IFL: Logicbite 8

'And', 'Or', 'Not'

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I have spent seven chapters of *IFL* talking informally about very general logical notions like validity and form. I now narrow the focus considerably, while the approach begins to be more formal (in ways which will become clear). I'm going to be talking – for more than ten chapters in all – about the core of propositional logic, which discusses arguments whose logically relevant materials are just 'and', 'or' and 'not'.

Here at the outset it must seem crazy to be spending so much time on so very narrow a class of arguments. But there's method in my madness! Propositional logic gives us a simple and uncluttered context in which I can introduce a whole swathe of concepts and techniques of formal logic. Most students find it a *lot* easier to first master this cut-down fragment of 'first-order logic' (our ultimate target), before going on to tackle the full-strength version. So be patient: you'll in fact learn a great deal from looking in depth at our seemingly very limited initial topic. Promise!

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You might reasonably suppose that there can be nothing very interesting or controversial to say about 'and', 'or' and 'not' in their role as so-called propositional connectives. So let me quote passages from a couple of good authors, who in fact – by my lights – say controvertible things about each of our three connectives!

To start, here's Greg Restall near the start of his short book *Logic: An Introduction* (2006):[1]

> Consider the two propositions
>
> > An action is good when it makes people happy.
> > Keeping your promises is always good.
>
> You might believe both of these propositions. You can assert both in one go by asserting their conjunction:
>
> > An action is good when it makes people happy, and keeping your promises is always good.
>
> This is a *single* proposition – you may believe it or reject it, or simply be undecided. It is a special proposition, because it is related to two other propositions. It is said to be the *conjunction* of the original propositions. The conjunction is true just when both conjuncts are true. If either of the original propositions is false, the conjunction is false.

So this is one central use of 'and': to connect two complete sentences to form a third, with the result expressing a conjunction – i.e. the composite proposition is true just when the propositions expressed by the conjoined sentences are both true together. (Note that conjunction so defined is commutative – i.e. the order of the conjoined sentences doesn't make a difference to truth or falsehood.)

Of course, 'and' can also be used in to connect parts of sentences, as in 'Fred is slim and handsome', 'Fred and Ginger danced the night away', 'Ginger danced gracefully and expressively'. There are complications here. The first case seems simply equivalent to a sentential conjunction, 'Fred is slim and Fred is handsome'. But the second case isn't similarly equivalent to 'Fred

[1] Greg Restall is an Australian logician who has written extensively, including a now standard book on so-called substructural logic and other topics in proof theory and its philosophy.

danced the night away and Ginger danced the night away', since the original version, on the natural reading, conveys that Fred and Ginger were dancing together.

Here's Restall making similar points:

> Sometimes a sentence uses the word 'and' to connect parts of speech other than phrases. For example the 'and' in the sentence
>
> > Justice and tolerance are valuable.
>
> connects the words 'justice' and 'tolerance', and these words do not (by themselves) express propositions. However, the sentence is still a *kind* of conjunction. At the very least, it seems to have the same meaning as the conjunction of the two sentences
>
> > Justice is valuable.
> > Tolerance is valuable.
>
> because saying 'Justice and tolerance are valuable' is a shorter way of saying the more long winded 'Justice is valuable and tolerance is valuable'.
>
> However, you must be careful! Sometimes sentences feature the word 'and', without expressing conjunctions of propositions. ... For example, if I say
>
> > Colleen and Errol got married.
>
> this is not a conjunction of two propositions. It certainly is not the conjunction of the propositions 'Colleen got married' and 'Errol got married', since that proposition
>
> > Colleen got married and Errol got married.
>
> means something else. That conjunction does not say that Colleen and Errol married each other, whereas (at least in the colloquial speech familiar to me) to say Colleen and Errol got married is to say that they did marry each other.

So far so good. But concentrate now on cases where 'and' *does* conjoin sentences. We may wonder: does it always express conjunction at least in these cases? Here's Restall, arguing that it doesn't (with a tiny change to his example):

> Furthermore, sometimes we use 'and' to join two propositions and it still does not express a simple conjunction. Sometimes we use 'and' to indicate a kind of order between the two propositions. For example, the two sentences
>
> > I went out and I had dinner.
> > I had dinner and I went out.
>
> say very different things. The first indicates that you went out and then had dinner. The second indicates that you had dinner and then went out. These are not propositional conjunctions in our sense, because making sure that 'I had dinner' and that 'I went out' are true is not enough to make it true that 'I went out and I had dinner'. For that, we require the right kind of order.

However, on second thoughts, it is far from clear that these considerations – similar examples are used by other authors too – really show that 'and' as a propositional connective sometimes has a more-than-conjunctive meaning. In *IFL* I offer a reason for doubting this. (Hint: suppose in the last pair of examples you replace the 'and' with a full stop. What then?)

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Next let's consider 'or', again in its use as a connective joining two propositions. This time, Warren Goldfarb can introduce the discussion (the quotation is from his *Deductive Logic*):

> If I assert the statement
>
> (1) Callas gave an uninspired performance or the audience was predisposed against her

> then I must agree that at least one of the statements
>
> (1) Callas gave an uninspired performance
> (2) The audience was predisposed against Callas
>
> is true, even though I might not know which. Moreover, I do not preclude the possibility that both (2) and (3) are true; I am denying only that (2) and (3) are both false. Statement (1) is the *disjunction* of (2) and (3); it is true if at least one of (2) and (3) is true, and is false otherwise. Statements (2) and (3) are the *disjuncts* of (1).
>
> This account of the logical behavior of (1) might perhaps be met with some hesitancy. The hesitancy arises because "or" has two precise but conflicting senses in English. The sense just ascribed to the "or" in (1) is called the *inclusive* sense. The contrasting exclusive sense is that under which "*p* or *q*" counts as true if and only if exactly one of "*p*" and "*q*" is true. Inclusive "or" and exclusive "or" differ only in the case that both constituent statements are true; in this case "*p* or *q*" is true when "or" is inclusive, and is false when "or" is exclusive.
>
> To find an instance in which "or" must be interpreted exclusively, we must provide circumstances in which a person is using the statement containing "or" explicitly to deny that both constituent statements are true. Here is a well-worn example. Suppose a child is pleading to be taken both to the beach and to the movies, and the parent replies:
>
> > We will go to the beach or we will go to the movies.
>
> The exclusive nature of "or" here is clear: the parent is promising one outing but precluding both.
>
> More common are instances in which "or" should be interpreted inclusively. It seems to me that (1) is such an instance. . . . Similarly, suppose the rule-book says that a student satisfies the logic requirement on the condition that
>
> > The student takes the Deductive Logic course or the student passes the departmental examination.
>
> If the student overzealously does both, then clearly the condition would still be considered true, and the student taken to have satisfied the requirement. Thus "or" is inclusive here.

Goldfarb goes on to suggest that there are 'don't care' cases where we can construe "or" either way: but he judges that inclusive disjunction is overall the most common use.

This is a common position, that everyday "or" has two different senses, inclusive and exclusive. But again, just to be awkward, I don't find Goldfarb's beach-or-movies example a particularly convincing argument for this. The child pleads to be taken to both. Various scenarios:

(1) The parent baldly texts back "We will go to the beach or we will go to the movies" (no emoticons or other extras). This isn't a decisively exclusive case: the parent is just promising to do at least one.

(2) The parent says 'No, we haven't time. We will go to the beach or we will go to the movies. You choose'. In another, they explicitly say 'We will go to the beach or we will go to the movies, but not both'. In these cases the parent indeed precludes a double treat – but it is of course the 'No'/'not both' that does *that*, and there is no need to read an extra denial into the 'or'. In other words, we can understand the 'or' as still allowing three cases as in (1) – one or other or both disjuncts are true – and then the additional 'no' or 'not both' are there to rule one case out.

(3) This time, the 'No' is unspoken but can be clearly already be taken as read e.g. from the parent's demeanour. So a double treat is still already ruled out, before anything is said.

> If the parent then goes on to say "We will go to the beach or we will go to the movies" why should the 'or' be read as having a different sense from in (1) and (2)?

What Goldfarb needs, it seems, is a case which isn't like (1), (2) or (3), but is a case where the double treat is ruled out, and is ruled out in virtue of the special exclusive meaning of "or" and not because of a spoken or unspoken prior exclusion. Are there obviously such cases?

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I'll leave that last question hanging just as I left hanging the question whether 'and' sometimes in fact means 'and then'. For the moment I'm in business of noting complications, not resolving them! So let's instead turn to consider 'not'. Back to Restall:

> [A third] way to form new propositions from old is provided by *negation*. We can assert the negation of a proposition simply by using a judiciously placed 'not'. Here is an example:
>
> > Keeping your promises is not always good.
>
> This is the negation of
>
> > Keeping your promises is always good.
>
> You have to be careful in where you place the 'not' in a proposition in order to negate it. What we want is a proposition that simply states that the original proposition is false. We get a different proposition if we place the 'not' somewhere else. If I say
>
> > Keeping your promises is always not good.
>
> I say something much stronger than the original negation. I'm saying that keeping promises is always bad—it is never good. This goes much further than simply denying the claim that keeping your promises is always a good thing. In general, given a proposition *p*, you can express the negation of *p* by saying 'it's not the case that *p*'. This is cumbersome, but it works in every case. We say that this proposition is the negation of *p*. . .
>
> Strictly speaking, negation is not a connective, as it does not connect different propositions. It is an operator, as it operates on the original proposition to provide another. However, we will abuse the terminology just a little, and call each of conjunction, disjunction, negation, . . . connectives.

True, prefixing 'It is not the case that' to a proposition p usually gives us a composite proposition which says the exact opposite, i.e. which is true just when p is false and vice versa. But does this *always* work, as Restall says? Well, there's a provocation! When a logician says that something is always the case in natural language, it is always(!) worth pausing to try to think up counterexamples. In this case, it is pretty easy. Can you find some?

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More ordinary-language complications. Here's an example from Goldfarb, slightly adjusted:

> Fred danced and Fred sang or Ginger sang.

Take it that 'and' and 'or' in this case express conjunction and inclusive disjunction respectively; our sentence is still ambiguous. Which of the following are we saying?

> Fred danced and sang, or Ginger sang.
> Fred danced, and Fred or Ginger sang.

We could punctuate to disambiguate the original, or – less informally but even more clearly – use brackets like this:

> (Fred danced and Fred sang) or Ginger sang.
> Fred danced and (Fred sang or Ginger sang).

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We've seen then that 'and' (used as a connective between whole propositions) usually expresses conjunction, but perhaps sometimes more. 'Or' (used as a connective between whole propositions) usually expresses inclusive disjunction, but perhaps sometimes exclusive disjunction. Inserted 'not' may or may not express the negation of the original proposition; prefixed 'It is not the case that' can be used to express negation more reliably, but still not always. And when it comes to putting together 'and's, 'or's and 'not's in logical contexts we'll at least sometimes need to use punctuation or bracketing to make clear how to group clauses together in an unambiguous way.

Now, these are all pretty tedious annoyances, not of serious logical interest! If our concern is with the assessment of arguments for validity, we really want to clear out of the way questions about whether in a particular case we mean minimal conjunction or *and then*, whether we mean inclusive or exclusive disjunction, etc. *before* turning to the assessment of the clarified argument. So here's an inviting strategy.

(1) Introduce some nicely tidied-up connectives *and**, *or**, *not** which *always* unambiguously express conjunction, inclusive disjunction, and negation; and insist that we use brackets with them so we can't get into Fred'n'Ginger ambiguities.

(2) Assess an ordinary language argument which relies on the connectives by regimenting it (transcribing it, translating it) using the unambiguous *and**, *or**, *not**, and then (with the clarification done) evaluate the tidily regimented version.

And this two stage procedure is just what we do in standard propositional logic.

As you will see, we introduce conventional symbols for the connectives instead of *and**, *or**, *not**. And for brevity's sake, we start abbreviating the basic propositions which are being connected by using single symbolic letters for them. But do be clear: the use of formal symbols is *not* of the essence: it is the tidying job they do which matters.

Now read *IFL* Chapter 8.