Answers to Exercises 8

A *Show the following expressions are wffs by producing construction trees for them. Which is the main connective of each wff? Also, list all the subformulae of the last two examples.*

1. $((P \vee P) \wedge R)$
2. $(\neg(R \wedge S) \vee \neg Q)$
3. $\neg\neg((P \wedge Q) \vee (\neg P \vee \neg Q))$
4. $((((P \vee P) \wedge R) \wedge Q) \vee (\neg(R \wedge S) \vee \neg Q))$
5. $(\neg(\neg(P \wedge Q) \wedge \neg(P \wedge R)) \vee \neg(P \wedge (Q \vee R)))$
6. $(\neg((R \vee \neg Q) \wedge \neg S) \wedge (\neg(\neg P \wedge Q) \wedge S))$

1

$$\dfrac{\dfrac{P \quad P}{(P \vee P)} \qquad R}{((P \vee P) \wedge R)}$$

main connective '∧'

2

$$\dfrac{\dfrac{\dfrac{R \quad S}{(R \wedge S)}}{\neg(R \wedge S)} \qquad \dfrac{Q}{\neg Q}}{(\neg(R \wedge S) \vee \neg Q)}$$

main connective '∨'

3

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P \quad Q}{(P \wedge Q)} \qquad \dfrac{\dfrac{P}{\neg P} \quad \dfrac{Q}{\neg Q}}{(\neg P \vee \neg Q)}}{((P \wedge Q) \vee (\neg P \vee \neg Q))}}{\neg((P \wedge Q) \vee (\neg P \vee \neg Q))}}{\neg\neg((P \wedge Q) \vee (\neg P \vee \neg Q))}$$

main connective '¬'

4

$$\dfrac{\dfrac{\dfrac{\dfrac{P \quad P}{(P \vee P)} \quad R}{((P \vee P) \wedge R)} \quad Q}{(((P \vee P) \wedge R) \wedge Q)} \qquad \dfrac{\dfrac{\dfrac{R \quad S}{(R \wedge S)}}{\neg(R \wedge S)} \quad \neg Q}{(\neg(R \wedge S) \vee \neg Q)}}{((((P \vee P) \wedge R) \wedge Q) \vee (\neg(R \wedge S) \vee \neg Q))}$$

main connective '∨'

5

$$\dfrac{\dfrac{\dfrac{\dfrac{P \quad Q}{(P \wedge Q)}}{\neg(P \wedge Q)} \quad \dfrac{\dfrac{P \quad R}{(P \wedge R)}}{\neg(P \wedge R)}}{(\neg(P \wedge Q) \wedge \neg(P \wedge R))}}{\neg(\neg(P \wedge Q) \wedge \neg(P \wedge R))} \qquad \dfrac{P \quad \dfrac{\dfrac{Q \quad R}{(Q \vee R)}}{(P \wedge (Q \vee R))}}{\neg(P \wedge (Q \vee R))}$$

$$(\neg(\neg(P \wedge Q) \wedge \neg(P \wedge R)) \vee \neg(P \wedge (Q \vee R)))$$

main connective '∨'

subformulae: P , Q, R, $(P \wedge Q)$, $(P \wedge R)$, $(Q \vee R)$, $\neg(P \wedge Q)$, $\neg(P \wedge R)$, $(P \wedge (Q \vee R))$, $\neg(P \wedge (Q \vee R))$, $(\neg(P \wedge Q) \wedge \neg(P \wedge R))$, $\neg(\neg(P \wedge Q) \wedge \neg(P \wedge R))$ $(\neg(\neg(P \wedge Q) \wedge \neg(P \wedge R)) \vee \neg(P \wedge (Q \vee R)))$

(NB a wff counts as a limiting case of a sub-formula of itself).

6

$$\dfrac{\dfrac{\dfrac{R \qquad \dfrac{Q}{\neg Q}}{(R \vee \neg Q)} \qquad \dfrac{S}{\neg S}}{((R \vee \neg Q) \wedge \neg S)}}{\neg((R \vee \neg Q) \wedge \neg S)} \qquad \dfrac{\dfrac{\dfrac{P}{\neg P} \qquad Q}{(\neg P \wedge Q)}}{\neg(\neg P \wedge Q)} \qquad \dfrac{S}{(\neg(\neg P \wedge Q) \wedge S)}$$

$$(\neg((R \vee \neg Q) \wedge \neg S) \wedge (\neg(\neg P \wedge Q) \wedge S))$$

main connective '$\wedge$', and again every wff on the tree is a subformula.

B *Which of the following expressions are wffs of* ***PL****?*

1. $((P \vee Q) \wedge \neg R))$
2. $((P \vee (Q \wedge \neg R) \vee ((Q \wedge \neg R) \vee P))$
3. $\neg(\neg P \vee (Q \wedge (R \vee \neg P))$
4. $((P \wedge (Q \vee R)) \wedge (Q \vee R))$
5. $(((P \wedge (Q \wedge \neg R)) \vee \neg\neg\neg(R \wedge Q)) \vee (P \wedge R))$
6. $((P \wedge (Q \wedge \neg R)) \vee \neg\neg\neg((R \wedge Q) \vee (P \wedge R)))$
7. $(\neg(P \vee \neg(Q \wedge R)) \vee (P \wedge Q \wedge R)$
8. $\neg\neg((\neg P \wedge (S \vee \neg S)) \wedge ((Q \wedge (P \wedge \neg R)) \vee \neg S))$

Repair the defective expressions by adding/removing the minimum number of brackets needed to do the job. Show the results are indeed wffs by producing construction trees in each case. What is the main connective in each case?

1. Not a wff (more right hand brackets than left): a simple repair deletes the last bracket

   $((P \vee Q) \wedge \neg R)$

   Tree is simple, and main connective is '$\wedge$'

2. Not a wff (more left hand brackets than right): a simple repair is

   $((P \vee (Q \wedge \neg R)) \vee ((Q \wedge \neg R) \vee P))$

   Again, the tree for simple for the repaired wff: here's the final stages

   $$\dfrac{\dfrac{P \qquad (Q \wedge \neg R)}{(P \vee (Q \wedge \neg R))} \qquad \dfrac{(Q \wedge \neg R) \qquad P}{((Q \wedge \neg R) \vee P)}}{((P \vee (Q \wedge \neg R)) \vee ((Q \wedge \neg R) \vee P))}$$

   And so main connective '$\vee$'

3. Not a wff (more left hand brackets than right): a simple repair is

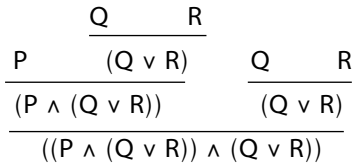
   $\neg(\neg P \vee (Q \wedge (R \vee \neg P)))$

   The corresponding tree, of course is,

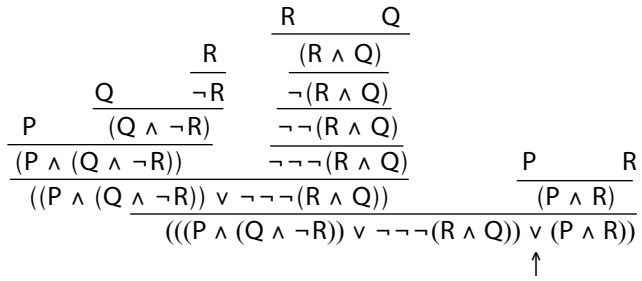
   $$\dfrac{\dfrac{\dfrac{P}{\neg P} \qquad \dfrac{Q \qquad \dfrac{\dfrac{P}{\neg P} \qquad R}{(R \vee \neg P)}}{(Q \wedge (R \vee \neg P))}}{(\neg P \vee (Q \wedge (R \vee \neg P)))}}{\neg(\neg P \vee (Q \wedge (R \vee \neg P)))}$$

   And so the main connective is the initial '$\neg$'

4. A wff as it stands: the main connective is the second '∧'.

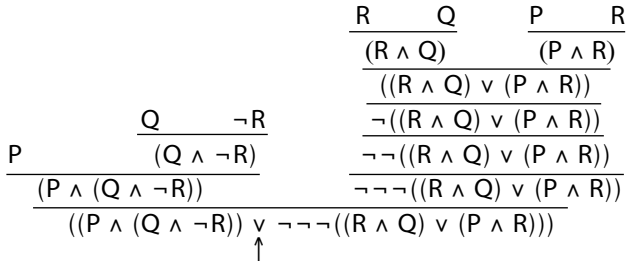


5. A wff as it stands



Main connective the last 'v'.

6 Also a wff.



Main connective: the first 'v'.

7. '(¬(P v ¬(Q ∧ R)) v (P ∧ Q ∧ R)' is not a wff. One repair is:

(¬(P v ¬(Q ∧ R)) v (P ∧ (Q ∧ R)))

which is the disjunction of '¬(P v ¬(Q ∧ R))' and '(P ∧ (Q ∧ R))'. The tree then is obvious and the main connective the second 'v'.

8. A wff with main connective '¬':