Answers to Exercises 9

A We are given that ‘P’ *means Plato is a great philosopher; ‘Q’ means Quine is a great philosopher; ‘R’ means Russell is a great philosopher.* With that manual, the translations go into **PL** as follows:

1. Either Quine is a great philosopher or Russell is ⇒ (Q ∨ R)
2. Neither Plato nor Quine is a great philosopher ⇒ ¬(P ∨ Q): or (¬P ∧ ¬Q)
3. Plato, Russell and Quine are great philosophers ⇒ (P ∧ (R ∧ Q)): or ((P ∧ R) ∧ Q)
4. Not both Quine and Russell are great philosophers ⇒ ¬(Q ∧ R)
5. Quine is a great philosopher and Russell isn’t ⇒ (Q ∧ ¬R)
6. Either Quine and Russell are great philosophers, or Plato is ⇒ ((Q ∧ R) ∨ P)
7. It isn’t the case the Quine is and Russell isn’t a great philosopher ⇒ ¬(Q ∧ ¬R)

B Suppose ‘P’ means *Fred is a fool*; ‘Q’ means *Fred knows some logic*; ‘R’ means *Fred is a rocket scientist*. Then the best we can do by way of translating the following sentences into **PL** is as follows:

1. Fred is a rocket scientist, but he knows no logic ⇒ (R ∧ ¬Q)
2. Fred’s a fool, even though he knows some logic ⇒ (P ∧ Q)
3. Although Fred’s a rocket scientist, he’s a fool and even knows no logic ⇒ (R ∧ (P ∧ ¬Q))
4. Fred’s a fool, yet he’s a rocket scientist who knows some logic ⇒ (P ∧ (R ∧ Q))
5. Fred is not a rocket scientist who knows some logic ⇒ ¬(R ∧ Q)
6. Fred is a fool despite the fact that he knows some logic ⇒ (P ∧ Q)
7. Fred knows some logic unless he is a fool ⇒ ((Q ∧ ¬P) ∨ (P ∧ ¬Q))

Question: What do you think is lost in the translations, given that **PL** *only has the ‘colourless’ connectives ‘∧’, ‘∨’ and ‘¬’?*

(1) Roughly speaking, in using ‘A but B’ rather than ‘A and B’ the speaker signals that (she takes it that) there is some kind of tension or opposition between the truth of A and the truth of B—the truth of A might incline you to suppose B is false. It is arguable, however, that the existence of such a tension or opposition is not part of what the speaker is strictly speaking asserting. Someone who says ‘Jancis is a woman but she can do logic’ may be revealing the daft prejudice that that being a woman is in some sort of tension with being good at logic—but if the facts of the matter are that she is a woman and is good at logic, then it seems that our speaker has the facts right, has said something true to the facts even if in an objectionable way. Likewise, someone who says ‘Fred is a rocket scientist, but he can’t do logic’ is arguably saying something that is true to the facts just so long as ‘(R ∧ ¬Q)’ is true. So the translation arguably gets the truth-conditions right, but misses the hint of a contrast. (‘Yet’ seems to work like ‘but’: so similar remarks apply to 4)

(2) Similar remarks apply to ‘even though’. Someone who says ‘A even though B’ is committed to A, and to B, so is committed to ‘A and B’. What else is asserted as being a fact of the matter? Evidently the speaker implies that the truth of B might lead you to expect that not-A: but is that part of what is actually asserted about the world, or is ‘even though’ (as it were) mood music, colouring the assertion of the conjunction with a signal about how surprising the speaker finds it, without making any additional factual claim over a plain, uncoloured conjection? (3 is similar. ‘Despite the fact that’ seems to work like ‘even though’ so similar remarks apply to 6.)

(7) ‘A unless B’ is naturally heard as an exclusive disjunction—either you’ve got A (without B) or else B obtains and A fails.

C *Confirm that all the following strings are wffs by producing construction trees. Suppose that* ‘P’ *and* ‘R’ *are both true and* ‘Q’ *false. Evaluate the wffs by working down the trees. Then do the working again in the short form.*

1. ¬(P ∧ R)

$$\dfrac{\dfrac{P \qquad\qquad R}{(P \wedge R)}}{\neg(P \wedge R)} \qquad\qquad \dfrac{\dfrac{\text{‘}P\text{’} \Rightarrow T \qquad \text{‘}R\text{’} \Rightarrow T}{\text{‘}(P \wedge R)\text{’} \Rightarrow T}}{\text{‘}\neg(P \wedge R)\text{’} \Rightarrow F}$$

P	R	¬	(P	∧	R)
T	T	F	T	T	T
		4	1	3	2

The numbers here are optional!

2. ¬¬¬¬(Q ∨ ¬R)

R
Q ¬R
(Q ∨ ¬R)
¬(Q ∨ ¬R)
¬¬(Q ∨ ¬R)
¬¬¬(Q ∨ ¬R
¬¬¬¬(Q ∨ ¬R)

‘R’ ⇒ T
‘Q’ ⇒ F ‘¬R’ ⇒ F
‘(Q ∨ ¬R)’ ⇒ F
‘¬(Q ∨ ¬R)’ ⇒ T
‘¬¬(Q ∨ ¬R)’ ⇒ F
‘¬¬¬(Q ∨ ¬R)’ ⇒ T
‘¬¬¬¬(Q ∨ ¬R)’ ⇒ F

Q	R	¬¬¬¬(Q ∨ ¬R)
F	T	<u>F</u>T F T F F FT
		8 7 6 5 1 4 3 2

or, if we don’t repeat the valuations of the atomic letters on the right (as we suggested that we don’t at p. 79, a susggestion we’ll follow henceforth) we get simply

Q	R	¬¬¬¬(Q ∨ ¬R)
F	T	<u>F</u>T F T F FT
		6 5 4 3 2 1

3. (¬(P ∨ ¬R) ∧ Q)

R
P ¬R
(P ∨ ¬R)
¬(P ∨ ¬R) Q
(¬(P ∨ ¬R) ∧ Q)

‘R’ ⇒ T
‘P’ ⇒ T ‘¬R’ ⇒ F
‘(P ∨ ¬R)’ ⇒ T
‘¬(P ∨ ¬R)’ ⇒ F ‘Q’ ⇒ F
‘(¬(P ∨ ¬R) ∧ Q)’ ⇒ F

P	Q	R	(¬(P ∨ ¬R) ∧ Q)
T	F	T	F T F <u>F</u>
			3 2 1 4

4. ((R ∨ ¬Q) ∧ (Q ∨ P))

Q
R ¬Q Q P
(R ∨ ¬Q) (Q ∨ P)
((R ∨ ¬Q) ∧ (Q ∨ P))

‘Q’ ⇒ F
‘R ’ ⇒ T ‘¬Q ’ ⇒ T ‘Q ’ ⇒ F ‘P’ ⇒ T
‘(R ∨ ¬Q)’ ⇒ T ‘(Q ∨ P)’ ⇒ T
‘((R ∨ ¬Q) ∧ (Q ∨ P))’ ⇒ T

P	Q	R	((R ∨ ¬Q) ∧ (Q ∨ P))
T	F	T	T T <u>T</u> T
			2 1 4 3

5. ¬(P ∨ ((Q ∧ ¬P) ∨ R))

P
Q ¬P
(Q ∧ ¬P) R
P ((Q ∧ ¬P) ∨ R)
(P ∨ ((Q ∧ ¬P) ∨ R))
¬(P ∨ ((Q ∧ ¬P) ∨ R))

‘P’ ⇒ T
‘Q’ ⇒ F ‘¬P’ ⇒ F
‘(Q ∧ ¬P)’ ⇒ F ‘R’ ⇒ T
‘P’ ⇒ T ‘((Q ∧ ¬P) ∨ R)’ ⇒ T
‘(P ∨ ((Q ∧ ¬P) ∨ R))’ ⇒ T
‘¬(P ∨ ((Q ∧ ¬P) ∨ R))’ ⇒ F

P	Q	R	¬(P ∨ ((Q ∧ ¬P) ∨ R))
T	F	T	<u>F</u> T F F T
			5 4 2 1 3

6. $\neg(\neg P \vee \neg(Q \wedge \neg R))$

$$\dfrac{\dfrac{\dfrac{P}{\neg P}\qquad \dfrac{\dfrac{Q\qquad \dfrac{R}{\neg R}}{(Q \wedge \neg R)}}{\neg(Q \wedge \neg R)}}{(\neg P \vee \neg(Q \wedge \neg R))}}{\neg(\neg P \vee \neg(Q \wedge \neg R))}$$

$$\dfrac{\dfrac{\dfrac{\text{‘}P\text{’} \Rightarrow T}{\text{‘}\neg P\text{’} \Rightarrow F}\qquad \dfrac{\dfrac{\text{‘}Q\text{ ’} \Rightarrow F\qquad \dfrac{\text{‘}R\text{’} \Rightarrow T}{\text{‘}\neg R\text{’} \Rightarrow F}}{(Q \wedge \neg R)\text{’} \Rightarrow F}}{\neg(Q \wedge \neg R)\text{’} \Rightarrow T}}{\text{‘}(\neg P \vee \neg(Q \wedge \neg R))\text{’} \Rightarrow T}}{\text{‘}\neg(\neg P \vee \neg(Q \wedge \neg R))\text{’} \Rightarrow F}$$

(If you still need the short form given by this stage, you aren't following! Go back and reread Chapter 9!)

7. $(\neg(P \wedge \neg Q) \wedge \neg\neg R)$

$$\dfrac{\dfrac{\dfrac{P\qquad \dfrac{Q}{\neg Q}}{(P \wedge \neg Q)}}{\neg(P \wedge \neg Q)}\qquad \dfrac{\dfrac{R}{\neg R}}{\neg\neg R}}{(\neg(P \wedge \neg Q) \wedge \neg\neg R)}$$

$$\dfrac{\dfrac{\dfrac{P\text{’} \Rightarrow T\qquad \dfrac{\text{‘}Q\text{’} \Rightarrow F}{\text{‘}\neg Q\text{’} \Rightarrow T}}{\text{‘}(P \wedge \neg Q)\text{’} \Rightarrow T}}{\text{‘}\neg(P \wedge \neg Q)\text{’} \Rightarrow F}\qquad \dfrac{\dfrac{\text{‘}R\text{’} \Rightarrow T}{\text{‘}\neg R\text{’} \Rightarrow F}}{\text{‘}\neg\neg R\text{’} \Rightarrow T}}{\text{‘}(\neg(P \wedge \neg Q) \wedge \neg\neg R)\text{’} \Rightarrow F}$$

P	Q	R	$(\neg(P \wedge \neg Q) \wedge \neg\neg R)$
T	F	T	F T T $\underline{F}$ T F
			3 2 1 6 5 4

8. $(((P \vee \neg Q) \wedge (Q \vee R)) \vee \neg\neg(Q \vee \neg R))$

$$\dfrac{\dfrac{\dfrac{P\qquad \dfrac{Q}{\neg Q}}{(P \vee \neg Q)}\qquad \dfrac{Q\qquad R}{(Q \vee R)}}{((P \vee \neg Q) \wedge (Q \vee R))}\qquad \dfrac{\dfrac{\dfrac{Q\qquad \dfrac{R}{\neg R}}{(Q \vee \neg R)}}{\neg(Q \vee \neg R)}}{\neg\neg(Q \vee \neg R)}}{(((P \vee \neg Q) \wedge (Q \vee R)) \vee \neg\neg(Q \vee \neg R))}$$

$$\dfrac{\dfrac{\dfrac{\text{‘}P\text{’} \Rightarrow T\qquad \dfrac{\text{‘}Q\text{’} \Rightarrow F}{\text{‘}\neg Q\text{’} \Rightarrow T}}{\text{‘}(P \vee \neg Q)\text{’} \Rightarrow T}\qquad \dfrac{\text{‘}Q\text{ ’} \Rightarrow F\qquad \text{‘}R\text{’} \Rightarrow T}{\text{‘}(Q \vee R)\text{’} \Rightarrow T}}{\text{‘}((P \vee \neg Q) \wedge (Q \vee R))\text{’} \Rightarrow T}\qquad \dfrac{\dfrac{\dfrac{\text{‘}Q\text{’} \Rightarrow F\qquad \dfrac{\text{‘}R\text{’} \Rightarrow T}{\text{‘}\neg R\text{’} \Rightarrow F}}{\text{‘}(Q \vee \neg R)\text{’} \Rightarrow F}}{\text{‘}\neg(Q \vee \neg R)\text{’} \Rightarrow T}}{\text{‘}\neg\neg(Q \vee \neg R)\text{’} \Rightarrow F}}{\text{‘}(((P \vee \neg Q) \wedge (Q \vee R)) \vee \neg\neg(Q \vee \neg R))\text{’} \Rightarrow T}$$

In practice, evaluating a wff like this 'in one's head' without writing down this kind of valuation tree, you'd spot that the wff is disjunction of $((P \vee \neg Q) \wedge (Q \vee R))$ and $\neg\neg(Q \vee \neg R)$. You'd then see the *first* disjunct conjoins $(P \vee \neg Q)$ with $(Q \vee R)$, both of which are true on the given valuation. So the first disjunct is true – which makes the whole disjunction true, however things turn out for the second disjunct.

9. $(\neg(\neg P \vee \neg(Q \wedge \neg R)) \vee \neg\neg(Q \vee \neg P))$

As explained in §8.3, the construction tree is easily built if you work from the bottom up. So the tree starts

$$\dfrac{\neg(\neg P \vee \neg(Q \wedge \neg R))\qquad\qquad \neg\neg(Q \vee \neg P)}{(\neg(\neg P \vee \neg(Q \wedge \neg R)) \vee \neg\neg(Q \vee \neg P))}$$

If it isn't obvious where to divide the wff into components, see our rule on pp.67–68! And in fact as soon as we've made this initial divide, it should be obvious how we have to continue:

R
Q ¬R P
P (Q ∧ ¬R) Q ¬P
¬P ¬(Q ∧ ¬R) (Q ∨ ¬P)
(¬P ∨ ¬(Q ∧ ¬R)) ¬(Q ∨ ¬P)
¬(¬P ∨ ¬(Q ∧ ¬R)) ¬¬(Q ∨ ¬P)
(¬(¬P ∨ ¬(Q ∧ ¬R)) ∨ ¬¬(Q ∨ ¬P))

‘R’ ⇒ T
‘Q’ ⇒ F ‘¬R’ ⇒ F ‘P’ ⇒ T
‘P’ ⇒ T ‘(Q ∧ ¬R)’ ⇒ F ‘Q’ ⇒ F ‘¬P’ ⇒ F
‘¬P’ ⇒ F ‘¬(Q ∧ ¬R)’ ⇒ T ‘(Q ∨ ¬P)’ ⇒ F
‘(¬P ∨ ¬(Q ∧ ¬R))’ ⇒ T ‘¬(Q ∨ ¬P)’ ⇒ T
‘¬(¬P ∨ ¬(Q ∧ ¬R))’ ⇒ F ‘¬¬(Q ∨ ¬P)’ ⇒ F
‘(¬(¬P ∨ ¬(Q ∧ ¬R)) ∨ ¬¬(Q ∨ ¬P))’ ⇒ F

P	Q	R	(¬(¬P ∨ ¬(Q ∧ ¬R)) ∨ ¬¬(Q ∨ ¬P))
T	F	T	F F T T F F <u>F</u> F T F F
			6 1 5 4 3 2 11 10 9 8 7

And finally …

10. ¬((¬(P ∧ ¬Q) ∧ ¬¬R) ∧ ¬(¬(P ∨ ¬R) ∧ Q))

R
Q P ¬R
P ¬Q R (P ∨ ¬R)
(P ∧ ¬Q) ¬R ¬(P ∨ ¬R) Q
¬(P ∧ ¬Q) ¬¬R (¬(P ∨ ¬R) ∧ Q)
(¬(P ∧ ¬Q) ∧ ¬¬R) ¬(¬(P ∨ ¬R) ∧ Q)
((¬(P ∧ ¬Q) ∧ ¬¬R) ∧ ¬(¬(P ∨ ¬R) ∧ Q))
¬((¬(P ∧ ¬Q) ∧ ¬¬R) ∧ ¬(¬(P ∨ ¬R) ∧ Q))

‘R’ ⇒ T
‘Q’ ⇒ F ‘P’ ⇒ T ‘¬R’ ⇒ F
‘P’ ⇒ T ‘¬Q ’ ⇒ T ‘R’ ⇒ T ‘(P ∨ ¬R)’ ⇒ T
‘(P ∧ ¬Q) ’ ⇒ T ‘¬R’ ⇒ F ‘¬(P ∨ ¬R)’ ⇒ F ‘Q’ ⇒ F
‘¬(P ∧ ¬Q)’ ⇒ F ‘¬¬R’ ⇒ T ‘(¬(P ∨ ¬R) ∧ Q)’ ⇒ F
‘(¬(P ∧ ¬Q) ∧ ¬¬R)’ ⇒ F ‘¬(¬(P ∨ ¬R) ∧ Q)’ ⇒ T
‘((¬(P ∧ ¬Q) ∧ ¬¬R) ∧ ¬(¬(P ∨ ¬R) ∧ Q))’ ⇒ F
‘¬((¬(P ∧ ¬Q) ∧ ¬¬R) ∧ ¬(¬(P ∨ ¬R) ∧ Q))’ ⇒ T

P	Q	R	¬((¬(P ∧ ¬Q) ∧ ¬¬R) ∧ ¬(¬(P ∨ ¬R) ∧ Q))
T	F	T	<u>T</u> F T T F T F F T F T F F
			13 3 2 1 6 5 4 12 11 9 8 7 10