
Faculty of Philosophy

Formal Logic

Lecture 1

Peter Smith

One who wishes to attain human perfection must first study Logic.

Maimonides, *12th Century*

- Preliminaries
- What is Logic?
- Deduction and Induction
- Validity and Soundness
- Systematicity and Formality

Why Do Elementary Logic At All?

- ▶ Because getting clear about fundamental ideas like deduction, validity, consistency, etc. etc. is absolutely crucial to understanding how to evaluate philosophical arguments ...
- ▶ ... and a reading knowledge of formal logical symbolism is essential too. (Philosophers often use bits of logical symbolism to clarify their arguments.)
- ▶ Because the artificial and simply formal languages of logic give us highly illuminating objects of comparison when we come thinking about how natural languages work. (Relevant to topics in 'philosophical logic' and the philosophy of language.)
- ▶ But mainly because it us the point of entry into the study of one of the major intellectual achievements by philosophers of the 20th – i.e. the development of mathematical logic. (Not least in Cambridge: Russell and Whitehead, Ramsey, Turing.)

Why So Many Lectures/Classes?

- ▶ NOT because the Faculty thinks that logic is vastly more important than other 1A topics.
- ▶ We DO think that – perhaps unlike philosophy in general – it can usefully be systematically taught by lectures, because it lends itself to a cumulative, step by step, presentation.
- ▶ But you also need lots of checks as you are going along to confirm that you are indeed understanding – hence the back-up logic-classes.

The Course Text and These Lectures

- ▶ Peter Smith *Introduction to Formal Logic*, CUP 2003/2009. NB: get heavily revised reprint.
- ▶ The lectures will follow the organization of the book (up to you how many lecture notes you think it worth taking) ...
- ▶ ... and are mainly intended to be helpful back-up for the book (though I'll try to throw in plenty of asides and extras).
- ▶ Serious mathematicians can just read the book in about three days and then follow the classes, while reading some other more advanced logic books. These lectures are intended for the rest of you.
- ▶ **Anyone** who gets into Cambridge can do elementary formal logic if they put in enough effort. Read the book carefully, come to lectures especially if you have the slightest difficulties with the book, do the exercises in the book, go to logic classes!

- Preliminaries
- What is Logic?
- Deduction and Induction
- Validity and Soundness
- Systematicity and Formality

What's it all about, then?

The business of logic is the development of systematic techniques for the assessment of arguments for deductive validity.

What's an argument? – 1

The *Michael Palin analysis* (from the Monty Python 'Argument clinic' sketch!)

MP: Is this the right room for an argument?

JC: I've told you once.

MP: No you haven't.

JC: Yes I have.

MP: When?

JC: Just now!

MP: No you didn't.

JC: Yes I did!

MP: Didn't.

JC: Did. ...

What's an argument? – 2

...

MP: Oh, look this is futile.

JC: No it isn't.

MP: I came here for a good argument.

JC: No you didn't, you came here for an argument.

MP: Well, an argument's not the same as contradiction.

JC: It can be.

MP: No it can't. **An argument is a connected series of statements intended to establish a definite proposition.**

An argument (in the sense that concerns us) is a passage of *reasoning* in which considerations are offered in support of a conclusion.

The evaluation of arguments - 1

- ▶ What kind of evaluation are we interested in?
- ▶ Consider [A]:
 - Bertrand Russell is an Oxford philosopher.
 - All Oxford philosophers are bad logicians.
 - So: Bertrand Russell is a bad logician.

Premises are false: but the **inferential move** is cogent.
- ▶ Compare
 - Socrates is a man.
 - All men are mortal.
 - So: Socrates is mortal.

Obviously both arguments use the same type of inference move which is **guaranteed** to preserve truth (i.e. if the premisses are true, then the conclusion has to be true too).

The evaluation of arguments - 2

- ▶ What kind of evaluation are we interested in?
- ▶ Consider [B]:

Russell loves logic.

All good philosophers love logic.

So: Russell is a good philosopher.

Premises and conclusions are true: but inferential move is a bad one – the premisses do not support the conclusion.

- ▶ Compare

Socrates is a human being.

All women are human beings.

So: Socrates is a woman.

Same type of inference move, and here it leads from true premisses to false conclusion – so is obviously unreliable.

The evaluation of arguments - 3

- ▶ We can **assess the premisses** – ‘does the argument start off from truths’?
- ▶ We can **assess the reliability of the inference** – ‘is the move from the premisses to the conclusion a reliable one (one that would give us good reason to believe the conclusion if we accept the premisses)?’
- ▶ Logic is concerned with the SECOND kind of evaluation, the evaluation of an argument for **internal cogency**.

Probabilifying inferences – 1

- ▶ Consider the argument [A]
 1. I have bought just one ticket for the next lottery draw.
 2. 10 million lottery tickets have been sold for the next lottery draw.
 3. The next lottery draw will be fair.
 4. So: I won't win.

- ▶ The premisses give me very strong reason to believe the conclusion – they make it very probable. But the premisses don't absolutely guarantee the conclusion. It might be my lucky week. It is possible, even if very, very unlikely, that premisses are true and the conclusion false.

Probabilifying inferences – 2

- ▶ Why do you believe that this lecture won't last until midnight?
- ▶ On the basis of some such reasoning as [B]
 1. No previous logic lecture time-tabled for an hour has lasted for a whole day.
 2. This logic lecture is time-tabled for an hour.
 3. So: this lecture isn't now going to last for a whole day.
- ▶ Again: the premisses give you very strong reason to believe the conclusion (make it probable) – but they don't absolutely guarantee the conclusion.

Absolutely watertight inferences

- ▶ Contrast the 'lottery' and 'lecture' arguments with the 'Russell argument':
 1. Bertrand Russell is an Oxford philosopher.
 2. All Oxford philosophers are bad logicians.
 3. So: Bertrand Russell is a bad logician.
- ▶ The inference in this argument is **absolutely watertight**, the premisses (if they are true) absolutely guarantee the truth of the conclusion.
- ▶ If the premisses of this argument are true, the conclusion **must** be true (there is no possibility, not even a remote chance, that the premisses are true and the conclusion is false).
- ▶ In any conceivable situation (in any 'possible world') in which the premisses are true the conclusion is true too.

Probabilifying vs. absolutely watertight inferences

- ▶ NB Our probabilifying arguments give you VERY good reason to believe their conclusions. You are entirely rational to believe that you won't win the lottery, that this lecture will stop before midnight.
- ▶ So our probabilifying arguments are good arguments, in the sense that their premisses really DO strongly support their conclusions.
- ▶ Still, there is a difference between 'strongly support' and 'absolutely guarantee'.

Jargon time!!

- ▶ An inference move which is such that if its premisses are true then the conclusion **MUST** be true – i.e. an absolutely watertight inference – is said to be **deductively valid**.
- ▶ The notion goes back to Aristotle's *Prior Analytics*:

A deduction (syllogismos) is speech (logos) in which, certain things having been supposed, something different from those supposed results of necessity because of their being so.
- ▶ The key notion here is '**results of necessity**'.
- ▶ Aristotle builds in three more ideas which we might query:
 1. Need a valid deduction have more than one premiss?
 2. Need the conclusion of a valid deduction be different from each premiss?
 3. Need the conclusion of a valid deduction be true *because of* the truth of each premiss?



Aristotle was born in Stagira, Macedonia in 384 B.C, and died in Chalcis, on the Aegean island of Euboea, in 322 B.C. Depicted here (on the right) in Raphael's *School of Athens* (1510/11).

Jargon time (continued)

- ▶ We said: an inference move which is such that if its premisses are true then the conclusion **MUST** be true – i.e. an absolutely watertight inference – is said to be **deductively valid**.
- ▶ An inference move which is such that its premisses strongly probabilify but don't absolutely guarantee its conclusion is said to be **inductively strong**.
- ▶ For shorthand: a *deductive argument* is one that aims to use a deductively valid inference; an *inductive argument* is one that aims to use an inductively strong inference.
- ▶ (NB 'aims': there can be invalid deductive arguments, i.e. arguments that purport to use deductively valid inferences but don't – like fallacious mathematical proofs. Likewise there can be weak inductive arguments.)

Deductive logic

- ▶ Our focus in this course is entirely on *deductive* arguments.
- ▶ Evaluating a deductive argument – one that aims to use deductively valid inference(s) – is a matter of seeing whether the inference moves are indeed deductively valid.
- ▶ We said ‘the business of logic is the evaluation of arguments’. We can now say that (at least for the purposes of this course) ‘the business of logic is the evaluation of arguments to see whether their inferences are deductively valid’.

- Preliminaries
- What is Logic?
- Deduction and Induction
- Validity and Soundness
- Systematicity and Formality

The dimensions of evaluation again

- ▶ A deductive argument can be evaluated in two independent ways, for the TRUTH of the premisses, for the VALIDITY of its inferential move.
- ▶ An argument can have *true* premiss/premisses and be *valid*:
Bertrand Russell had a brother.
So: Bertrand Russell wasn't an only child.
- ▶ An argument can have *false* premisses and be *valid*:
Bertrand Russell was Greek.
All Greeks are great philosophers.
So: Bertrand Russell was a great philosopher.
- ▶ An argument can have *true* premisses and be *invalid*:
Bertrand Russell was a Trinity philosopher.
So: Bertrand Russell was a great philosopher.
- ▶ An argument can have *false* premisses and be *invalid*.

The dimensions of evaluation: jargon again – 1

- ▶ It is crucial to keep these dimensions of assessment sharply separate. TRUTH is a property of successful statements: VALIDITY is a property of successful inferences. NEVER say that a statement is 'valid' or an inference is 'true'.
- ▶ It is conventional to say – for shorthand – that an argument is valid when the inference(s) in it is/are valid.
- ▶ But NB: on this conventional usage, to say an argument is valid is ONLY to make a positive evaluation of its inferential move(s). It ISN'T to make an all-in positive evaluation.
- ▶ NB again: there can, on the conventional usage of the term, be valid arguments with (some) false premisses, for example
Tony Blair says that God exists
Everything Tony Blair says is true.
So: God exists.

The dimensions of evaluation: jargon again – 2

- ▶ To repeat: in the standard logician's sense

Tony Blair says that God exists

Everything Tony Blair says is true.

So: God exists.

is a VALID argument. Having a valid argument for a conclusion does NOT establish that conclusion as true.

- ▶ Of course, in the street sense, if someone claims to have a 'valid' argument that God exists they are usually claiming to have proved that God exists. To do that they need more than valid inferences, they need true premisses as well.
- ▶ The standard logician's term for an argument which DOES evaluate positively on both dimensions – i.e. makes a VALID inference move from TRUE premisses – is SOUND.
- ▶ The Blair argument is logically valid, but not sound.

- Preliminaries
- What is Logic?
- Deduction and Induction
- Validity and Soundness
- Systematicity and Formality

Patterns of argument

Consider the following arguments:

All philosophers are neurotic.

Russell is a philosopher.

So Russell is neurotic.

All existentialists write bosh.

Sartre is an existentialist.

So Sartre writes bosh.

All Prime Ministers tell lies.

Gordon Brown is a Prime Minister.

So Gordon Brown tells lies.

Evidently these all involve the SAME type of inferential move, and are **valid** for the same reason. (Are they **sound**??)

Generalizing about arguments like

All philosophers are neurotic.

Russell is a philosopher.

So Russell is neurotic.

we can say

Any inference step which has one premiss which says that everything which has a certain property has a second property, and whose other premiss says that a particular individual has the first property, and whose conclusion says that individual has the second property, is valid.

But it is a LOT easier to say:

Any inference of the following form

All F are G

n is F

So, n is G.

is valid.

Similarly

No woman is a great poet.

Homer is a woman.

So Homer is not a great poet.

is valid. And we can generalize:

Any inference step which has one premiss which says that nothing which has a certain property has a second property, and whose other premiss says that a particular individual has the first property, and whose conclusion says that individual lacks the second property, is valid.

It is a LOT easier to say:

Any inference of the following form

No F is G

n is F

So, n is not G.

is valid.

Aside: getting more complex in two ways

- ▶ Multi-premiss inferences. We've of course just taken two very simple examples of reliable patterns of argument. And not all reliable patterns depend on quantifiers like 'All' or 'No'.

Consider e.g. argument by cases (three premisses):

Either P or Q

If P then R

If Q then R

So, R

- ▶ Multi-step arguments. We'll of course be interested not just in one-step arguments but in longer **proofs** that aim to chain together more than one valid steps.

Systematicity and formality

- ▶ Even our simple examples show that, as soon as we want to study deduction systematically, we'll want to talk about FORMS of argument.
- ▶ And forms of argument are naturally represented symbolically. (Even Aristotle, in his logical works, uses a primitive symbolic apparatus.)
- ▶ So logic – as a systematic study of deduction – immediately and entirely naturally becomes 'SYMBOLIC LOGIC' or 'FORMAL LOGIC'.
- ▶ And as we'll see later, there is another entirely natural pressure that pushes us towards seeing formal symbolism as an essential tool of logical theory.

Final examples

We've taken examples so far where it is obvious that the argument is valid/invalid as the case may be. Some cases aren't quite so obvious. Which of these are valid?

- ▶ Most Irish are Catholics.
Most Catholics oppose abortion.
So, at least some Irish oppose abortion.
- ▶ Every pigeon is in a pigeon-hole.
There are more pigeons than pigeon-holes.
So, at least two pigeon-holes contain more than one pigeon.
- ▶ There are more people than there are hairs on anyone's head.
So, at least two people have the same number of hairs on their head.