Answers to Exercises 24

A Suppose 'm' denotes Myfanwy, 'n' denotes Ninian, 'o' denotes Olwen, 'Fx' means *x is a philosopher*, 'Gx' means *x speaks Welsh*, 'Lxy' means *x loves y*, and 'Rxyz' means that *x is a child of y and z*. Take the domain of discourse to consist of human beings. Translate the following into **QL**:

1 Ninian is loved by Myfanwy and Olwen
(Lmn ∧ Lon)

2 Neither Myfanwy nor Ninian love Olwen
¬(Lmo ∨ Lno) *or* (¬Lmo ∧ ¬Lno)

3 Someone is a child of Myfanwy and Ninian
∃xRxmn

4 No philosopher loves Olwen
¬∃x(Fx ∧ Lxo) *or* ∀x(Fx ⊃ ¬Lxo)

5 Myfanwy and Ninian love everyone
∀x(Lmx ∧ Lnx) *or* (∀xLmx ∧ ∀xLnx)

6 Some philosophers speak Welsh
∃x(Fx ∧ Gx)

7 No Welsh-speaker who loves Myfanwy is a philosopher
¬∃x((Gx ∧ Lxm)) ∧ Fx) *or* ∀x((Gx ∧ Lxm)) ⊃ ¬Fx)

8 Some philosophers love both Myfanwy and Olwen
∃x(Fx ∧ (Lxm ∧ Lxo))

9 Some philosophers love every Welsh speaker
∃x(Fx ∧ ∀y(Gy ⊃ Lxy))

10 Everyone who loves Ninian is a philosopher who loves Myfanwy
∀x(Lxn ⊃ (Fx ∧ Lxm))

11 Some philosopher is a child of Olwen and someone or other
∃x(Fx ∧ ∃yRxoy)

12 Whoever is a child of Myfanwy and Ninian loves them both
∀x(Rxmn ⊃ (Lxm ∧ Lxn))

13 Everyone speaks Welsh only if Olwen speaks Welsh
(∀xGx ⊃ Go) [*not* ∀x(Gx ⊃ Go), *which isn't equivalent*]

14 Myfanwy is a child of Ninian and of someone who loves Ninian
∃x(Rmnx ∧ Lxn)

[oops, first printing has 'Bethan' etc. Sorry! Using obvious translation, that would be
∃x(Rbcx ∧ Lxc)]

15 Some philosophers love no Welsh speakers
∃x(Fx ∧ ¬∃y(Gy∧ Lxy)) *or* ∃x(Fx ∧ ∀y(Gy ⊃ ¬Lxy))

16 Every philosopher who speaks Welsh loves Olwen
∀x((Fx ∧ Gx) ⊃ Lxo))

17 Every philosopher who speaks Welsh loves someone who loves Olwen
∀x((Fx ∧ Gx) ⊃ ∃y(Lxy ∧ Lyo))

18 If Ninian loves every Welsh speaker, then Ninian loves Myfanwy
(∀x(Gx ⊃ Lnx) ⊃ Lnm)

19 No Welsh speaker is loved by every philosopher
¬∃x(Gx ∧ ∀y(Fy ⊃ Lyx)) *or* ∀x(Gx ⊃ ∃y(Fy ∧ ¬Lyx))

20 Every Welsh speaker who loves Ninian loves no one who loves Olwen
∀x((Gx ∧ Lxn) ⊃ ¬∃y(Lxy ∧ Lyo))

21 Whoever loves Myfanwy, loves a philosopher only if the latter loves Myfanwy too
∀x(Lxm ⊃ ∀y((Lxy ∧ Fy) ⊃ Lym))

22 Anyone whose parents are a philosopher and someone who loves a philosopher is a philosopher too.
∀x({∃y∃zRxyz ∧ [y is a philosopher and z loves a philosopher]} ⊃ Fx)
∀x((∃y∃zRxyz ∧ (Fy ∧ ∃w(Lzw ∧ Fw))) ⊃ Fx)

23 Only if Ninian loves every Welsh-speaking philosopher does Myfanwy love him
(Lmn ⊃ ∀x((Fx ∧ Gx) ⊃ Lnx))

24 No philosophers love any Welsh-speaker who has no children
∀x(Fx ⊃ ¬∃y(Lxy ∧ {Gy ∧ y has no children}))
∀x(Fx ⊃ ¬∃y(Lxy ∧ (Gy ∧ ¬∃z∃wRzyw)))

B Take the domain of quantification to be the (positive whole) numbers, and let ‘n’ denote the number one, ‘Fx’ mean *x is odd*, ‘Gx’ mean *x is even*, ‘Hx’ mean *x is prime*, ‘Lxy’ mean *x is greater than y*, ‘Rxyz’ mean that *x is the sum of y and z*. Then translate the following from **QL** into natural English:

1 ¬∃x(Fx ∧ ¬Gx)
⇒ No odd number is not even [*which is false! to get a truth, which is what I'd intended, delete the second negation in both (1) and its translation!*]

2 ∀x∀y∃zRzxy
⇒ Every pair of numbers has a sum

3 ∀x∃yLyx
⇒ For any number, there's a larger one

4 ∀x∀y((Fx ∧ Ryxn) ⊃ Gy)
⇒ If a number is one more than an odd number, then it is even.

5 ∀x∀y((Gx ∧ Rxyn) ⊃ Fy)
⇒ If a number is one less than an even number, then it is odd.

6 ∀x∃y((Gx ∧ Fy) ∧ Rxyy)
⇒ Any even number is equal to twice some odd number (*more literally*: any even number is equal to some odd number added to itself – false of course!

7 ∀x∀y(∃z(Rzxn ∧ Ryzn) ⊃ (Gx ⊃ Gy))
⇒ If two numbers differ by two, then if one is even, so is the other.

8 ∀x∀y∀z(((Fx ∧ Fy) ∧ Rzxy) ⊃ Gz)
⇒ The sum of two odd numbers is even.

9 ∀x(Gx ⊃ ∃y∃z((Hy ∧ Hz) ∧ Rxyz))
⇒ Every even number is the sum of two primes. [*Goldbach's conjecture*]

10 ∀w∃x∃y(((Hx ∧ Hy) ∧ (Lxw ∧ Lyw)) ∧ ∃z(Rzxn ∧ Ryzn))
⇒ Take any number, then there is a pair of primes larger than it which differ by two. [*The twin primes conjecture*]

C Which of the following pairs are equivalent, and why?

1. ∀x(Fx ⊃ Gx); (∀xFx ⊃ ∀xGx)

Interpret ‘F’ as *man*, ‘G’ as *woman*, and take the domain to be *people*. Then ‘∀x(Fx ⊃ Gx)’ is false; but ‘∀xFx’ and ‘∀xGx’ are both false so ‘(∀xFx ⊃ ∀xGx)’ is true. So these wffs are not equivalent.

2. ∃x(Fx ⊃ Gx); (∃xFx ⊃ ∃xGx)

Interpret ‘F’ as *horse*, ‘G’ as *unicorn*, and take the domain to be *living creatures*. Then ‘∃xFx’ is true, and ‘∃xGx’ is false so ‘(∃xFx ⊃ ∃xGx)’ is false. Suppose ‘a’ denotes a dog in the domain; then ‘Fa’ is false, as is ‘Ga’, so ‘(Fa ⊃ Ga)’ is true, so ‘∃x(Fx ⊃ Gx)’ is true. So these wffs are not equivalent.

3. ∃x(Fx ⊃ Gx); (∀xFx ⊃ ∃xGx)

Equivalent: for consider this chain ∃x(Fx ⊃ Gx) ≡ ¬∀x¬(Fx ⊃ Gx) ≡ ¬∀x(Fx ∧ ¬Gx) ≡ ¬(∀xFx ∧ ∀x¬Gx) ≡ (∀xFx ⊃ ¬∀x¬Gx) ≡ (∀xFx ⊃ ∃xGx) – which relies on the equivalence of

$\forall x(Ax \wedge Bx)$ and $(\forall xAx \wedge \forall xBx)$.

4. $\forall x(Fx \supset Gx)$; $(\exists xFx \supset \forall xGx)$

Take the domain to be living things, interpret 'F' as *man*, 'G' as *human*. Then '$\forall x(Fx \supset Gx)$' is true and '$(\exists xFx \supset \forall xGx)$' false, so the wffs are not equivalent.

Q: The claim that, e.g., that a wff of the form $(A \vee \exists xFx)$ is equivalent to one of the form $\exists x(A \vee Fx)$ depends on our stipulation that the domain of quantification isn't empty. Why?

A: Because in an empty domain, $\exists xC$ is always false; so if A is true, $(A \vee \exists xFx)$ is true but $\exists x(A \vee Fx)$ is false; so the wffs aren't equivalent.

Q: Which other equivalences we stated in §24.3 above also depend on that stipulation?

A: Similarly, if A is false, $(A \supset \exists xFx)$ is true and $\exists x(A \supset Fx)$ false, so those are no equivalent in empty domains. The other equivalences stated remain correct.