why the supposed counterexample fails to make the case, and must tell us what principle of inference really is being used in the intended argument here. If Aristotle isn’t relying on the fallacious inference-type as displayed, then what \( is \) he up to?

The same goes for other challenges by the counterexample method. Once we’ve agreed a statement of some argument, if you want to resist its demolition by a counterexample to the reliability of an inference step it uses, then your only hope is to show that the inference step in the argument has been mis-identified.

(c) Expressions like ‘every’ and ‘some’ are standardly termed quantifiers by logicians (see §[ref]): what we’ve just noted is that in general we can’t shift around the order of quantifiers in a proposition. It is perhaps worth noting that a number of supposed arguments for the existence of God commit the same quantifier-shift fallacy. Even if you grant that

\[ D \]

(1) Every causal chain has an uncaused first link,

it does not follow that

(2) There is something (God) that is the uncaused first link of every causal chain.

Again, even if you suppose that

\[ E \]

(1) Every ecological system has an intelligent designer,

(though that’s a premiss which Darwin exploded), it still does not follow that
There is some intelligent designer (God) which designed every ecological system.

There may, consistently with the premiss, be no one Master Designer of all the ecosystems; perhaps each system was produced by a competing designer.

4.4 Summary

- The counterexample method to show invalidity is applicable to a target argument when (1) we can find a pattern of inference that the argument is depending on, and
- (2) we can show that the pattern is not a reliable one by finding a counterexample to its reliability, i.e. find an argument exemplifying this pattern which has (or evidently could have) true premisses and a false conclusion.

- This method is familiar in the everyday evaluation of arguments as the ‘But you might as well argue . . .’ gambit.

Exercises 4

Some of the following arguments are invalid. Which? Why? In particular, use the counterexample method to prove invalid the ones that are.

1. Many ordinary people are corrupt, and politicians are ordinary people. So, some politicians are corrupt.
3. Everyone who admires Bach loves the Goldberg Variations; some who admire Chopin do not love the Goldberg Variations; so some admirers of Chopin do not admire Bach.
4. Some nerds are trainspotters. Some nerds wear parkas. So some trainspotters wear parkas.
5. Anyone who is good at logic is good at assessing philosophical arguments. Anyone who is mathematically competent is good at logic. Anyone who is good at assessing philosophical arguments admires Bertrand Russell. Hence no-one who admires Bertrand Russell lacks mathematical competence.
6. (Lewis Carroll) Everyone who is not a lunatic can do logic. No lunatics are fit to serve on a jury. None of your cousins can do logic. Therefore none of your cousins is fit to serve on a jury.
7. Most logicians are philosophers; few philosophers are unwise; so at least some logicians are wise.
8. Few Sicilians approve of abortion; many atheists approve of abortion; so few atheists are Sicilians.
9. All logicians are rational; no existentialists are logicians; so if Sartre is an existentialist, he isn’t rational.

10. If Sartre is an existentialist, he isn’t rational; so if he is irrational, he is an existentialist.
5 Proofs

The counterexample method gives us a way of demonstrating the invalidity of some inferences. This chapter introduces a way of showing the validity of various other inferences which aren’t obviously valid at first sight.

5.1 Three sample proofs

(a) Suppose we are wondering whether a certain argument is valid. If we can find another argument which relies on the same form of inference and which is plainly invalid, then that settles it – the original argument is not deductively valid either. But what if we have tried to find such a counterexample and failed. What does that show? Perhaps nothing very much. Maybe there is no counterexample to be found, because the original argument is indeed valid. But it could be that the argument is invalid and we have just not spotted a suitable counterexample to prove the point.

What to do? In particular, if failure to find a counterexample doesn’t settle the matter, how can we demonstrate that a challenged inference really is a valid one?

Take this charmingly daft example from Lewis Carroll:

A Babies are illogical; nobody is despised who can manage a crocodile; illogical persons are despised; so babies cannot manage a crocodile.

This three-premiss, one-step, argument is in fact valid. How can we demonstrate its validity? Well, consider now the following two-step argument:

A’ (1) Babies are illogical. (premiss)
     (2) Nobody is despised who can manage a crocodile. (premiss)
     (3) Illogical persons are despised. (premiss)
     (4) Babies are despised. (from 1, 3)
     (5) Babies cannot manage a crocodile. (from 2, 4)

Here, we have inserted an extra step between the original premisses and the target conclusion (and we can omit writing ‘So’ before lines (4) and (5), as the commentary on the right suffices to indicate that they are the results of inferences).

The inference from the original premisses to the interim conclusion (4), and then the further inference from the (one of) original premisses plus that interim claim (4) to the final conclusion (5), are both evidently valid. So we can indeed get from the initial premisses to the final conclusion by a necessarily truth-preserving route. Which shows that argument A is indeed valid.
Here’s a second quick example, equally daft (due to Richard Jeffrey):

\[ \text{B} \]
Everyone loves a lover; Romeo loves Juliet; so everyone loves Juliet.

Take the first premiss to mean ‘everyone loves anyone who is a lover’ (where a lover is, of course, a person who loves someone). Then this inference too is, slightly surprisingly, deductively valid! Here is a multi-step proof:

\[ \text{B'} \]
(1) Everyone loves a lover (premiss)
(2) Romeo loves Juliet. (premiss)
(3) Romeo is a lover. (from 2)
(4) Everyone loves Romeo. (from 1, 3)
(5) Juliet loves Romeo. (from 4)
(6) Juliet is a lover. (from 5)
(7) Everyone loves Juliet. (from 1, 6)

We have again indicated on the right the ‘provenance’ of each new statement as the argument unfolds. And by inspection we can see that each small inference step is valid, is necessarily truth-preserving. As the argument grows, then, we are adding new propositions which must be true if the original premisses are true. These new propositions can then serve in turn as inputs to further valid inferences. Everything is chained together so that, if the original premisses are true, each added proposition must be true too, and therefore the final conclusion in particular must be true. Hence the original inference \( \text{B} \) that jumps straight across the intermediate stages must also be valid.

This pair of simple examples illustrates, then, a standard technique for establishing the validity of a perhaps unobviously valid inference:

We can demonstrate that an inferential leap from given premisses to a certain conclusion is valid by breaking down the big leap into smaller steps, each one of which is clearly deductively cogent.

There’s a familiar shorthand for a chain of inferences put together in such a way to be deductively cogent, namely a proof. So the idea is that we can establish that an unobvious inference step is valid by providing a more detailed proof filling in between premisses and conclusion, where each step in the proof is evidently in good order. (Careful: a proof in our sense doesn’t necessarily establish the truth of its conclusion outright, only the truth of the conclusion assuming the truth of the premisses.)

More or less carefully articulated proofs, defending inferential leaps by reducing them to a sequence of smaller steps, occur naturally in everyday contexts, not just in mathematics or philosophy. As we will see later, some systems of formal logic – so-called ‘natural deduction’ systems – aim to regiment such everyday proofs into tidy deductive structures governed by strict rules. Our examples in this chapter, however, fall somewhere between loose everyday argumentation and the rigorous presentations of natural deduction systems. For want of a better label, these are semi-formal proofs.

In our two examples so far, we have indicated at each new step which earlier statements the inference depended on. But we can do even better by also indicating
what type of inference move is being invoked at each stage. Take another argument from Lewis Carroll who is an inexhaustible source of silly but instructive examples:

\[ \text{C} \]

Anyone who understands human nature is clever; every true poet can stir the heart; Shakespeare wrote Hamlet; no one who does not understand human nature can stir the heart; no one other than a true poet wrote Hamlet; so Shakespeare is clever.

The big inferential leap from those five premisses to the conclusion is in fact valid. You might like to give a proof that shows that it is indeed valid, before reading on!

We can go by a number of routes here. But grant that the following two labelled patterns of inference are deductively reliable ones (which of course they are):

\[ U: \text{Any/every F is G; n is F; so n is G} \]
\[ V: \text{No one who isn’t F is G; so any G is F} \]

Then we can argue as follows, using these two types of valid inference:

\[ C' \]

(1) Anyone who understands human nature is clever. (premiss)
(2) Every true poet can stir the heart. (premiss)
(3) Shakespeare wrote Hamlet. (premiss)
(4) No one who does not understand human nature can stir the heart. (premiss)
(5) No one other than a true poet wrote Hamlet. (premiss)
(6) Anyone who wrote Hamlet is a true poet. (from 5, by V)
(7) Shakespeare is a true poet. (from 3, 6, by U)
(8) Shakespeare can stir the heart. (from 2, 7, by U)
(9) Anyone who can stir the heart understands human nature. (from 4, by V)
(10) Shakespeare understands human nature. (from 8, 9, by U)
(11) Shakespeare is clever. (from 1, 10, by U)

That’s all very, very laborious, to be sure: but now we have the provenance of every move fully documented. Each of the inference steps is an instance of a clearly truth-preserving pattern, and together they get us from the initial premisses of the argument to its final conclusion; so if the initial premisses are true, so must be the conclusion. We have a proof here which shows that the original inference C must be valid.

(d) In a single-step argument, the inputs to the only inference step are of course also the premisses of the argument. That’s trivial! In the case of multi-step arguments it is only a little more complicated: still, just to be really clear, let’s spell things out.

Each inference-step in our multi-step examples has its own immediate inputs or premisses and delivers its own immediate conclusion: the inputs needed for the conclusion are indicated in the commentary on the right. But these inputs need not be among the initial propositions marked ‘premiss’ (those are the original premisses of the argument, i.e. propositions that have no further backing at least within this argument). For, crucially, the conclusion of an inference step along the way is also available to be the input to a later inference in the argument.
5.2 Glimpsing an ideal

We have now glimpsed an ideally explicit way of setting out a multi-step semi-formal proof – in $C'$ every premiss we are going to need is given at the outset and marked as such, and each further proposition then comes with a certificate which tells us what earlier proposition(s) it is inferred from and what principle of inference is being invoked.

Needless to say, everyday arguments (and even not-so-everyday proofs in mathematics books) usually fall well short of meeting these ideal standards of explicitness! They don’t come ready-chunked into numbered statements, with each new inference step bearing a supposed certificate of excellence. However, faced with a puzzling multi-step argument, massaging it into something nearer this ideal fully documented shape should help us to assess the argument. On the one hand, an explicitly documented proof gives a critic a particularly clear target to fire at. If someone wants to reject the conclusion, then they will have to rebut one of the premisses or come up with a counterexample to the general reliability of one of the forms of inference that is explicitly called on. On the other hand, looking on the bright side, if the premisses are agreed and if the inference moves are uncontentiously valid, then the proof will indeed establish the conclusion beyond further dispute.

The exercise of regimenting an argument into a more ideally explicit form will also reveal redundancies (premisses or inferential detours that are not really needed to get to the final conclusion) and expose where there are suppressed premisses (i.e. premisses that aren’t stated but which are really needed if we are to get a well-constructed proof).

Arguments with suppressed premisses are traditionally called enthymemes. Here’s a toy example:

**D** The constants of nature have to take values in an extremely narrow range (have to be ‘fine-tuned’) to permit the evolution of intelligent life. So the universe was intelligently designed.

This as it stands is plainly gappy: there are unspoken assumptions lurking in the background. But we can make it into a deductively valid inference, if we make explicit the currently suppressed premisses. One premiss will be uncontroversial, just noting that intelligent life has evolved. The other premiss will be much more problematic: roughly, intelligent design is needed in order for the universe to be fine-tuned enough. But only with some such additions can we get a cogent argument. (Exercise: construct a fully annotated proof with valid inference steps using a tidied versions of the old and new premisses.)

5.3 Indirect arguments

(a) Let’s return to another toy example, the crocodile argument $A'$. Suppose someone is puzzled by the final step where we inferred that babies cannot manage a crocodile from the original premiss that nobody is despised who can manage a crocodile together with the interim conclusion that babies are despised. How could we convince them that this step really is valid?

We might try amplifying that final step, so the expanded proof now runs as follows:
A

(1) Babies are illogical. (premiss)
(2) Nobody is despised who can manage a crocodile. (premiss)
(3) Illogical persons are despised. (premiss)
(4) Babies are despised. (from 1, 3)

Suppose temporarily, for the sake of argument,
(5) Babies can manage a crocodile. (supposition)
(6) Babies are not despised. (from 2, 5)
(7) Contradiction! (from 4, 6)

Our supposition leads to absurdity, hence
(8) Babies cannot manage a crocodile. (RAA)

What is going on here at the end of this proof? We want to establish (8). But instead of aiming directly for the conclusion, we branch off by temporarily supposing the exact opposite is true, i.e. we suppose (5). However, this supposition immediately leads to something that contradicts an earlier claim. Hence the supposition (5) is, as they say, ‘reduced to absurdity’ (given the original premisses): therefore its opposite (8) must therefore be true after all. The label ‘RAA’ indicates that the argument terminates with a *reductio ad absurdum* inference.

Another quick example. Take the argument:

E

(1) No girl loves any unreconstructed sexist. (premiss)
(2) Caroline is a girl who loves whoever loves her. (premiss)
(3) Henry loves Caroline. (premiss)
(4) Caroline is a girl who loves Henry. (from 2, 3)
(5) Caroline is a girl. (from 4)
(6) Caroline loves Henry. (from 4)

Suppose temporarily, for the sake of argument,
(7) Henry is a unreconstructed sexist. (supposition)
(8) No girl loves Henry. (from 1, 7)
(9) Caroline does not love Henry. (from 5, 8)
(10) Contradiction! (from 6, 9)

Our supposition leads to absurdity, hence
(11) Henry is not a unreconstructed sexist. (RAA)

And we are done.

(b) Let’s spell out the RAA principle invoked in these two proofs.

We will say that an (explicit) *contradiction* is a pair of propositions, C together with its exact opposite not-C (or what comes to the same thing, a single proposition of the type C and not-C). Then we have the following, where the ‘A₂’ and ‘S’ are schematic variables for whole propositions:
5.4 Deductively cogent multi-step arguments

(RAA) *Reductio ad absurdum*. If $A_1, A_2, \ldots, A_n$ plus the temporary supposition $S$ logically entail a contradiction then, then from $A_1, A_2, \ldots, A_n$ by themselves we can validly infer not-$S$.

Why does this principle hold? Suppose that a bunch of premisses $A_1, A_2, \ldots, A_n, S$ do entail a contradiction. Then there can be no situation in which these premisses are all true together (or else this would be a situation in which the entailed contradiction would also have to be true, and it can’t be). So any situation in which the other premisses are all true is one in which $S$ has to be false. Hence those other premisses logically entail not-$S$.

(c) *Reductio* arguments are often called *indirect* arguments. We don’t go straight from premisses to conclusion, but take a side-step via some additional supposition which we temporarily add for the sake of argument and then eventually ‘discharge’ (i.e. drop again). We will soon be meeting other kinds of indirect argument.

For now we just make one quick comment. When we are giving proofs where temporary suppositions are made en route, we evidently need to have some way of clearly indicating which steps of the argument are being made while that temporary supposition is in play. We have used above the common device of indenting the argument to the right while the supposition is in play; and then we go back left when the supposition is discharged and is finished with. This gives us a neat visual display of the argument’s structure. Later, we’ll be putting this device to use in formal proofs, and will then explain it more carefully.

### 5.4 Deductively cogent multi-step arguments

(a) We initially characterized deductive validity as being, in the first place, a property of individual inference steps. We then fell in with the habit of calling a one-step argument valid if it involves a valid inference step from initial premisses to final conclusion. That qualification ‘one-step’ is important.

For consider the following mini-argument:

**F**

1. All philosophers are logicians. (premiss)
2. All logicians are philosophers. (from 1!?)

This inference is horribly fallacious. It just doesn’t follow from the premiss that all philosophers are logicians that only philosophers are logicians. (You might as well argue ‘All women are human beings, hence all human beings are women’.)

Here’s another really bad inference:

**G**

1. All existentialists are philosophers. (premiss)
2. All logicians are philosophers. (premiss)
3. All existentialists are logicians. (from 1, 2!?)

It plainly doesn’t follow from the claims that the existentialists and logicians are both among the philosophers that any of the existentialists are logicians, let alone that all of
them are. (You might as well argue ‘All women are human beings, all men are human beings, hence all women are men’.)

But now imagine someone who chains this pair of rotten inferences together into a two-step argument as follows:

\[
\begin{align*}
H & \quad (1) \text{All existentialists are philosophers.} \\
& \quad (2) \text{All philosophers are logicians.} \\
& \quad (3) \text{All logicians are philosophers.} \\
& \quad (4) \text{All existentialists are logicians.}
\end{align*}
\]

Here, our reasoner first makes the same fallacious inference as in \(F\), and then compounds the sin by committing the same inferential howler as in \(G\). So they have got from the initial premisses \(H(1)\) and \(H(2)\) to their final conclusion \(H(4)\) by two quite terrible moves: it would therefore be extremely odd to dignify this supposed ‘proof’ as valid, meaning deductively cogent.

Note, however, that in this case the two howlers by luck happen to cancel each other out, and there are no possible circumstances in which the initial premisses \(H(1)\) and \(H(2)\) are both true and yet the final conclusion \(H(4)\) is false. If the existentialists are all philosophers, and all philosophers are logicians, then the existentialists must of course be logicians. Hence, the inferential jump from initial premisses to final conclusion is in fact valid.

In sum, then, if we were to say that a multi-step argument is valid if the initial premisses absolutely guarantee the final conclusion, then we’d have to count the two-step argument \(H\) as valid. Which is, as we noted, would be a very unhappy way of describing the situation, given that \(H\) involves a couple of nasty fallacies!

(b) Doesn’t this just show that we must define a genuine proof – a deductively virtuous multi-step argument, where luck doesn’t enter into it – to be an argument whose individual inferential steps are all valid?

But again things aren’t quite that easy. Not only must individual steps be valid, but they must be chained together in the right kind of way.

To illustrate, consider this one-step inference:

\[
\begin{align*}
I & \quad (1) \text{Socrates is a philosopher.} \\
& \quad (2) \text{All philosophers have snub noses.} \\
& \quad (3) \text{Socrates is a philosopher and all philosophers have snub noses.}
\end{align*}
\]

That’s trivially valid (if you are given \(A\) and \(B\), you can infer \(A\-and-B\)). And here is another equally trivial valid inference (since from \(A\-and-B\) you can infer \(B\)):

\[
\begin{align*}
J & \quad (1) \text{Socrates is a philosopher and all philosophers have snub noses.} \\
& \quad (2) \text{All philosophers have snub noses.}
\end{align*}
\]

And thirdly, this too is plainly valid, an instance of a now very familiar form of inference:

\[
\begin{align*}
K & \quad (1) \text{Socrates is a philosopher.} \\
& \quad (2) \text{All philosophers have snub noses.} \\
& \quad (3) \text{Socrates has a snub nose.}
\end{align*}
\]
Taken separately, then, those three little inferences are quite unproblematic. But now imagine someone chains them together into the following unholy tangle:

L
(1) Socrates is a philosopher. (premiss)
(2) Socrates is a philosopher and all philosophers have snub noses. (from 1, 3 as in I)
(3) All philosophers have snub noses. (from 2 as in J)
(4) Socrates has a snub nose. (from 1, 3 as in K)

By separately valid steps we seem to have deduced the shape of Socrates’ nose just from the premiss that he is a philosopher! What has gone wrong?

The answer is obvious enough. In the middle of the argument we have gone round in a circle. L(2) is derived from L(3), and then L(3) is derived from L(2). Circular arguments can’t take us anywhere. So that suggests we need an improved account of what makes for a deductively cogent multi-step argument. We might say: for cogency, the individual inferential steps must all be valid, and the steps must be chained together in a non-circular way, with steps only depending on what is already available in the argument.

(c) That’s an improvement, but more still needs to be said. Because as it stands, this doesn’t allow e.g. the reductio arguments we met in the previous arguments.

For remember, in such arguments, we can introduce a new temporary supposition that doesn’t follow from what’s gone before, so long as we later ‘discharge’ the assumption. And note too that at the point of discharge, when we use the rule (RAA) – to take the one rule for an indirect proof that we know about – we have an inference step whose ‘input’ is not a proposition or two but a prior ‘sub-proof’ (one of those indented passages of argument). Allowing such cases will evidently complicate our definition of a cogent multi-step argument.

We signalled back in §2.3 that multi-step arguments are going to be tricky to deal with. But for our purposes in this book, we don’t need to attempt a once-and-for-all general story that attempts to cover every sort of argumentative structure that has been recognized by logicians as codifying a cogent way of producing absolutely compelling proofs. It will be enough to spell out some key ‘natural deduction’ structures and explain their cogency. A lot more on this in due course.

5.5 Summary

• To establish the validity of a perhaps unobvious inferential leap we can use deductively cogent multi-step arguments, i.e. proofs, filling in the gap between premisses and conclusion.

• Simple, direct, proofs chain together inference steps that are valid – ideally, are obviously valid – building up from the initial premisses to the desired conclusion, with each new step depending on what’s gone before.

• However, some common methods of proof are ‘indirect’, like reductio ad absurdum, and involve making new temporary suppositions for the sake of argument, suppositions that are later discharged.
• The availability of indirect modes of inference complicates the story about what makes for a deductively cogent multi-step argument; we will have to return to this.

**Exercises 5**

Which of the following arguments are valid? Where an argument is valid, provide a proof. Some of the examples are enthymemes that need repair.

1. No philosopher is illogical. Jones keeps making argumentative blunders. No logical person keeps making argumentative blunders. All existentialists are philosophers. So, Jones is not an existentialist.

2. Jane has a first cousin. Jane’s father had no siblings. So, if Jane’s mother had no sisters, she had a brother.

3. Every event is causally determined. No action should be punished if the agent isn’t responsible for it. Agents are only responsible for actions they can avoid doing. Hence no action should be punished.

4. Something is an elementary particle only if it has no parts. Nothing which has no parts can disintegrate. An object that cannot be destroyed must continue to exist. So an elementary particle cannot cease to exist.

5. No experienced person is incompetent. Jenkins is always blundering. No competent person is always blundering. So, Jenkins is inexperienced.

6. Only logicians are good philosophers. No existentialists are logicians. Some existentialists are French philosophers. So, some French philosophers are not good philosophers.

7. Either the butler or the cook committed the murder. The victim died from poison if the cook did the murder. The butler did the murder only if the victim was stabbed. The victim didn’t die from poison. So, the victim was stabbed.

8. Promise-breakers are untrustworthy. Beer-drinkers are very communicative. A man who keeps his promises is honest. No one who doesn’t drink beer runs a bar. One can always trust a very communicative person. So, no one who keeps a bar is dishonest.

9. When I do an example without grumbling, it is one that I can understand. No easy logic example ever makes my head ache. This logic example is not arranged in regular order, like the examples I am used to. I can’t understand these examples that are not arranged in regular order, like the examples I am used to. I never grumble at an example, unless it gives me a headache. So, this logic example is difficult.
What have we done so far? In bare headlines,

- We have explained, at least in an introductory way, the notion of a valid inference-step, and the corresponding notions of a deductively valid/sound (one-step) argument.
- We have distinguished deductive validity from other virtues that an argument might have (like being an inductively strong argument).
- We have noted how different arguments may share the same form of inference. And we exploited this fact when we developed the counterexample method for demonstrating invalidity.
- We have seen some simple examples of direct multi-step proofs, where we show that a conclusion really can be validly inferred from certain premisses by filling in the gap between premisses and conclusion with evidently valid intermediate inference steps. We briefly looked at one kind of indirect method of proof, reductio ad absurdum.

All this has been fairly informal, and we have quietly skated past some issues and explicitly shelved others. But hopefully, you will have gained at least a preliminary understanding of some key logical concepts.

What next? At this point, we could go in a number of directions. We could remain content with an informal level of discussion, and spend quite a bit of time exploring techniques for teasing out the inferences involved in passages of extended prose argumentation – after all, as we have noted before, real-life arguments are rarely given to us with their propositions neatly numbered, all served up as a fully documented proof! Then we could explore a variety of rather common ways in which everyday arguments can go wrong. And that kind of study in informal logic (as it is often called) would be a highly profitable enterprise.

But this book takes a different direction. We are going to assume that we are already competent enough at filleting out the bones of an argument from everyday prose; and we are going to be focusing on a couple of broad classes of deductive arguments, with the aim of constructing rigorous, comprehensive, formal theories about them.

In short, we are going for depth rather than breadth. Our approach will indeed give us a powerful set of tools for dealing with certain kinds of real-life arguments. But that isn’t the only reason for taking the formal route. For a start, in the course of these discussions
we are going to be developing a whole range of ideas which belong in the toolkit of any analytic philosopher.

I would claim, in fact, that the now very well understood theory of ‘first-order quantification theory’ – the main branch of formal logic which we are eventually introducing in this book – is one of the great intellectual achievements of formally minded philosophers and of philosophically minded mathematicians, delightful in itself. And it opens the door into a very rich and fascinating field. Although, in this introductory book, there will only be very occasional glimpses further through the door, what follows will at least take us to the threshold.

There are, however, different possible routes to the threshold, different ways of starting to develop formal logical systems. Once upon a time, so-called axiomatic systems were all the rage – indeed, many mathematicians still love them. Then it became common for philosophy students to be introduced to serious logic via natural deduction systems: and there are at least two different main styles of these. But ‘natural’ doesn’t always equate with ‘easy’: which is perhaps why so-called ‘tree’ or ‘tableau’ systems became popular, being indeed particularly easy to understand and manipulate by those without a mathematical bent. Given world enough and time, we’d develop all three approaches, and so-called sequent calculi too, and discuss their pros and cons.

But we have to make some choices. In what follows you will meet both trees and one flavour of natural deduction. Or at least you will if you decide to take the full set menu. However, that’s not compulsory. You can, if you like, pick and choose à la carte, and concentrate on trees or alternatively concentrate on natural deduction (though either way, you’ll miss out on some instructive fun). There will be plenty of signposting at possible choice points.