Answers to Exercises 16

1. We start assuming that the premisses and the negation of the conclusion are all true:

(1) $(R \lor P) \Rightarrow T$
(2) $(R \lor Q) \Rightarrow T$
(3) $\neg(P \lor Q) \Rightarrow T$

A negated disjunction is true (i.e. the disjunction is false) if each disjunct is false, i.e.

(4) $\neg P \Rightarrow T$
(5) $\neg Q \Rightarrow T$

The truth of the disjunction (1) gives us the alternatives

(6)	$R \Rightarrow T$	$P \Rightarrow T$
		✻

The right-hand branch immediately yields a contradiction: then we can use the disjunction (2) to get

(7)	$R \Rightarrow T$	$Q \Rightarrow T$
		✻

Which leaves the open branch where $P \Rightarrow F$, $Q \Rightarrow F$ and $R \Rightarrow T$ (which indeed makes the original premisses of the argument true and conclusion false – check that!). *Invalid argument*

2. Again, assume the premisses and the negation of the conclusion are all true:

(1) $\neg(P \land \neg Q) \Rightarrow T$
(2) $P \Rightarrow T$
(3) $\neg Q \Rightarrow T$

A negated conjunction is true (i.e. the conjunction is false) if one of the conjuncts is false, i.e.

(4)	$\neg P \Rightarrow T$	$\neg\neg Q \Rightarrow T$
	✻	✻

Both branches immediately lead to contradiction, since each branch contains a pair of wffs of the form *A*, ¬*A* each supposed to be true. *Valid argument*

3. As always, assume the premisses and the negation of the conclusion are all true:

(1) $(\neg P \lor Q) \Rightarrow T$
(2) $(R \lor P) \Rightarrow T$
(3) $\neg(R \land Q) \Rightarrow T$

There is no right order for 'unpacking' these wffs: start, say, with the last …. That yields

(4)	$\neg R \Rightarrow T$	$\neg Q \Rightarrow T$

Now we have to explore these alternative ways of making (3) correct; unpacking the first wff, we consider its implications for each alternative:

(5)	$\neg P \Rightarrow T$	$Q \Rightarrow T$	$\neg P \Rightarrow T$	$Q \Rightarrow T$
				✻

We can close off one branch (only one!). Now to explore the remaining three branches, we unpack (2), adding its implications to every open branch:

(6)	$P \Rightarrow T$	$R \Rightarrow T$	$P \Rightarrow T$	$R \Rightarrow T$	$P \Rightarrow T$	$R \Rightarrow T$
	✻	✻		✻	✻	

That leaves two open branches: one on which $P \Rightarrow T$, $Q \Rightarrow T$, $R \Rightarrow F$, the other where $P \Rightarrow F$, $Q \Rightarrow F$, $R \Rightarrow T$. A quick check confirms that those two valuations make the premisses true and conclusion false. *Invalid argument*

4.

(1)	$((P \wedge \neg Q) \vee R) \Rightarrow T$	
(2)	$(Q \vee S) \Rightarrow T$	
(3)	$\neg(R \wedge S) \Rightarrow T$	
(4)	$(P \wedge \neg Q) \Rightarrow T$	$R \Rightarrow T$
(5)	$P \Rightarrow T$	
(6)	$\neg Q \Rightarrow T$	

Here we've unpacked the wff on the right at line (4) [why do we only add those conjuncts to the right-hand branch?]. Now we consider the implications of (2) for each branch

(7)	$Q \Rightarrow T$ ✲	$S \Rightarrow T$	$Q \Rightarrow T$	$S \Rightarrow T$

Which leaves us to unpack (3)

(8)	$\neg R \Rightarrow T$	$\neg S \Rightarrow T$ ✲	$\neg R \Rightarrow T$ ✲	$\neg S \Rightarrow T$	$\neg R \Rightarrow T$ ✲	$\neg S \Rightarrow T$ ✲

Again there are open branches, so this is an *invalid argument*.

5. This time, let's leave off the explicit assignments of truth:

(1)	P
(2)	$\neg(P \wedge \neg Q)$
(3)	$\neg(Q \wedge \neg R)$
(4)	$(\neg R \wedge S)$
(5)	$\neg S$

Unpacking (4) gives us

(6)	$\neg R$
(7)	S ✲

And we are done already! A wise choice about what to unpack first closes the tree fast! (I doubt that this was intended; a better example would have had $(\neg R \vee S)$ – leading still to a closed tree, but a more complex one.)

6. Again, we start with the premisses and negated conclusion:

(1)	$((P \wedge Q) \vee (R \wedge P))$
(2)	$\neg((Q \wedge R) \vee P)$

Let's unpack the second first, as this doesn't immediately lead to a branching tree

(3)	$\neg(Q \wedge R)$	
(4)	$\neg P$	
(5)	$(P \wedge Q)$	$(R \wedge P)$
(6)	P	R
(7)	Q ✲	P ✲

A valid argument.