

Which of the following wffs are tautologies, which contradictions, which neither?

- 1 $(P \wedge ((\neg P \vee Q) \wedge \neg Q))$
- 2 $((Q \wedge (\neg\neg P \vee R)) \vee \neg\{P \wedge Q\})$
- 3 $((P \vee \neg(Q \wedge R)) \vee ((\neg P \wedge Q) \wedge R))$
- 4 $((P \vee (Q \wedge \neg R)) \vee \neg((\neg P \vee R) \vee Q))$
- 5 $((P \wedge (\neg Q \vee \neg R)) \vee \neg((P \vee \neg R) \vee \neg Q))$
- 6 $\neg(\{(\neg(P \wedge \neg R) \wedge \neg(Q \wedge \neg S)) \wedge \neg\{(\neg(P \vee Q) \vee (R \vee S))\})$

1 is a contradiction:

P	Q	$(P \wedge ((\neg P \vee Q) \wedge \neg Q))$
T	T	T F F
T	F	T F F
F	T	F F F
F	F	F F F

1 2,5 3 4 2

2 is a tautology

P	Q	R	$((Q \wedge (\neg\neg P \vee R)) \vee \neg\{P \wedge Q\})$
T	T	T	T T T T F
T	T	F	T T T T F
T	F	T	T T T T
T	F	F	T T T T
F	T	T	F F F T T
F	T	F	F F F T T
F	F	T	F F F T T
F	F	F	F F F T T

5 6 4 2,7 1

3 is a tautology

P	Q	R	$((P \vee \neg(Q \wedge R)) \vee ((\neg P \wedge Q) \wedge R))$
T	T	T	T T T
T	T	F	T T T
T	F	T	T T T
T	F	F	T T T
F	T	T	F F F T T
F	T	F	F F F T T
F	F	T	F F F T T
F	F	F	F F F T T

1 3 2 4,7 6 5

4 is neither a tautology nor a contradiction

P	Q	R	$((P \vee (Q \wedge \neg R)) \vee \neg((\neg P \vee R) \vee Q))$
T	T	T	T T T
T	T	F	T T T
T	F	T	T T T
T	F	F	T T T
F	T	T	F F F F T
F	T	F	F F F T
F	F	T	F F F F T
F	F	F	F F F F T

1 3 2 4,7 6 5

5 is neither—and strictly, to settle the question, all we need is one line that comes out true and one line that comes out false, thus:

P	Q	R	$((P \wedge (\neg Q \vee \neg R)) \vee \neg((P \vee \neg R) \vee \neg Q))$			
T	T	T				
T	T	F				
T	F	T	T	T	T	<u>I</u>
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F	F	F	<u>F</u>	F T T

6 is a tautology

P	Q	R	S	$\neg(\{\neg(P \wedge \neg R) \wedge \neg(Q \wedge \neg S)\} \wedge \neg\{\neg(P \vee Q) \vee (R \vee S)\})$									
T	T	T	T	<u>I</u>			F	F		T	T		
T	T	T	F	<u>I</u>			F	F		T	T		
T	T	F	T	<u>I</u>			F	F		T	T		
T	T	F	F	<u>I</u>	F	F	F	T	F	T	F	F	
T	F	T	T	<u>I</u>			F	F		T	T		
T	F	T	F	<u>I</u>			F	F		T	T		
T	F	F	T	<u>I</u>			F	F		T	T		
T	F	F	F	<u>I</u>	F	F	F	T	F	T	F	F	
F	T	T	T	<u>I</u>			F	F		T	T		
F	T	T	F	<u>I</u>			F	F		T	T		
F	T	F	T	<u>I</u>			F	F		T	T		
F	T	F	F	<u>I</u>	T	FF	F	T	F	T	F	F	
F	F	T	T	<u>I</u>			F	F		T	T		
F	F	T	F	<u>I</u>			F	F		T	T		
F	F	F	T	<u>I</u>			F	F		T	T		
F	F	F	F	<u>I</u>			F	F	T	F	T	F	
				12	8	10	9	7,11	6	4	3	2,5	1