A *Use* **QL**$^=$ *trees to show the following inferences are valid:*

1. *Jack* (m) *is Fingers* (n). *Fingers is never caught* (F). *Whoever is never caught* (F) *escapes justice* (G). *So Jack escapes justice.*

(1)		$m = n$			
(2)		Fn			
(3)		$\forall x(Fx \supset Gx)$			
(4)		$\neg Gm$			Negated conclusion
(5)		$(Fn \supset Gn)$		√	
(6)	$\neg Fn$		Gn		
(7)	✻		Gm		By LL, from 1, 6
			✻		

where LL is, of course, Leibniz's Law. Check that you understand why there is just the single check mark here!

2. *There is a wise philosopher. There is a philosopher who isn't wise. So there are at least two philosophers.*

We could choose the domain of quantification to be philosophers. Then the translation would be simply:

$\exists xFx,\ \exists x\neg Fx\ \therefore\ \exists x\exists y\neg x = y$

If we'd chosen our usual domain of humans, we'd need to carve out the philosophers by some predicate, as in:

$\exists x(Gx \wedge Fx),\ \exists x(Gx \wedge \neg Fx)\ \therefore\ \exists x\exists y((Gx \wedge Gy) \wedge \neg x = y)$

Here's the tree for the first, simpler translation:

(1)	$\exists xFx$		
(2)	$\exists x\neg Fx$		
(3)	$\neg\exists x\exists y\neg x = y$		Negated conclusion

We follow the usual rule-of-thumb, deal with negated quantifiers first, then instantiate the existentials (each with a new name of course) – thereby now checking each of (1) to (3):

(4)	$\forall x\neg\exists y\neg x = y$		
(5)	Fa		
(6)	$\neg Fb$		

Now instantiate the universal quantification of (4)

(7)	$\neg\exists y\neg a = y$	√	
(8)	$\forall y\neg\neg a = y$		From 7
(9)	$\neg\neg a = b$	√	From 8
(10)	$a = b$		
(11)	Fb		From 5, 10 by LL
	✻		

The tree for the more complex translation involves basically the same moves, but with a little more detail added …:

(1)	$\exists x(Gx \wedge Fx)$	√	
(2)	$\exists x(Gx \wedge \neg Fx)$	√	
(3)	$\neg\exists x\exists y((Gx \wedge Gy) \wedge \neg x = y)$	√	Negated conclusion
(4)	$\forall x\neg\exists y((Gx \wedge Gy) \wedge \neg x = y)$		
(5)	$(Ga \wedge Fa)$	√	
(6)	$(Gb \wedge \neg Fb)$	√	
(7)	Ga		

(8) Fa
(9) Gb
(10) $\neg Fb$
(11) $\neg\exists y((Ga \wedge Gy) \wedge \neg a = y)$ √
(12) $\forall y\neg((Ga \wedge Gy) \wedge \neg a = y)$
(13) $\neg((Ga \wedge Gb) \wedge \neg a = b)$ √

(14) $\neg(Ga \wedge Gb)$ √ | $a = b$
 | Fb

(15) $\neg Ga$ | $\neg Gb$ | *
 * | * |

3. *Whoever stole the goods* (F)*, knew the safe combination* (G)*. Someone stole the goods, and only Jack knew the safe combination. Hence Jack stole the goods.*

(1) $\forall x(Fx \supset Gx)$
(2) $(\exists xFx \wedge (Gn \wedge \forall x(Gx \supset x = n)))$
(3) $\neg Fn$ Negated conclusion

Now check off (2) and unpack the conjuncts in (2)

(4) $\exists xFx$
(5) $(Gn \wedge \forall x(Gx \supset x = n))$ √
(6) Gn
(7) $\forall x(Gx \supset x = n)$

We have universal quantifications at (1) and (7), and an existential at (6). So we follow the usual rule of thumb, instantiate existentials first to get a new name in play, and use that to instantiate the universals, thus…

(8) Fa
(9) $(Fa \supset Ga)$
(10) $(Ga \supset a = n)$

And now the tree quickly closes:

(11) $\neg Fa$ | Ga
 * |

(12) $\neg Ga$ | $a = n$ From 8, 12 by LL
(13) * | Fn
 | *

4. *For every number, there's a larger one. No number is larger than itself. So for every number, there's a distinct number which is larger than it.*

Take the domain of quantification to be numbers. Use 'Rxy' for 'x is larger than y'. Then the argument translates:

$\forall x\exists yRyx,\ \neg\exists xRxx \therefore \forall x\exists y(\neg x = y \wedge Ryx)$

So the tree is

(1) $\forall x\exists yRyx$
(2) $\neg\exists xRxx$ √
(3) $\neg\forall x\exists y(\neg x = y \wedge Ryx)$ √ Negated conclusion
(4) $\forall x\neg Rxx$ From 2
(5) $\exists x\neg\exists y(\neg x = y \wedge Ryx)$ From 3

This leaves us with two universals and an existential quantification as unchecked propositions. So of course we instantiate the existential first to give …

(6)	$\neg\exists y(\neg a = y \wedge Rya)$		From 5
(7)	$\forall y\neg(\neg a = y \wedge Rya)$		From 6
(8)	$\exists y Rya$		From 1
(9)	$\neg Raa$		From 4

This again leaves us with two universals and an existential quantification as unchecked propositions. So again we instantiate the existential first to give …

(10)	Rba	From 8

We have a new name to play with and the obvious next move is to instantiate (7) with it to get

(11)	$\neg(\neg a = b \wedge Rba)$		From 7
(12)	$\neg\neg a = b$	$\neg Rba$	
(13)	$a = b$	✻	From 12
(14)	Raa		From 10, 13 by LL
	✻		

5. *No one who isn't Bryn loves Angharad. At least one person loves Angharad. So Bryn loves Angharad.*

We can just use 'Fx' for 'x loves Angharad' (as the internal structure of that predicate isn't relevant to the argument):

(1)	$\forall x(\neg x = b \supset \neg Fx)$		
(2)	$\exists x Fx$	√	
(3)	$\neg Fb$		Negated conclusion
(4)	Fa		From 2
(5)	$(\neg a = b \supset \neg Fa)$		From 1
(6)	$\neg\neg a = b$	$\neg Fa$	
(7)	$a = b$	✻	
(8)	Fa		
	✻		

6. *Exactly one person admires Frank. All and only those who admire Frank love him. Hence exactly one person loves Frank.*

Use 'F' for 'admires Frank', and 'G' for 'loves Frank'. 'All and only Fs are Gs' can be translated as '$\forall x(Fx \equiv Gx)$' or equivalently $\forall x((Fx \supset Gx) \wedge (Gx \supset Fx))$.

(1)	$\exists x(Fx \wedge \forall y(Fy \supset y = x))$		
(2)	$\forall x(Fx \equiv Gx)$		
(3)	$\neg\exists x(Gx \wedge \forall y(Gy \supset y = x))$	√	Negated conclusion
(4)	$\forall x\neg(Gx \wedge \forall y(Gy \supset y = x))$		From 3

Once again, this leaves us with two universals and an existential quantification as unchecked propositions. So again we instantiate the existential first to give …

(5)	$(Fa \wedge \forall y(Fy \supset y = a))$	From 1
(6)	$(Fa \equiv Ga)$	From 2
(7)	$\neg(Ga \wedge \forall y(Gy \supset y = a))$	From 4

Next, on the principle 'apply non-branching connective rules first', we must next check off (5) then (6):

(8)	Fa		
(9)	$\forall y(Fy \supset y = a)$		
(10)	Fa	$\neg Fa$	From 6
(11)	Ga	$\neg Ga$	From 6
		✻	

Now we continue the left-hand branch by unpacking (7):

(12)	¬Ga	¬∀y(Gy ⊃ y = a)	
(13)	*	∃y¬(Gy ⊃ y = a)	From 12

Next, to make use of (13) we instantiate the quantifier with a new name

(14)	¬(Gb ⊃ b = a)	From 13
(15)	Gb	From 14
(16)	¬b = a	From 14

We have two universal quantifications, at (9) and (2) to instantiate with the new name 'b'.

(17)	(Fb ⊃ b = a)		From 9
(18)	¬Fb	b = a	
(19)	(Fb ≡ Gb)	*	

(20)	Fb	¬Fb
(21)	Gb	¬Gb
	*	*

Here, without much commentary, is another tree for an alternative translation of the original argument:

(1)	∃x∀y(Fy ≡ y = x)	√	
(2)	∀x(Fx ≡ Gx)		
(3)	¬∃x∀y(Gy ≡ y = x)	√	Negated conclusion
(4)	∀x¬∀y(Gy ≡ y = x)		From 3
(5)	∀y(Fy ≡ y = a)		From 1
(6)	¬∀y(Gy ≡ y = a)	√	From 4
(7)	∃x¬(Gy ≡ y = a)	√	From 6
(8)	¬(Gb ≡ b = a)		From 7
(9)	(Fb ≡ Gb)		From 2
(10)	(Fb ≡ b = a)		From 5

(12)	Fb			¬Fb			From 9
(13)	Gb			¬Gb			From 9
(14)	Gb		¬Gb	Gb	¬Gb		From 8
(15)	¬b = a		b = a	¬b = a	b = a		From 8
			*	*			
(16)	Fb	¬Fb			Fb	¬Fb	From 10
(17)	b = a	¬b = a			b = a	¬b = a	From 10
	*	*			*	*	

7. *The present King of France is bald. Bald men are sexy. Hence whoever is a present King of France is sexy.*

(1)	∃x((Fx ∧ ∀y(Fy ⊃ y = x)) ∧ Gx)		
(2)	∀x(Gx ⊃ Hx)		
(3)	¬∀x(Fx ⊃ Hx)	√	Negated conclusion
(4)	∃x¬(Fx ⊃ Hx)		From 3

We now have two existential quantifications to instantiate, with different names of course:

(5)	¬(Fa ⊃ Ha)	From 1
(6)	((Fb ∧ ∀y(Fy ⊃ y = b)) ∧ Gb)	From 4

We can now disassemble (5) and (6) and check them off to get …

(7)	Fa	
(8)	$\neg Ha$	
(9)	$(Fb \wedge \forall y(Fy \supset y = b))$	√
(10)	Gb	
(11)	Fb	
(12)	$\forall y(Fy \supset y = b)$	

We have two universal quantifications to use, at lines (2) and (12): the obvious instantiations to make are …

(13)	$(Gb \supset Hb)$
(14)	$(Fa \supset a = b)$

And the rest is straightforward:

(16)	$\neg Gb$ ✲	Hb	
(17)		Fa / $a = b$	
(18)		✲ / $\neg Hb$ ✲	From 8 and 18, by LL

Or, starting from an alternative translation for *The present King of France is bald* …

(1)	$\exists x \forall y((Fy \equiv y = x) \wedge Gx)$	√	
(2)	$\forall x(Gx \supset Hx)$		
(3)	$\neg \forall x(Fx \supset Hx)$	√	Negated conclusion
(4)	$\exists x \neg (Fx \supset Hx)$	√	From 3
(5)	$\neg (Fa \supset Ha)$	√	From 1
(6)	$\forall y((Fy \equiv y = b) \wedge Gb)$		From 4
(7)	$((Fa \equiv a = b) \wedge Gb)$	√	From 6
(8)	$(Gb \supset Hb)$	√	From 2
(9)	$(Fa \equiv a = b)$		From 7
(10)	Gb		From 7
(11)	Fa		From 5
(12)	$\neg Ha$		From 5

(13)	$\neg Gb$ ✲	Hb	
(14)		Fa / $\neg Fa$	From 9
(15)		$a = b$ / $\neg a = b$	From 9
(16)		$\neg Hb$ / ✲	From 12, 15 by LL

8. *Angharad loves only Bryn and Caradoc, who are different people. So Angharad loves exactly two people.*

Let's use 'Fx' for 'Angharad loves *x*', then we have

(1)	$\{((Fb \wedge Fc) \wedge \neg b = c) \wedge \forall x(Fx \supset (x = b \vee x = c))\}$		
(2)	$\neg \exists y \exists z\{((Fy \wedge Fz) \wedge \neg y = z) \wedge \forall x(Fx \supset (x = y \vee x = z))\}$	√	Negated conclusion
(3)	$\forall y \neg \exists z\{((Fy \wedge Fz) \wedge \neg y = z) \wedge \forall x(Fx \supset (x = y \vee x = z))\}$		From 2
(4)	$\neg \exists z\{((Fb \wedge Fz) \wedge \neg b = z) \wedge \forall x(Fx \supset (x = b \vee x = z))\}$	√	From 3
(5)	$\forall z \neg\{((Fb \wedge Fz) \wedge \neg b = z) \wedge \forall x(Fx \supset (x = b \vee x = z))\}$		From 4
(6)	$\neg\{((Fb \wedge Fc) \wedge \neg b = c) \wedge \forall x(Fx \supset (x = b \vee x = c))\}$ ✲		From 5

This closes particularly quickly because of a cunning choice of variables in our initial translation. Suppose we had started with the translations

(1)	$\{((Fb \land Fc) \land \neg b = c) \land \forall x(Fx \supset (x = b \lor x = c))\}$		
(2)	$\neg\exists x\exists y\{((Fx \land Fy) \land \neg x = y) \land \forall z(Fz \supset (z = x \lor z = y))\}$	√	Negated conclusion
(3)	$\forall x\neg\exists y\{((Fx \land Fy) \land \neg x = y) \land \forall z(Fz \supset (z = x \lor z = y))\}$		From 2
(4)	$\neg\exists y\{((Fb \land Fy) \land \neg b = y) \land \forall z(Fz \supset (z = b \lor z = y))\}$	√	From 3
(5)	$\forall y\neg\{((Fb \land Fy) \land \neg b = y) \land \forall z(Fz \supset (z = b \lor z = y))\}$		From 4
(6)	$\neg\{((Fb \land Fc) \land \neg b = c) \land \forall z(Fz \supset (z = b \lor z = c))\}$		From 5

And now we can't immediately close the tree as (1) and (6) aren't literal contradictions. So we now check off (1) and (6) and unpack them to get

(7)	$((Fb \land Fc) \land \neg b = c)$			From 1
(8)	$\forall x(Fx \supset (x = b \lor x = c))$			From 1
(9)	$\neg((Fb \land Fc) \land \neg b = c)$	$\neg\forall z(Fz \supset (z = b \lor z = c))$	√	From 6
(10)	✻	$\exists z\neg(Fz \supset (z = b \lor z = c))$	√	From 9
(11)		$\neg(Fa \supset (a = b \lor a = c))$		From 10
(12)		$(Fa \supset (a = b \lor a = c))$		From 8
		✻		

9. *Juliet kisses exactly two philosophers. Hence there is more than one philosopher.*

Let's use 'Fx' for 'Juliet kisses *x*' and take the domain to be philosophers. Then we get the simple translation

(1)	$\exists x\exists y\{((Fx \land Fy) \land \neg x = y) \land \forall z(Fz \supset (z = x \lor z = y))\}$	
(2)	$\neg\exists x\exists y\,\neg x = y$	Negated conclusion

Instantiating (1) and then instantiating the result gives us

(3)	$\exists y\{((Fa \land Fy) \land \neg a = y) \land \forall z(Fz \supset (z = a \lor z = y))\}$
(4)	$\{((Fa \land Fb) \land \neg a = b) \land \forall z(Fz \supset (z = a \lor z = b))\}$

Unpacking the conjuntions in (4) gives us

(5)	$((Fa \land Fb) \land \neg a = b)$
(6)	$\forall z(Fz \supset (z = a \lor z = b))$
(7)	$(Fa \land Fb)$
(8)	$\neg a = b$

Now we can quickly expose a contradiction between (8) and (2): for from (2) we get …

(9)	$\forall x\neg\exists y\,\neg x = y$
(10)	$\neg\exists y\,\neg a = y$
(11)	$\forall y\neg\neg a = y$
(12)	$\neg\neg x = b$
	✻

Note we couldn't have used 'Kjx' for 'Juliet kisses *x*' and still taken the domain to be philosophers. For any name must name something in the domain, and we aren't given that Juliet is a philosopher! So we'd have to take a wider domain and then explicitly pick out the philosophers by a predicate (say 'G'), as in …

(1) $\exists x\exists y\{(([Ljx \land Gx] \land [Ljy \land Gy]) \land \neg x = y) \land \forall z([Ljz \land Gz] \supset (z = x \lor z = y))\}$

and everything gets more longwinded.

B *Show that the following wffs are q-logical truths*

1. ∀x∀y(x = y ⊃ (Fx ⊃ Fy))
2. ∀y∀z(y = z ⊃ (∀x(Lxy ∧ Fy) ⊃ ∀x(Lxz ∧ Fz))) [final closing bracket added!]

Thinking about the structure of those proofs, conclude that each way of filling out the following schema from §33.1 yields a q-logical truth:

(LS) ∀*v*∀*w*(*v* = *w* ⊃ (C(…*v*…*v*…) ⊃ C(…*w*…*w*…)))

We establish something is a q-logical truth using trees by assuming its negation and proving a contradiction, thus …

(1)	¬∀x∀y(x = y ⊃ (Fx ⊃ Fy))	√
(2)	∃x¬∀y(x = y ⊃ (Fx ⊃ Fy))	√
(3)	¬∀y(a = y ⊃ (Fa ⊃ Fy))	√
(4)	∃y¬(a = y ⊃ (Fa ⊃ Fy))	√
(5)	¬(a = b ⊃ (Fa ⊃ Fb))	√
(6)	a = b	
(7)	¬(Fa ⊃ Fb)	√
(8)	Fa	
(9)	¬Fb	
(10)	Fb	From 6, 8 by LL
	*	

(1)	¬∀y∀z(y = z ⊃ (∀x(Lxy ∧ Fy) ⊃ ∀x(Lxz ∧ Fz)))	√
(2)	∃y¬∀z(y = z ⊃ (∀x(Lxy ∧ Fy) ⊃ ∀x(Lxz ∧ Fz)))	√
(3)	¬∀z(a = z ⊃ (∀x(Lxa ∧ Fa) ⊃ ∀x(Lxz ∧ Fz)))	√
(4)	∃z¬(a = z ⊃ (∀x(Lxa ∧ Fa) ⊃ ∀x(Lxz ∧ Fz)))	√
(5)	¬(a = b ⊃ (∀x(Lxa ∧ Fa) ⊃ ∀x(Lxb ∧ Fb)))	√
(6)	a = b	
(7)	¬ (∀x(Lxa ∧ Fa) ⊃ ∀x(Lxb ∧ Fb))	√
(8)	∀x(Lxa ∧ Fa)	
(9)	¬∀x(Lxb ∧ Fb)	
(10)	∀x(Lxb ∧ Fb)	From 6, 8 by LL
	*	

The common pattern here can be exposed like this …

(1)	¬∀*v*∀*w*(*v* = *w* ⊃ (C(…*v*…*v*…) ⊃ C(…*w*…*w*…)))	√
(2)	∃*v*¬∀*w*(*v* = *w* ⊃ (C(…*v*…*v*…) ⊃ C(…*w*…*w*…)))	√
(3)	¬∀*w*(a = *w* ⊃ (C(…a…a…) ⊃ C(…*w*…*w*…)))	√
(4)	∃*w*(a = *w* ⊃ (C(…a…a…) ⊃ C(…*w*…*w*…)))	√
(5)	(a = b ⊃ (C(…a…a…) ⊃ C(…b…b…)))	√
(6)	a = b	
(7)	¬ (C(…a…a…) ⊃ C(…a…a…))	√
(8)	C(…a…a…)	
(9)	¬C(…a…a…)	
(10)	C(…a…a…)	From 6, 8 by LL
	*	

assuming 'a' and 'b' are new names, not occurring in the context C (if they do, we can use different names of course – and so in every case show an instance of (LS) is indeed a q-logical truth.

C *Returning to the labelled wffs and inferences earlier in this chapter, show that*

1. **G** *q-entails* **F**.
2. **L** *is q-valid.*
3. **M** *is q-valid.*

Finally, confirm that the two Russellian ways of translating The F is G, *via the schemas (R) and (R′) of §34.1, are indeed equivalent, by showing*

4. '∃x∀y((Fy ≡ y = x) ∧ Gx)' *q-entails* '∃x((Fx ∧ ∀y(Fy ⊃ y = x)) ∧ Gx)'.
5. '∃x((Fx ∧ ∀y(Fy ⊃ y = x)) ∧ Gx)' *q-entails* '∃x∀y((Fy ≡ y = x) ∧ Gx)'.

1. **G** q-entails **F**:

(1)	∃x∀y(Fy ≡ y = x)	√		**G**
(2)	¬∃x(Fx ∧ ∀y(Fy ⊃ y = x))	√		Negation of **F**
(3)	∀x¬(Fx ∧ ∀y(Fy ⊃ y = x))			
(4)	∀y(Fy ≡ y = a)			From 1
(5)	¬(Fa ∧ ∀y(Fy ⊃ y = a))	√		From 3

Branching at (5):

(6)	¬Fa		¬∀y(Fy ⊃ y = a)	
(7)	(Fa ≡ a = a) √ From 4		∃y¬(Fy ⊃ y = a)	

Left branch, branching at (7):

(8)	Fa	¬Fa	From 7
(9)	a = a	¬a = a	From 7
(10)	✻	✻	

Right branch:

(8)	¬(Fb ⊃ b = a) √		
(9)	Fb		
(10)	¬b = a		
(11)	(Fb ≡ b = a)		From 4

Branching at (11):

(12)	Fb	¬Fb	From 11
(13)	b = a	¬b = a	From 11
	✻	✻	

2. **L** *is q-valid (where* **L** *is the argument 'The author of the Iliad wrote the Odyssey. Hence at least one person wrote the Iliad'*).

(1)	∃x((Fx ∧ ∀y(Fy ⊃ y = x)) ∧ Gx)	√	
(2)	¬∃xFx	√	Negation of conclusion
(3)	∀x¬Fx		
(4)	((Fa ∧ ∀y(Fy ⊃ y = a)) ∧ Ga)	√	From 1
(5)	(Fa ∧ ∀y(Fy ⊃ y = a))	√	From 4
(6)	Ga		From 4
(7)	Fa		From 5
(8)	∀y(Fy ⊃ y = a)		From 5
(9)	¬Fa		From 3
	✻		

3. **M** *is q-valid (where* **M** *is the argument 'The author of the Iliad wrote the Odyssey. Hence whoever wrote the Iliad wrote the Odyssey.'*).

(1)	∃x((Fx ∧ ∀y(Fy ⊃ y = x)) ∧ Gx)	√	
(2)	¬∀x(Fx ⊃ Gx)	√	Negation of conclusion
(3)	∃x¬(Fx ⊃ Gx)		
(4)	((Fa ∧ ∀y(Fy ⊃ y = a)) ∧ Ga)	√	From 1
(5)	(Fa ∧ ∀y(Fy ⊃ y = a))	√	From 4
(6)	Ga		From 4
(7)	Fa		From 5
(8)	∀y(Fy ⊃ y = a)		From 5

(9)	¬(Fb ⊃ Gb) √		From 3
(10)	Fb		From 9
(11)	¬Gb		From 9
(12)	(Fb ⊃ b = a) √		From 8
(13)	¬Fb	b = a	From 12
(14)	*	¬Ga	From 11, 13 by LL
		*	

4. ‘∃x∀y((Fy ≡ y = x) ∧ Gx)’ *q-entails* ‘∃x((Fx ∧ ∀y(Fy ⊃ y = x)) ∧ Gx)’.

(1)	∃x∀y((Fy ≡ y = x) ∧ Gx) √			
(2)	¬∃x((Fx ∧ ∀y(Fy ⊃ y = x)) ∧ Gx) √			Negation of conclusion
(3)	∀x¬((Fx ∧ ∀y(Fy ⊃ y = x)) ∧ Gx)			
(4)	∀y((Fy ≡ y = a) ∧ Ga)			From 1
(5)	¬((Fa ∧ ∀y(Fy ⊃ y = a)) ∧ Ga) √			From 3
(6)	((Fa ≡ y = a) ∧ Ga) √			
(7)	(Fa ≡ y = a)			From 6
(8)	Ga			From 6
(13)	¬(Fa ∧ ∀y(Fy ⊃ y = a)) √		¬Ga	From 12
			*	
(14)	¬Fa	¬∀y(Fy ⊃ y = a) √		From 13
(15)	*	∃y¬(Fy ⊃ y = a) √		From 14
(16)		¬(Fb ⊃ b = a) √		From 15
(17)		Fb		From 16
(18)		¬b = a		From 16

So far, that’s pretty much all automatic. To finish off, we need more ‘b’-involving info so we have to use one of our universally quantified wffs, and the obvious one to go for is (4):

(19)	((Fb ≡ b = a) ∧ Ga) √		From 4
(20)	(Fb ≡ b = a)		
(21)	Ga		
(22)	Fb	¬Fb	From 20
(23)	b = a	¬b = a	From 20
	*	*	

And finally …

5. ‘∃x((Fx ∧ ∀y(Fy ⊃ y = x)) ∧ Gx)’ *q-entails* ‘∃x∀y((Fy ≡ y = x) ∧ Gx)’.

(1)	∃x((Fx ∧ ∀y(Fy ⊃ y = x)) ∧ Gx) √	
(2)	¬∃x∀y((Fy ≡ y = x) ∧ Gx) √	Negation of conclusion
(3)	∀x¬∀y((Fy ≡ y = x) ∧ Gx)	
(4)	((Fa ∧ ∀y(Fy ⊃ y = a)) ∧ Ga) √	From 1
(5)	(Fa ∧ ∀y(Fy ⊃ y = a)) √	From 4
(6)	Ga	From 4
(7)	Fa	From 5
(8)	∀y(Fy ⊃ y = a)	From 5
(9)	¬∀y((Fy ≡ y = a) ∧ Ga) √	From 3
(10)	∃y¬((Fy ≡ y = a) ∧ Ga) √	From 9
(11)	¬((Fb ≡ b = a) ∧ Ga)	From 10

(12)	¬(Fb ≡ b = a) √	¬Ga ✲	From 12

(13)	Fb	¬Fb	From 12
(14)	¬b = a	b = a	From 12
(15)	|	¬Fa	From 13, 14 by LL
	|	✲	From 17, 15

To finish the left-most branch we need more more ‘b’-involving info, so we have to use one of our universally quantified wffs, and this time nthe obvious one to go for is (8):

(15)	(Fb ⊃ b = a) √		From 8
(16)	¬Fb ✲	b = a ✲	