Corrections made between the first and fourth printings of *IGT*

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An Introduction to Gödel's Theorems now exists in three official versions and (just to complicate matters!) a withdrawn rogue reprinting. The quick way of telling these apart is to glance at the imprints page (the verso of the title page).

1. The original 2007 printing (imprints page simply says "First published 2007").
2. The first corrected reprinting, 2008 (imprints page says "First published 2007 / Reprinted with corrections 2008").
3. There was a rogue third printing, which by error was of the uncorrected original version, and which was fairly speedily withdrawn by CUP (imprints page says "Third printing 2008").
4. A 2009 reprinting with further corrections (imprints page says "First published 2007 / Fourth printing with corrections 2009").

This document describes most of the corrections made between the original 2007 printing and the fourth printing (if you have the second printing, return to the website's corrections page and select the relevant document).

This list of corrections is divided, for convenience, into two sections:

1. **More serious errors/needed improvements**. These are the errors that could lead readers astray, or passages that need improvement. (Obviously, I can by now see various ways that I'd do the book differently, if ever I have the opportunity to write a new edition. Here I'm noting the sort of local improvements that can be absorbed into a corrected reprint.)
2. **Minor errors/improvements**. The rest are obvious typos, or correct other very minor errors or infelicities.

1 More serious errors/needed improvements

P. 14 (line 6). *The printed presentation of the Cantorian diagonal argument carelessly doesn't make it absolutely explicit that the diagonal argument generalizes to show that* any *purported enumerating function fails. To rectify this . . .*

Replace from "There's obviously" with "There's an infinite number of different such strings. Suppose, for reductio, that there is an enumerating function f which maps the natural numbers onto the strings, for example like this:"

And then, four lines after the displayed table, replace from "the initial string" to the end of the proof with in the first place; it differs from the next string s_1 in the next place; and so on. So our diagonal construction defines a new string s that differs from each of the s_j, contradicting the assumption that our map f is 'onto' and enumerates *all* the binary strings. Now, our supposed enumerating function f was just one example map; but the same 'flipped diagonal' construction will evidently work to show that any other candidate map f' must also fail to enumerate all the strings. So $\mathbb{B}$ is infinite, but not enumerable. In a word, it is *indenumerable*.

P. 23 (para (c)). *The characterization of axiomatized theories could confuse some who have come across other definitions of theories in other books. To rectify this ...*

Replace para (c) up to the indented passage with

> (c) Given an axiomatized formal theory T, it is decidable which wffs are its logical or non-logical axioms and which arrays of wffs conform to the derivation rules of T's proof system. It will therefore also be decidable which arrays of wffs are axiomatic T-proofs – i.e. which arrays are properly constructed proofs, all of whose premisses are indeed T-axioms.
>
> So, to summarize these constraints on decidability:

Add after the indented passage

> (d) The point needs highlighting. Logic books often define a theory very generously to be *any set of sentences*, or to be *any set of sentences which are consequences of some set of sentences* Σ. In this book, however, when we talk of theories, we are only going to be interested in *axiomatized formal theories* in the sense of theories where it is *decidable* what's in a set of axioms Σ. For it is only theories of this kind to which Gödel's theorems can be applied. [*Footnote* It can be left as a useful exercise to ask yourself, as you read through, which claims we make about theories in fact still apply on a more generous understand of the notion.]

P. 37 (indented definition). *It is very misleading to use "express" informally in (ii) when it has a more technical sense in (i).*

Replace "(ii) it can express quantifications over numbers." with "(ii) it can form wffs which quantify over numbers."

Pp. 40–41 (paras (ii) to (v)). *The main change here arises because L's native quantifiers might run over more than the numbers, so need to be explicitly restricted to get quantifications over numbers. Because the chapter finishes at the very bottom of a page, there are other small deletions/rearrangements made to ensure that the pagination doesn't change.*

Replace from para. (ii) at the bottom of p. 40 through to and including para (v) on p. 41 with:

> (ii) Since R is decidable, then in any given sufficiently expressive arithmetical language L there will be some wff of L which expresses R: let's abbreviate that wff $\mathsf{R}(\mathsf{x}, \mathsf{y})$. Then Rmn just when $\mathsf{R}(\overline{\mathsf{m}}, \overline{\mathsf{n}})$ is true.
>
> (iii) By definition, a sufficiently expressive language can form wffs which quantify over numbers. So $\exists x R(x, n)$ just when $\exists \mathsf{x}(\mathsf{Nat}(\mathsf{x}) \wedge \mathsf{R}(\mathsf{x}, \overline{\mathsf{n}}))$ is true (where $\mathsf{Nat}(\mathsf{x})$ stands in for whatever L-predicate might be needed explicitly to restrict L's quantifiers to numbers).
>
> (iv) So from (i) and (iii) we have

$n \in \overline{K}$ if and only if $\neg\exists \mathsf{x}(\mathsf{Nat}(\mathsf{x}) \wedge \mathsf{R}(\mathsf{x}, \overline{\mathsf{n}}))$ is true.

(v) Now suppose for a moment that the set $\mathcal{T}$ of truths of arithmetic expressible in L *is* effectively enumerable. Then, given a description of the expression R, we could run through the supposed effective enumeration of $\mathcal{T}$, and whenever we come across a truth of the type $\neg\exists \mathsf{x}(\mathsf{Nat}(\mathsf{x}) \wedge \mathsf{R}(\mathsf{x}, \overline{\mathsf{n}}))$ – and it will be effectively decidable if a wff has that particular syntactic form – list the number n. That procedure would give us an effectively generated list of all the members of $\overline{K}$.

P. 48 (footnote) *To harmonize with the revisions on pp. 40–41, . . .*

Replace "$\neg\exists \mathsf{j}\mathsf{R}(\mathsf{j}, \overline{\mathsf{n}})$" with "$\neg\exists \mathsf{x}(\mathsf{Nat}(\mathsf{x}) \wedge \mathsf{R}(\mathsf{x}, \overline{\mathsf{n}}))$".

P. 103 (last paragraph of §12.3). *The qualification 'has a smidgin of induction' is unnecessary and should be deleted. The fourth printing gives the induction-free proof.*

P. 176 (second paragraph of proof at top of the page). *The proof in the book is not wrong: but it doesn't get to its conclusion by quite the shortest route, and the one-line unnecessary detour could I suppose be distracting.*

Replace from "Now assume . . . " to the end of proof, with

> Now assume T is ω-consistent and hence plain consistent, and suppose $T \vdash \neg\gamma$. Since $T \vdash \gamma \leftrightarrow \neg\mathsf{Prov}(\ulcorner\gamma\urcorner)$, it follows $T \vdash \mathsf{Prov}(\ulcorner\gamma\urcorner)$. Hence, by (C$\omega$), $T \vdash \gamma$. Contradiction. So, assuming T *is* ω-consistent, we can't have $T \vdash \neg\gamma$. ⊠

P. 178 (Proof sketch near foot of the page). *The claim that the* Σ_1 *wff* $\overline{\mathsf{Prf}}(\mathsf{w}, \mathsf{x})$ *is equivalent to the* Π_1 *wff* $\forall \mathsf{z}(\overline{\mathsf{Prf}}(\mathsf{w}, \mathsf{z}) \rightarrow \mathsf{z} = \mathsf{x})$ *is false! Suppose* w *is replaced by a numeral* $\overline{\mathsf{m}}$ *for a number which isn't a super-Gödel number. Then* $\overline{\mathsf{Prf}}(\overline{\mathsf{m}}, \overline{\mathsf{n}})$ *must be false, though* $\forall \mathsf{z}(\overline{\mathsf{Prf}}(\overline{\mathsf{m}}, \mathsf{z}) \rightarrow \mathsf{z} = \overline{\mathsf{n}})$ *is vacuously true. Thanks to Adil Sanaulla for noting this!*

In fact, to repair this silly mistake is non-trivial. See the 'Correction to Rosser's Theorem' for details.

P. 183 (Last sentence before (b)). Replace "So our theorem means, roughly speaking, that there will inevitably be theorems which can be stated briefly but which only have relatively enormous proofs." by "So our theorem means, roughly speaking, that there will inevitably be T-theorems which only have enormous proofs but can, relative to the length of the proof, be stated briefly." And then in first sentence of (b) replace "some short wffs with relatively enormous proofs" by "some relatively short wffs with enormous proofs". [Thanks to Duncan Watson.]

P. 184 (Proof sketch). *The proof as printed twice says "for every/any wff* φ*" when it should say "for every/any* T*-theorem* φ*". Thanks to Peter Milne.*

Replace first two paragraphs of the proof sketch with:

> Suppose the theorem is false. So there is a sentence γ which is undecided by T, but such that $T + \gamma$ does *not* exhibit speed-up over T. Then there is a p.r. function f such that for every T-theorem φ, if φ has a proof in $T + \gamma$ with g.n. p, then it has a proof in the original T with g.n. number no greater than $f(p)$.
>
> Well, take any T-theorem φ. Then $(\gamma \vee \varphi)$ is also a T-theorem. But $(\gamma \vee \varphi)$ is trivially provable in $T + \gamma$. And there will be a very simple computation, with no open-ended searching, that takes us from the g.n. of φ to the g.n. of the trivial proof in $T + \gamma$ of $(\gamma \vee \varphi)$. In other words, the g.n. of the proof will be $h(\ulcorner\varphi\urcorner)$,

for some p.r. function h.[10] So, by our supposition with $(\gamma \vee \varphi)$ for φ, $(\gamma \vee \varphi)$ must have a proof in T with g.n. no greater than $f(h(\ulcorner\varphi\urcorner))$.

P. 197 (§22.7, line 12). *It is plainly* not *the case, as a stray clause wrongly says, that an* L_{2A} *wff* $\varphi(\mathsf{x})$*'s lacking second-order quantifiers is equivalent to its being already a wff of* L_A*. For* $\varphi(\mathsf{x})$ *could contain second-order free variables/parameters. And it is important that* $\mathsf{ACA_0}$ *allows such wffs into the Comprehension Scheme. An essay saying much more about* $\mathsf{ACA_0}$ *appears on the book's website.*

Delete "already belong to L_A, i.e. be purely arithmetical". Add to footnote 13 reference to essay about $\mathsf{ACA_0}$ on the website.

P. 266 (Footnote, first sentence) Delete "at most".

P. 270 (line 16 up). Delete sentence "A little reflection ... diagonals)". Add to previous paragraph:

> A little reflection on the patterns in the displayed calculation should convince you that this procedure *does* always terminate.

P. 286 (after statement of Theorem 30.10). *The passage as written very oddly misses out the best proof of the theorem from materials already to hand, as was pointed out to me by Arnon Avron. So replace the nineteen lines from "How do we prove this?" to "general-purpose programming framework." with this:*

> How do we prove this? Well, we already have the following argument to hand:
>
> *Proof.* Suppose for reductio there is a recursive function f that enumerates (the Gödel numbers for) the truths of L_A. We know that any recursive function can be captured in Q (by Theorem 30.1); now just appeal to the fact that Q is sound and apply the last remark in §4.7 to conclude that Q's language L_A suffices to express any recursive function in the sense of §12.1. So in particular there is a formal wff $\mathsf{F(x,y)}$ which expresses that enumerating function f. But then the formal wff $\exists\mathsf{x}\,\mathsf{F(x,y)}$ will be satisfied by a number if and only if it numbers a truth of L_A. But by Theorem 21.5 there cannot be such a wff. ⊠
>
> It is instructive, however, to consider whether we can give an alternative proof which stays closer to the informal argument in Chapter 5, which depended on various intuitive claims about computer programs. But if we going to sharpen up *that* line of argument and make it rigorous, we'll have to give some theoretical treatment of a general-purpose programming framework.

P. 299 (line 6). Replace "recursive" with "regular".

P. 309. *The printed proof of Theorem 33.5 is unnecessarily complicated.*

Replace subsection (a) up to the Theorem by

> (a) We've just seen that, using a Gödel-numbering scheme to code up facts about Turing machines, there will be a purely arithmetical L_A-sentence $\mathsf{H}(\bar{\mathsf{e}})$ which 'says' that the program Π_e halts when run on input e.
>
> Now let's suppose, for reductio, that the truths of L_A are recursively enumerable. That means that there is an effectively computable function that given

successive inputs 0, 1, 2 ... delivers as output a listing of (the Gödel numbers of) the truths of L_A.

So start effectively generating that listing. Since exactly one of $\mathsf{H}(\overline{\mathsf{e}})$ and $\neg\mathsf{H}(\overline{\mathsf{e}})$ is true, the Gödel number for one of them must eventually turn up – by hypothesis the effective enumeration lists (the Gödel numbers of) *all* the truths. So setting the enumeration going will be an effective way of deciding which of $\mathsf{H}(\overline{\mathsf{e}})$ and $\neg\mathsf{H}(\overline{\mathsf{e}})$ is true – wait and see which turns up! That gives us an effective way of solving the self-halting problem and determining whether the program Π_e halts when run on input e.

And now we can either make a labour-saving appeal to Church's Thesis, or we can fairly easily rigorize this informal line of thought, to get the result that if the truths of L_A are recursively enumerable then the self-halting problem would be *recursively* solvable. Hence, contraposing, we have

P. 323 (second indented passage) Insert (after "effective calculation") "together with mathematical argument".

P. 341 *To tighten and complete the argument here ...*

Replace paragraph beginning "But we can ..." with

> But we can in fact be a bit more concessive to Black. To establish the argument for the third premiss of the squeezing argument, it is actually enough that $c(n, j)$ be μ-recursive (it doesn't have to be p.r.; and note we've already fixed that c is a regular function that eventually, for some value of j, defaults to zero). Suppose, then, that there is a transition function tr that takes us from the code for the j-th step of the computation for input n to the code for the $j + 1$-th step, so $c(n, Sj) = tr(n, j, c(n, j))$. Then $c(n, j)$ will certainly be μ-recursive if tr is. Now suppose that the definition of tr involves the use of a restricted search operator $(\mu z \leq a(n, j))$, where $a(n, j)$ – whose value is to be extracted from $c(n, j)$ – is a μ-recursive function giving the size of the active patch of dataspace that needs to be looked around in applying a transition rule for the next step. *Then tr can still be μ-recursive if $a(n, j)$ is μ-recursive – yet $a(n, j)$ could grow as fast as the wildly fast-growing Ackermann-Péter function!* So in fact we could revise the definition of a KU-algorithm to allow the size of the active patch to grow as fast as you like, so long the growth is governed by some μ-recursive function, and yet c would *still* remain μ-recursive as needed for the squeezing argument for the Church–Turing Thesis.
>
> You might object: "What right do we have to build *recursive* bounds into the (re)definition of a KU-algorithm, rather than perhaps faster-growing *effectively computable* bounds? We can't assume these are the same without assuming the Thesis which we are trying to prove!" But against *this* we can rerun our earlier thought with even more confidence. It surely is quite inconsistent with the intuitive idea of an idiot-proof algorithm which proceeds by *small* steps, accessible to limited agents, that what is processed at each step should not just get ever bigger, but get bigger *even faster than Ackermann-fast!* [*Footnote* I am indebted here to discussions with Tim Button and Bruno Whittle.]

(References to Button and Whittle should then be indexed.)

2 Minor errors/minor improvements

The following are obvious typos, or other very minor errors or infelicities that shouldn't cause a reader any problems (if noticed at all!).

P. x (first line of contents for Ch. 30). Replace "recursive" by "μ-recursive".

P. 3 (four lines up from bottom of main text). "before than" should read "before". [Thanks to Guido Rooms.]

P. 9 (line 20). Replace "But plainly . . . deliver a result!" by "But plainly, if an algorithmic procedure is actually to decide whether some property holds or actually to compute a function, for any input, more is required. It needs always to terminate after a finite number of steps and deliver a result!". Then add a new footnote: "Note, then, it isn't part of the very idea of an algorithm that its execution always terminates: in general, an algorithm may only compute a partial function." [Thanks to Russell Pannier.]

P. 14 (line 4). Delete "s".

P. 27 (Proof). To better link up the proof here to the remark about 'do until' proofs in §29.1, para 2, rewrite the proof very slightly as follows:

> *Proof* We know from Theorem 3.1 that there's an algorithm for effectively enumerating the theorems of T. So to decide whether the sentence φ of T's language is a T-theorem, start effectively listing the theorems, and do this until either φ or $\neg\varphi$ turns up and then stop. If φ turns up, declare it to be a theorem. If $\neg\varphi$ turns up, declare that φ is *not* a theorem.
>
> Why does this work as a decision procedure? Well first, by hypothesis, T is negation complete, so either φ is a T-theorem or $\neg\varphi$ is. So it is guaranteed that – within a finite number of steps – either φ or $\neg\varphi$ will be produced in our enumeration of the theorems, and our 'do until' procedure terminates. And second, if φ is produced, φ is a theorem of course, while if $\neg\varphi$ is produced, we can conclude that φ is not a theorem, since the theory is assumed to be consistent.
>
> Hence, in this case, there *is* a dumbly mechanical procedure for deciding whether φ is a theorem. ⊠

P. 29 (sec. (c) line 1). Replace "punctilious" by "pedantic".

P. 33 (line 12). After "open wff ^{3}" insert "$\psi(\mathsf{x})$".

P. 33 (line 9 up). After "open wff" insert "$\chi(\mathsf{x})$".

P. 38 (§5.2, first line of Proof). "The theorem hold" should read "The theorem holds". [Thanks to Matthew Mead.]

P. 38 (§5.2, fifth line of Proof). "just be evaluating" should read "just by evaluating". [Thanks to Saeed Salehi.]

P. 42 (footnote). "Zielger" should read "Ziegler". The entry in the Bibliography should be emended similarly. [Thanks to Richard Zach.]

P. 48 (para A.). Replace "(i)", "(ii)", "(i′)", "(ii′)" with "(a)", "(b)", "(a′)", "(b′)". [To avoid any possible confusion with (i) and (ii) earlier in the section.]

Pp. 85–115, passim (Running heads). The spacing after "p.r." in running heads is end-of-sentence spacing rather than word-gap spacing. [Thanks to the eagle-eyed Ed Snow.]

P. 96 (penultimate line of Proof for D). "$\mu x < g(n)$" should read "$\mu x \leq g(n)$". [Thanks to Saeed Salehi.]

p. 97 (second line of proof of (R1)). Replace "$sg(Sn \dot{-} m)$ and $sg(n \dot{-} m)$" by "$sg(Sm \dot{-} n)$ and $sg(m \dot{-} n)$". [Thanks to Curtis Brown.]

p. 111 (line 10, last line of indent). Replace "$\beta(b, i)$" by "$\beta(c, i)$". [Thanks to Curtis Brown.]

p. 115 (line 9). Replace "$f(m) = g(h(m))$" by "$f(m) = h(g(m))$". [Thanks to Russell Pannier.]

p. 115 (line 8 up). Replace "$\mathsf{G}(\vec{\mathsf{x}}, \mathsf{w})$ and $\mathsf{H}(\vec{\mathsf{x}}, \mathsf{u}, \mathsf{v}, \mathsf{w})$" by "$\mathsf{G}$ and H".

p. 115 (line 5 up). In first line of C*, replace $\mathsf{G}(\vec{\mathsf{x}}, \mathsf{w})$ by $\mathsf{G}(\vec{\mathsf{x}}, \mathsf{k})$ – compare C* at the top of the previous page. Also make corresponding change three lines up. [Thanks to Ed Snow.]

P. 129 (§15.4, lines 17, 20). On both occurrences, "$2^{21} \cdot 3^{21} \cdot 5^{19}$" should read "$2^{23} \cdot 3^{23} \cdot 5^{21}$". [Thanks to Michael Clark.]

P. 132 (first para. of Proof for (R7)). "$2^{11} \cdot 3^{4}$" should read "$2^{13} \cdot 3^{4}$"; "$2^{4} \cdot 3^{13} \cdot 5^{19}$" should read "$2^{4} \cdot 3^{15} \cdot 5^{21}$"; "$2^{11} \cdot 3^{4} \cdot 5^{4} \cdot 7^{13} \cdot 11^{19}$" should read $2^{13} \cdot 3^{4} \cdot 5^{4} \cdot 7^{15} \cdot 11^{21}$. [Thanks to Duncan Watson.]

P. 133 (Proof for (R8)). "2^{19}" should read "2^{21}"; and on the next line"2^{21}" should read "2^{23}". [Thanks to Duncan Watson.]

P. 132 (line 8 up). "$(\forall i \leq len(n))$" should read "$(\forall i < len(n))$". [Thanks to Richard Zach.]

P. 136 (footnote). Replace "$\neg\exists\xi\neg$" by "$\neg\forall\xi\neg$". [Thanks to Willemien Hoogendoorn.]

P. 141 (line 14). Replace "by the Σ_1/Π_1 lemma of Section 13.2" by "by the Σ_1/Π_1 theorem of Section 13.1". [Thanks to José F. Ruiz.]

P. 146 (line 2 of last main para). Replace "in such as way as" by "in such a way as". [Thanks to Hoss Parwas.]

P. 153 (line 5 of second para of §18.1). Replace "if would" by "it would". [Thanks to Hoss Parwas.]

P. 154 (first line after Theorem 17.1). Replace "now let's now" by "let's now". [Thanks to Hoss Parwas.]

P. 161 (first line) Replace "(i), (ii) and (iii)" with "(a), (b) and (c)". [Thanks to Willemien Hoogendoorn.]

P. 165 (line 5 up). Delete "This sentence turns out to be Π_1."

P. 165 (penultimate line). Replace "Assume T soundness" by "Assume T's soundness". [Thanks to Hoss Parwas.]

P. 166 (line 8). After "*is enough.*" insert "And Rosser's clever idea can in fact be used to construct a Π_1 undecidable sentence."

P. 173 (line 12). Replace "if and if only" by "if and only". [Thanks to José F. Ruiz.]

P. 176 (ine 11). Replace "the fixed point will be of Goldbach type" with "there will be a fixed point of Goldbach type".

P. 177 (first line of footnotes). Insert "a" after "showing me".

P. 178 (four lines up). There is a stray closing parenthesis. "$(\forall \mathsf{w} \leq \mathsf{v})\neg\overline{\mathsf{Prf}}_T(\mathsf{w}, \mathsf{x}))$" should be "$(\forall \mathsf{w} \leq \mathsf{v})\neg\overline{\mathsf{Prf}}_T(\mathsf{w}, \mathsf{x})$". [Thanks to Luca Incurvati.]

P. 184 (line 8). Replace "the proofs of *old* theorems" with "the proofs of some *old* theorems".

P. 187 (three quarters of the way down). Some Greek capital Ξ's have lost their serifs, thus Ξ. [Thanks to the eagle-eyed "MoeBlee".]

P. 196 (third line after Theorem 22.3). Replace "In other worlds" by "In other words". [Thanks to Hoss Parwas.]

P. 196 (last paragraph). Replace with

> Assuming $\mathcal{I}_{2A}$ is a model for PA_2 (so the theory is consistent), the axioms of PA_2 are enough to *semantically entail* all true sentences of L_{2A}. But the Gödel-Rosser Theorem tells us this formal deductive theory is not strong enough to *prove* all true L_{2A} sentences – and we can't expand PA_2 so as to prove them all either, so long as the expanded theory remains consistent and properly axiomatized. However, make the distinction between what is *semantically entailed* and what is *deductively proved*, and we reconcile the apparent conflict between the implication of Dedekind's categoricity result ('PA_2 settles all the truths') and Gödelian incompleteness ('PA_2 leaves some truths undecided').

[Thanks to Lee Corbin for spotting an errant 'entail' for 'prove'.]

P. 207 (eight lines up). Replace " wffs of an formal language" by "wffs of a formal language". [Thanks to Lee Corbin.]

P. 210 (line 18). Line ending "more L_A sentences than PA." should end with a question mark. [Thanks to Lee Corbin.]

P. 213 (three lines before §24.2). Replace "in such as way as" by "in such a way as". [Thanks to Hoss Parwas.]

P. 214 (line 4). Replace "so to be speak" by "so to speak", and move phrase to end of parenthesis. [Thanks to Hoss Parwas.]

P. 223 (line after D'). "we are prove" should read "we are to prove". [Thanks to Saeed Salehi.]

P. 229 (penultimate line of §25.6). "half that" should read "half of that". [Thanks to Saeed Salehi.]

P. 238. Both the formulae at lines 1 and 3 should end with a period. Similarly for the formula at line 8 of (c).

P. 242 (the two intended formulae both(!) numbered (ii)). There are misplaced subscript Ts. In the first, "$Prf(x, y)_T$" should read "$Prf_T(x, y)$"; in the second "$\mathsf{Prf}(\mathsf{x}, \mathsf{y})_T$" should read "$\mathsf{Prf}_T(\mathsf{x}, \mathsf{y})$". [Thanks to Saeed Salehi.]

P. 242. The numbering of formulae on this page should be rectified so (ii) isn't repeated!

P. 251 (line 19). Replace "were it turn out" by "were it to turn out". [Thanks to "MoeBlee".]

P. 260 (second line of (b)). Replace "Of we course" by "Of course". [Thanks to Dave Chambers.]

P. 262 (penultimate line of main text). Replace "Wang (1974," by "(Wang 1974,". [Thanks to Stuart Smith.]

P. 269 (line 12). Replace "as n increases" by "as x increases".

P. 271 (footnote). Add reference to Hedman (2004, pp. 307–309). Add corresponding item to Bibliography: Hedman, S., 2004. *A First Course in Logic*. Oxford, Oxford University Press.

P. 277 (line 11 up). Replace "recursive" by "μ-recursive".

P. 278 (§30.2). In section title, and in next line, replace "recursive" by "μ-recursive".

P. 291 (line 9 up of main text). Replace "$f(n)$" by "$g(m, n)$". [Thanks to Saeed Salehi.]

P. 299 (lines 3 and 5). Delete the redundant "total" on both occurrences.

P. 304 (line 6). Replace "recursive" by "μ-recursive".

P. 308 (lines 5–6 of Proof). Replace "And for each j, $\mathsf{Q} \vdash \neg\mathsf{C}(\bar{\mathsf{e}}, \bar{\mathsf{e}}, \bar{\mathsf{j}}, 0)$ if Π_e never halts with input e." by "And if Π_e never halts with input e then, for each j, $\mathsf{Q} \vdash \neg\mathsf{C}(\bar{\mathsf{e}}, \bar{\mathsf{e}}, \bar{\mathsf{j}}, 0)$."

P. 308 (first two lines of §33.4). Replace "is not recursively unsolvable" by "is not recursively solvable". [Thanks to Rob Trueman]

P. 309 (line after Theorem 33.5). Replace "gives us the proof" by "gives us the second proof". [That's to co-ordinate with the revised p. 286.]

P. 322 (last line of main text). Replace "recursive" by "μ-recursive".

P. 323 (middle line of second indented passage). Replace "recursive" by "μ-recursive".

P. 333 (line 5 up). Replace colon with full stop (to prevent a sentence having two colons).

P. 348 The entries for Frege (1891) and Gandy (1988) are lacking publication location. Add "Oxford" in each case.

P. 359 (line 4, column 2). The page reference for 'nice theory' is in fact 151.