IFL: Logicbite 18

Introducing the material conditional

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When we are dealing with the truth-functional connectives 'and', 'or' and 'not', we have a crisp and clear definition of what it is for an argument to be tautologically valid – i.e. valid in virtue of the way those connectives feature in the premisses and conclusion. And we have a simple-minded (if tedious) brute-force method of deciding whether an argument is indeed tautologically valid – just run a truth-table test. Great!

Obviously, it will be equally great if we can extend our methods to cover arguments involving other connectives. And you already know that we can do that. I pointed this out in §16.1: so long as we are dealing with truth-functional connectives it still makes sense to talk of arguments involving them as being tautologically valid (and we can still apply a truth table test to determine whether they *are* tautologically valid). However, my examples so far of truth-functional connectives beyond the basic three won't have got your logical pulses racing! We've dealt with exclusive disjunction, which I suppose counts for something. But otherwise we have met the cooked-up dollar connective (which is no use for anything), and the absurdity constant '$\bot$'; (ok, that will be technically useful later, but still . . .).

It's time for something a bit more exciting! There's a so-familiar ordinary-language connective which features crucially in a vast number of arguments – the conditional, as in *if P then Q* (or simply *if P, Q* – or the other way around *Q, if P*). The ordinary-language conditional has been conspicuously missing from our story so far. Can we regiment this too in a PL style language, using a truth-functional connective?

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If you have been dipping into other textbooks as you go along, you'll have seen that most do indeed introduce a fourth connective from the outset. Alongside '$\land$', '$\lor$' and '$\neg$', we meet '$\to$' (alternatively '$\supset$') which (i) is a truth-functional connective, defined by another truth table, but (ii) is supposed to regiment the ordinary language conditional well enough, so that we can render *if P then Q* by the corresponding (P $\to$ Q).

And what's the truth-table for this new connective '$\to$'? Here it is:

P	Q	(P $\to$ Q)
T	T	T
T	F	F
F	T	T
F	F	T

But why *this* table? Here's Patrick Hurley suggesting some motivation – you just need to know that in *if P then Q*, P is said to be the antecedent of the conditional, and Q is said to be the consequent:

> Imagine that your logic instructor made the following statement: "If you get an A on the final exam, then you will get an A for the course." Under what conditions would you say that your instructor had lied to you? Clearly, if you got an A on the final exam but did not get an A for the course, you would say that she had lied. This outcome corresponds to a true antecedent and a false consequent.

So true P and false Q in this case clearly gives us a false conditional *if P then Q*, corresponding to the second line of the truth table.

> On the other hand, if you got an A on the final exam and also got an A for the course, you would say that she had told the truth (true antecedent, true consequent).

So with true P and true Q, the instructor has told the truth in saying *if P then Q*, in accordance with the first line of the table.

> But what if you failed to get an A on the final exam? Two alternatives are then possible: Either you got an A for the course anyway (false antecedent, true consequent) or you did not get an A for the course (false antecedent, false consequent). In neither case, though, would you say that your instructor had lied to you. Giving her the benefit of the doubt, you would say that she had told the truth.

If we buy that last thought, then that motivates the remaining two lines of the truth table.

Hurley goes fast here, and in fact returns later to say more. But this much is uncontroversially true: if we want to find a truth-functional connective which is any sort of rough candidate for regimenting the ordinary-language connective, then '$\to$' with the given table (also known for historical reasons as the *material conditional*) is the only option. But how good an option is it?

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That's a horribly difficult and contentious question – one I didn't want to get entangled with earlier. That's why I wanted to introduce truth-functional logic in *IFL*, and get you to see its great qualities, *before* tackling the conditional! (I see Paul Teller takes a similar line: by my lights, most authors unwisely try to sell the truth-functional '$\to$' too soon in their books!)

The situation, then, is this:

(1) Yes, 'and', 'or' and 'not' in ordinary language do behave in somewhat messy ways (even when just in their roles as operators on whole propositions); I catalogued some of the complications in §§8.2–8.4. But the complications are easy to describe and understand. And it does seem pretty clear that the officially truth-functional connectives '$\wedge$', '$\vee$' and '$\neg$' do capture core senses of those ordinary-language connectives. There's some tidying-up going on, but not outrageously much. It isn't a travesty to say that '$\wedge$' is (very often) a decent translation of 'and', etc.!

(2) By contrast, as you'll see, the behaviour of ordinary language conditionals isn't at all easy to describe and understand. For a start, there are importantly different varieties of conditional claims. Compare, for example,

    If Lee Harvey Oswald didn't shoot Kennedy, someone else did.
    If Lee Harvey Oswald hadn't shot Kennedy, someone else would have.

These are quite different claims, and arguably the first is true (given that *someone* shot Kennedy) and the second false (assuming Oswald in fact did it, acting quite alone). It seems plausible to say this: to evaluate the first sort of conditional, we just need to look at the actual world; to evaluate the second sort of conditional, we need to think about other possible worlds, other ways things might have been. But it is, to say the least, contentious how to spell out that plausible thought. And contentious how to carve up conditionals into different families needing such different kinds of treatment.

(3) And then, even if we take the *best* candidate family of conditionals for being regimented by a truth-functional '$\to$', it is horribly contentious how good the regimentation is. Is '$\to$' a decent tidying up of the core content of some 'if's? Or is it really – despite Hurley's motivating comments – more or less a travesty to propose '$\to$' as a translation of some 'if's? People have written whole books on this sort of question: it is very unclear both what we should say here and exactly how it should affect our formal logical treatment of conditionals.

Short version: conditionals are a minefield!

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OK, let me signal how I deal with things in the next two chapters of *IFL*.

(1) I begin in §18.1 by giving quick reminders of some elementary arguments involving 'if' which are clearly valid or clearly invalid. We will want any formal treatment of conditionals at least to give the right verdict in these elementary cases.

(2) Thinking about these elementary examples, I draw out some seemingly-obvious basic general principles about conditionals (§18.2), and then it is very easy to show that the only possible truth-functional rendition which obeys these principles is indeed '→' ($18.3). This so-called material conditional is the best we can do if we want something both conditional-like and truth-functional.

(3) And things initially go well. We see in §18.4 that if we translate the elementary arguments of §18.1 by using '→' for the conditional, and then run a truth table test, the arguments come out (tautologically) valid or invalid in just the right cases.

So far, so good! There are now some asides from the core theme, which you can skip on a first reading. First, I note that as well as 'if' conditionals, there are 'only if' conditionals and also 'if and only if' conditionals. So I extend our truth-functional story about 'if' conditionals to get matching truth-functional stories about these other conditionals in §§18.5, 18.6. Second, I officially extend the specification of a PL language to allow our new connective in §18.7. And third, in §188, I pause to hammer home the message that '→', '∴', and '⊨' are different sorts of symbols with different sorts of meanings, two belonging to our formal PL-languages, one belonging to our English metalanguage.

To continue, however, with the main line of development:

(4) I say more about the distinction between different types of conditional (as with the Oswald/Kennedy cases) in §19.1. At best, the truth-functional '→' is a candidate for regimenting one kind of conditional, what I arm-wavingly call 'simple indicative conditionals;.

(5) §§19.2 and 19.3 give some strong-looking arguments *for* treating those simple conditionals as truth-functional.

(6) §19.4 gives an equally strong-looking argument *against* treating those simple conditionals as truth-functional.

(7) We are now in a mess! In §19.5 I briefly explore three different responses. I look with a quite kindly eye on the third response, but leave it an open question whether it can be further defended.

(8) In §19.6, however, I suggest that we might side-step the problems we've found by changing tack. Perhaps we shouldn't think of our task as one of *more-or-less capturing* the core content of the ordinary-language conditional (whatever exactly that is); rather we should take our task to be one of *replacing* the conditional with a cleanly behaved substitute which will serve us well e.g. for mathematical purposes. The material conditional can then be sold as a 'best buy' replacement.

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Now read *IFL* Chapters 18 and 19 (perhaps initially omitting §§18.5–18.8). I'll return to say a bit more in the next Logicbite.