*IFL*: Logicbite 15

# Tautologies

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We now come to two core chapters of *IFL*, on *tautologies* and on *tautological entailment*. As you'll see soon enough, there is a sense in which a tautology is just a special case of a tautological entailment. That is why some authors (e.g. Virginia Klenk in her *Understanding Symbolic Logic*) discuss the general case first; but we'll start with the special case.

A preliminary remark. In both the special case and the general case we need to keep separate three different strands of the discussion:

(1) The definitions of the technical notions (tautology, tautological entailment). Crucial but straightforward.

(2) Philosophical remarks about the relation between the technical notions and corresponding informal ideas (tautology vs logical truth, tautological entailment vs logical entailment). A bit of care is required.

(3) Remarks about how to reduce the tedium involved in using truth tables to decide whether a wff is a tautology, or whether some wffs tautologically entail another wff. Unexciting – and don't let the fiddly details distract you from the Big Ideas involved in (1) and (2).

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Let's meet another new author, Tim Button, in his version of *forall*$\mathcal{X}$:[1]

> [In the section corresponding to *IFL* Chapter 13] we introduced the idea of a valuation and showed how to determine the truth value of any TFL ['Truth-Functional Logic'] sentence, on any valuation, using a truth table. In this section, we will introduce some related ideas, and show how to use truth tables to test whether or not they apply.
>
> [Earlier, cf. *IFL* p. 10], we explained *necessary truth* and *necessary falsity*. Both notions have surrogates in TFL. We will start with a surrogate for necessary truth.
>
> $\mathcal{A}$ is a tautology if and only if it is true on every valuation.
>
> We can use truth tables to decide whether a sentence is a tautology. If the sentence is true on every line of its complete truth table, then it is true on every valuation, so it is a tautology. [Mini-example: $P \vee \neg P$.]
>
> This is only, though, a *surrogate* for necessary truth. There are some necessary truths that we cannot adequately symbolize in TFL. One example is '$2 + 2 = 4$'. This *must* be true, but if we try to symbolize it in TFL, the best we can offer is a sentence letter, and no sentence letter is a tautology. Still, if we can adequately symbolize some English sentence using a TFL sentence which is a tautology, then that English sentence expresses a necessary truth.
>
> We have a similar surrogate for necessary falsity:
>
> $\mathcal{A}$ is a contradiction (in TFL) if and only if it is false on every valuation.

[1]The open source project *forall*$\mathcal{X}$ was started by P. D. Magnus. But the version that can be freely downloaded from forallx.openlogicproject.org owes a great deal to Tim Button, and passages I'm quoting are due to him.

If there are occasional faint echoes of *IFL* in his version, that is perhaps because Tim was once upon a time in my first year logic class! He was for a some years in my university post after I retired. Among other things, he is, with Sean Walsh, the author of a splendid book on *Philosophy and Model Theory*.

> We can use truth tables to decide whether a sentence is a contradiction. If the sentence is false on every line of its complete truth table, then it is false on every valuation, so it is a contradiction. [Mini-example: $\neg(P \vee \neg P)$.]

Button could have added that this too is only a *surrogate* for necessary falsehood. There are some necessary falsehoods that we cannot adequately symbolize in TFL. One example is '$0 = 1$'. This *must* be false (read as an arithmetic claim, of course), but if we try to symbolize it in TFL, the best we can offer is a sentence letter, and no sentence letter is a contradiction. However, if we can adequately symbolize some English sentence using a TFL sentence which is a contradiction then that English sentence expresses a necessary falsehood.

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A terminological note. Instead of 'tautology' vs 'contradiction' (which *is* surely the majority usage), we also find 'tautology' vs 'inconsistency' (Lemmon), 'truth-functionally true' vs 'truth-functionally false' (Simpson), 'logical truth' vs 'contradiction' (Teller). Goldfarb uses 'truth-functionally valid' rather than 'tautology' (a usage I deprecate, given how emphatically I elsewhere want to contrast the notions of validity and truth!). While Quine contrasts being 'valid' with being 'inconsistent' – but for him, these are primarily properties of *schemas* not formal sentences.

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Button goes on define some related notions such as tautological consistency which we'll meet later. For the moment, though, let's stick just to the first two key ideas which he introduces. And he has snappily given the headline news, both on (1) the two official definitions we want, and on (2) the relation between our defined formal notions and the corresponding informal notions.

We could say a bit more under the second heading. It is natural to claim that a tautology is necessarily true in virtue of its form – true in virtue of the way the connectives appear in it, as opposed to the content its atoms might have. But what does that mean? I spell things out in *IFL* Chapter 14. I also explain the sense in which we can *generalize* results about tautologies (having confirmed that $P \vee \neg P$ is a tautology, we don't need to redo the truth table to show that $Q \vee \neg Q$ is a tautology, or indeed that $(P \wedge \neg Q) \vee \neg(P \wedge \neg Q)$ is a tautology).

So now read on!