Corrections made between the second and fourth printings of *IGT*

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An Introduction to Gödel's Theorems now exists in three official versions and (just to complicate matters!) a withdrawn rogue reprinting. The quick way of telling these apart is to glance at the imprints page (the verso of the title page).

1. The original 2007 printing (imprints page simply says "First published 2007").
2. The first corrected reprinting, 2008 (imprints page says "First published 2007 / Reprinted with corrections 2008").
3. There was a rogue third printing, which by error was of the uncorrected original version, and which was fairly speedily withdrawn by CUP (imprints page says "Third printing 2008").
4. A 2009 reprinting with further corrections (imprints page says "First published 2007 / Fourth printing with corrections 2009").

This document lists most of the correction made between the second printing and the fourth (if you have the first printing, return to the website's corrections page and select the relevant document).

This list of corrections is divided, for convenience, into two sections:

1. **More serious errors/needed improvements**. These are the errors that could lead readers astray, or passages that need improvement. (Obviously, I can by now see various ways that I'd do the book differently, if ever I have the opportunity to write a new edition. Here I'm noting the sort of local improvements that can be absorbed into a corrected reprint.)
2. **Minor errors/improvements**. The rest are obvious typos, or correct other very minor errors or infelicities.

1 More serious errors/needed improvements

P. 103 (last paragraph of §12.3). The qualification 'has a smidgin of induction' is unnecessary and should be deleted. The fourth printing gives the induction-free proof.

P. 178 (Proof sketch near foot of the page). The claim that the Σ_1 wff $\overline{\mathsf{Prf}}(\mathsf{w}, \mathsf{x})$ is equivalent to the Π_1 wff $\forall \mathsf{z}(\overline{\mathsf{Prf}}(\mathsf{w}, \mathsf{z}) \to \mathsf{z} = \mathsf{x})$ is false! Suppose w is replaced by a numeral $\overline{\mathsf{m}}$ for a number which isn't a super-Gödel number. Then $\overline{\mathsf{Prf}}(\overline{\mathsf{m}}, \overline{\mathsf{n}})$ must be false, though $\forall \mathsf{z}(\overline{\mathsf{Prf}}(\overline{\mathsf{m}}, \mathsf{z}) \to \mathsf{z} = \overline{\mathsf{n}})$ is vacuously true. Thanks to Adil Sanaulla for noting this!

In fact, to repair this silly mistake is non-trivial. See the 'Correction to Rosser's Theorem' for details.

P. 183 (Last sentence before (b)). Replace "So our theorem means, roughly speaking, that there will inevitably be theorems which can be stated briefly but which only have relatively enormous proofs." by "So our theorem means, roughly speaking, that there will inevitably be T-theorems which only have enormous proofs but can, relative to the length of the proof, be stated briefly." And then in first sentence of (b) replace "some short wffs with relatively enormous proofs" by "some relatively short wffs with enormous proofs". [Thanks to Duncan Watson.]

P. 266 (Footnote, first sentence) Delete "at most".

P. 270 (line 16 up). Delete sentence "A little reflection ... diagonals)". Add to previous paragraph:

> A little reflection on the patterns in the displayed calculation should convince you that this procedure *does* always terminate.

P. 286 (after statement of Theorem 30.10). The passage as written very oddly misses out the best proof of the theorem from materials already to hand, as was pointed out to me by Arnon Avron. So replace the nineteen lines from "How do we prove this?" to "general-purpose programming framework." with this:

> How do we prove this? Well, we already have the following argument to hand:
>
> *Proof.* Suppose for reductio there is a recursive function f that enumerates (the Gödel numbers for) the truths of L_A. We know that any recursive function can be captured in Q (by Theorem 30.1); now just appeal to the fact that Q is sound and apply the last remark in §4.7 to conclude that Q's language L_A suffices to express any recursive function in the sense of §12.1. So in particular there is a formal wff $\mathsf{F}(\mathsf{x}, \mathsf{y})$ which expresses that enumerating function f. But then the formal wff $\exists \mathsf{x}\, \mathsf{F}(\mathsf{x}, \mathsf{y})$ will be satisfied by a number if and only if it numbers a truth of L_A. But by Theorem 21.5 there cannot be such a wff. ⊠
>
> It is instructive, however, to consider whether we can give an alternative proof which stays closer to the informal argument in Chapter 5, which depended on various intuitive claims about computer programs. But if we going to sharpen up *that* line of argument and make it rigorous, we'll have to give some theoretical treatment of a general-purpose programming framework.

2 Minor errors/minor improvements

The following are obvious typos, or other very minor errors or infelicities that shouldn't cause a reader any problems (if noticed at all!).

P. 9 (line 20). Replace "But plainly ... deliver a result!" by "But plainly, if an algorithmic procedure is actually to decide whether some property holds or actually to compute a function, for any input, more is required. It needs always to terminate after a finite number of steps and deliver a result!". Then add a new footnote: "Note, then, it isn't part of the very idea of an algorithm that its execution always terminates: in general, an algorithm may only compute a partial function." [Thanks to Russell Pannier.]

P. 27 (Proof). To better link up the proof here to the remark about 'do until' proofs in §29.1, para 2, rewrite the proof very slightly as follows:

> *Proof* We know from Theorem 3.1 that there's an algorithm for effectively enumerating the theorems of T. So to decide whether the sentence φ of T's language is a T-theorem, start effectively listing the theorems, and do this until either φ or $\neg\varphi$ turns up and then stop. If φ turns up, declare it to be a theorem. If $\neg\varphi$ turns up, declare that φ is *not* a theorem.
>
> Why does this work as a decision procedure? Well first, by hypothesis, T is negation complete, so either φ is a T-theorem or $\neg\varphi$ is. So it is guaranteed that – within a finite number of steps – either φ or $\neg\varphi$ will be produced in our enumeration of the theorems, and our 'do until' procedure terminates. And second, if φ is produced, φ is a theorem of course, while if $\neg\varphi$ is produced, we can conclude that φ is not a theorem, since the theory is assumed to be consistent.
>
> Hence, in this case, there *is* a dumbly mechanical procedure for deciding whether φ is a theorem. ⊠

P. 38 (§5.2, fifth line of Proof). "just be evaluating" should read "just by evaluating". [Thanks to Saeed Salehi.]

P. 42 (footnote). "Zielger" should read "Ziegler". The entry in the Bibliography should be emended similarly. [Thanks to Richard Zach.]

P. 96 (penultimate line of Proof for D). "$\mu x < g(n)$" should read "$\mu x \leq g(n)$". [Thanks to Saeed Salehi.]

p. 97 (second line of proof of (R1)). Replace "$sg(Sn \dot{-} m)$ and $sg(n \dot{-} m)$" by "$sg(Sm \dot{-} n)$ and $sg(m \dot{-} n)$". [Thanks to Curtis Brown.]

p. 111 (line 10, last line of indent). Replace "$\beta(b, i)$" by "$\beta(c, i)$". [Thanks to Curtis Brown.]

p. 115 (line 9). Replace "$f(m) = g(h(m))$" by "$f(m) = h(g(m))$". [Thanks to Russell Pannier.]

p. 115 (line 8 up). Replace "$\mathsf{G}(\vec{\mathsf{x}}, \mathsf{w})$ and $\mathsf{H}(\vec{\mathsf{x}}, \mathsf{u}, \mathsf{v}, \mathsf{w})$" by "$\mathsf{G}$ and H".

P. 132 (first para. of Proof for (R7)). "$2^{11} \cdot 3^4$" should read "$2^{13} \cdot 3^4$"; "$2^4 \cdot 3^{13} \cdot 5^{19}$" should read "$2^4 \cdot 3^{15} \cdot 5^{21}$"; "$2^{11} \cdot 3^4 \cdot 5^4 \cdot 7^{13} \cdot 11^{19}$" should read $2^{13} \cdot 3^4 \cdot 5^4 \cdot 7^{15} \cdot 11^{21}$. [Thanks to Duncan Watson.]

P. 133 (Proof for (R8)). "2^{19}" should read "2^{21}"; and on the next line"2^{21}" should read "2^{23}". [Thanks to Duncan Watson.]

P. 132 (line 8 up). "$(\forall i \leq len(n))$" should read "$(\forall i < len(n))$". [Thanks to Richard Zach.]

P. 165 (line 5 up). Delete "This sentence turns out to be Π_1."

P. 166 (line 8). After "*is enough.*" insert "And Rosser's clever idea can in fact be used to construct a Π_1 undecidable sentence."

P. 176 (ine 11). Replace "the fixed point will be of Goldbach type" with "there will be a fixed point of Goldbach type".

P. 184 (line 8). Replace "the proofs of *old* theorems" with "the proofs of some *old* theorems".

P. 196 (last paragraph). Replace with

> Assuming $\mathcal{I}_{2A}$ is a model for PA_2 (so the theory is consistent), the axioms of PA_2 are enough to *semantically entail* all true sentences of L_{2A}. But the Gödel-Rosser Theorem tells us this formal deductive theory is not strong enough to *prove* all true L_{2A} sentences – and we can't expand PA_2 so as to prove them all either, so long as the expanded theory remains consistent and properly axiomatized. However, make the distinction between what is *semantically entailed* and what is *deductively proved*, and we reconcile the apparent conflict between the implication of Dedekind's categoricity result ('PA_2 settles all the truths') and Gödelian incompleteness ('PA_2 leaves some truths undecided').

[Thanks to Lee Corbin for spotting an errant 'entail' for 'prove'.]

P. 207 (eight lines up). Replace " wffs of an formal language" by "wffs of a formal language". [Thanks to Lee Corbin.]

P. 210 (line 18). Line ending "more L_A sentences than PA." should end with a question mark. [Thanks to Lee Corbin.]

P. 223 (line after D'). "we are prove" should read "we are to prove". [Thanks to Saeed Salehi.]

P. 229 (penultimate line of §25.6). "half that" should read "half of that". [Thanks to Saeed Salehi.]

P. 238. Both the formulae at lines 1 and 3 should end with a period. Similarly for the formula at line 8 of (c).

P. 242 (the two intended formulae both(!) numbered (ii)). There are misplaced subscript Ts. In the first, "$Prf(x, y)_T$" should read "$Prf_T(x, y)$"; in the second "$\mathsf{Prf}(\mathsf{x}, \mathsf{y})_T$" should read "$\mathsf{Prf}_T(\mathsf{x}, \mathsf{y})$". [Thanks to Saeed Salehi.]

P. 242. The numbering of formulae on this page should be rectified so (ii) isn't repeated!

P. 260 (second line of (b)). Replace "Of we course" by "Of course". [Thanks to Dave Chambers.]

P. 262 (penultimate line of main text). Replace "Wang (1974," by "(Wang 1974,". [Thanks to Stuart Smith.]

P. 269 (line 12). Replace "as n increases" by "as x increases".

P. 271 (footnote). Add reference to Hedman (2004, pp. 307–309). Add corresponding item to Bibliography: Hedman, S., 2004. *A First Course in Logic*. Oxford, Oxford University Press.

P. 291 (line 9 up of main text). Replace "$f(n)$" by "$g(m, n)$". [Thanks to Saeed Salehi.]

P. 308 (first two lines of §33.4). Replace "is not recursively unsolvable" by "is not recursively solvable". [Thanks to Rob Trueman]

P. 309 (line after Theorem 33.5). Replace "gives us the proof" by "gives us the second proof". [That's to co-ordinate with the revised p. 286.]

P. 348 The entries for Frege (1891) and Gandy (1988) are lacking publication location. Add "Oxford" in each case.

P. 359 (line 4, column 2). The page reference for 'nice theory' is in fact 151.