IFL: Logicbite 4

The counterexample method

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You are given a (one-step!) argument to assess. You suspect it is invalid. How could you show this?

In *IFL* §4.4, we looked at the mini-example

(1) All philosophers are logicians.
So (2) All logicians are philosophers.

This argument is evidently invalid. But, to press home the point, I remarked that you might as well argue

(3) All women are human beings.
So (4) All human beings are women.

I showed that the inference move in the first argument is unreliable by giving a parallel second argument which relies on the same pattern of inference but which is obviously hopeless. In other words, I offered a counterexample to the reliability of the inference.

The idea that we can challenge an inference by giving a counterexample in this sort of way is surely familiar enough before we get round to officially studying logic: the problem, as we will see, is in saying clearly just how the challenge works.

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Before proceeding, a general remark. When quoting from other authors' textbooks, I've occasionally quibbled and criticized, pointing out some less-than-ideal phrasing. I'm about to do that again, at some length. I'm not, I hope, just being captious. Rather, I want to encourage a habit in you: you need to read carefully, stay critical, even when tackling a logic book – especially including mine!

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Here is Patrick J. Hurley in his *A Concise Introduction to Logic*[1] (compare the similar quote from Lemmon in Logicbite 2):

> This section shows that the truth of a deductive argument's inferential claim (that is, the correctness of the argument's reasoning) is determined by the form of the argument. In other words, validity is determined by form. For these purposes, consider the following argument:
>
> All adlers are bobkins.
> All bobkins are crockers.
> Therefore, all adlers are crockers.
>
> Because the words "adlers," "bobkins," and "crockers" are nonsensical, we do not know whether any of the statements in this argument are true or false.

Really? If those three expressions *are* nonsensical, the putative premisses and conclusion here are nonsense too. So we *do* know their truth-status: they can't be either true or false. To proceed, we need to assume that the words are *not* nonsense, but meaningful though unfamiliar.

[1]The title is a little comical, as this book weighs in at over 600 pages, even before the answers to exercises start. Apparently it is the most widely used logic text in North America, and is now in its thirteenth edition. Which I find a rather depressing thought. Anyway, I'm quoting here from the seventh edition. The thirteenth edition does just a little better, though the main complaint still stands.

> Yet, we do know that if we *assume* that the premises are true, it is impossible for the conclusion to be false. That is, if we assume that the adlers, whatever they might be, are included in the bobkins and the bobkins in the crockers, then we must accept the conclusion that the adlers are included in the crockers. According to the definition of validity, therefore, the argument is valid.
>
> This fact is important for understanding the nature of validity because it shows that the validity of an argument has nothing to do with its specific subject matter.

That's badly ambiguous. Does Hurley mean (a) that the validity of an argument – *any* argument – has nothing to do with its specific subject matter? Or does he just mean (b) that we've found a case, this case, where the validity of the argument has nothing to do with its specific subject matter? He is of course only entitled to (b). But judging from his opening words about validity being "determined by the form [as opposed to the subject matter] of the argument", it looks as if Hurley might think he has established that (a) is true. He hasn't.

> Even though we know nothing about adlers, bobkins, and crockers, we still know that the argument is valid. The validity of the argument arises from the way the terms "adlers," "bobkins," and "crockers" are arranged in the statements. If we represent these terms by their first letters, we obtain the following *argument form*. We use a line to separate the premises from the conclusion.
>
> All *A* are *B*.
> All *B* are *C*.
> ―――――――
> All *A* are *C*.
>
> This is a valid argument form. Its validity rests purely upon the arrangement of the letters within the statements, and it has nothing to do with what the letters might stand for. In light of this fact, we can substitute any terms we choose in place of *A*, *B*, and *C*, and as long as we are consistent, we will obtain a valid argument. For example, we might substitute "daisies" for *A*, "flowers" for *B*, and "plants" for *C* and obtain the following valid argument:
>
> All daisies are flowers.
> All flowers are plants.
> Therefore, all daisies are plants.
>
> Any argument, such as this, that is produced by uniformly substituting terms or statements in place of the letters in an argument form is called a *substitution instance* of that form.

We could quibble that Hurley hasn't explicitly told us what he means by 'valid argument *form*': but presumabbly it is one all of whose substitution instances are valid arguments.

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Hurley now continues

> Let us turn now to the concept of invalidity. Consider the following argument:
>
> All adlers are bobkins.
> All crockers are bobkins.
> Therefore, all adlers are crockers.
>
> As with the previous argument, we do not know whether the premises and conclusion of this argument are true or false. But if we assume that the premises are true, it is possible for the conclusion to be false. It might be the case, for example, that the adlers make up one part of the bobkins, that the crockers make up another part, and that the adlers and the crockers are completely separate from each other. In this case the premises would be true and the conclusion false. The argument is therefore invalid.

> If we represent the terms in this argument by their first letters, we obtain the following argument form:
>
> All A are B.
> All C are B.
> ―――――
> All A are C.
>
> This is an invalid form, and any argument that has this form is an invalid argument.

But read as it stands, that last claim of course is simply wrong. Take the argument

All algebraists are mathematicians.
All mathematicians are mathematicians.
Therefore, all algebraists are mathematicians.

That's trivially valid, yet is an instance of the displayed form. I've simply substituted 'mathematician' for both 'B' and 'C' – and Hurley has explicitly told us that we can substitute *any* terms we choose in place of B and C.

To be fair, Hurley does immediately footnote that "some substitution instances of invalid forms are actually valid". His footnoted example is badly chosen as it raises a quite irrelevant issue, but let's not worry about that. Anyway, he goes on qualify the claim I've just complained about:

> An argument is said to have a certain form if it is a substitution instance of that form. In the case of invalid forms, we must add the proviso that an argument has an invalid form only if it is not a substitution instance of any valid form.

OK: Hurley has told us that being valid is a matter of having a valid form, i.e. being a substitution instance of a valid form. So he is now inviting us to say that an argument 'has' an invalid form just if (i) it is an instance of that invalid form *and* (ii) it is not valid. So we have to know that an argument isn't valid before we can say that it is 'has' an invalid form (in Hurley's stipulated double-barrelled sense). But then we can't use 'having' an invalid argument form to *explain* the invalidity of an argument which is a substitution instance of that form!

Hurley seems to trip himself up over this. Despite noting that some substitution instances of invalid forms are actually valid, he writes

> The following argument has the invalid form just discussed:
>
> All cats are animals.
> All dogs are animals.
> Therefore, all cats are dogs.
>
> Notice that this substitution instance has true premises and a false conclusion. It is therefore clearly invalid, and it constitutes proof that the original argument is invalid. The reasoning behind this proof is as follows. The substitution instance is invalid because it has true premises and a false conclusion. Therefore, the substitution instance has an invalid form. But the form of the substitution instance is identical to the form of the (second) adler-bobkin argument. Therefore, the adler-bobkin argument is invalid.
>
> A substitution instance having true premises and a false conclusion is called a counterexample, and the method we have just used to prove the adler-bobkin argument invalid is called the *counterexample method*. ... The counterexample method consists in isolating the form of the argument and then constructing a substitution instance having true premises and a false conclusion.

Yes, the cats argument is a substitution instance of that earlier displayed form, and is an invalid argument. Yes, the displayed form is therefore an invalid form (i.e. is a form that has some invalid instances). Yes, the adler-bobkin is also a substitution instance of that displayed invalid form. But that *doesn't* by itself entail that it is an invalid argument because some substitution instances of invalid forms are actually valid. Hurley seems to be sliding from the observation that the adler-bobkin argument is an instance of the displayed invalid form to the implicit claim that the adler-bobkin argument 'has' that invalid form in his special double-barrelled sense. And he isn't entitled to that as yet.

Nor is he entitled, in his final sentence, to that talk about 'the' form of an argument, given he has himself just remarked that an argument can be an instance of more than one form, some valid, some invalid.

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I said at the outset that the 'you might as well say' or counterexample method for showing an argument is invalid is common and familiar. We've just seen one author getting into some tangles in trying to explain how the counterexample method works. And it is difficult to find authors doing much better.[2] But we now know some pitfalls to avoid: do I do any better in *IFL* Chapter 5?

[2] Let me know who I have overlooked! Yes, looking ahead, lots of authors say just the right things in *formal* contexts when talking about counterexamples to claims of tautological entailment, for example. But what about authors talking about more *informal* uses of the counterexample method?