Faculty of Philosophy

Formal Logic

Lecture 2

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- Validity again
- Systematicity and formality
- Modality and the invalidity principle
- The counterexample method

Valid deductions as absolutely watertight inferences

- Consider this inference

  Every good philosopher knows some logic.
  No post-modernist knows any logic.
  Jacques is a post-modernist.
  So: Jacques isn't a good philosopher.

  Who knows whether the premisses are true or false? But *if* the premisses are true, the conclusion has to be true too.
- The inferential move in this argument from premisses to conclusion is absolutely watertight. There is not even a remote chance, there's no possibility at all, that the premisses are true and the conclusion false.
- Such an inference, we said, is deductively valid.
- It is conventional to call a (one-step) argument valid if its inferential move is valid. An argument which is valid and has true premisses is called sound.

Logicians' deductions vs Sherlock's deductions

- The detective gathers evidence, and 'deduces' who did the dastardly deed.
- But this (normally) is not a valid deduction in the logician's sense. The detective's account of the murder may fit the facts and strike us as obviously the best explanation. But the best (= most plausible, most likely) explanation doesn't have to be the only one that is logically consistent with the facts.
- So the 'deduction' from the evidential facts to the detective's proposed explanation isn't absolutely guaranteed to be truth-preserving, i.e. is not deductively valid in the logician's sense.

Deduction in science – 1

- You might worry: "If deductive inference is to be contrasted with inductive, probabilistic, inference and with inference to the best explanation, does that mean that deductive inference only really features e.g. in mathematics and not in empirical science (which is surely based on induction and IBE)?"
- Here's a picture of science. We make theoretical conjectures; we use these conjectures (plus initial conditions) to make predictions; we test the predictions. If the predictions are right, the conjectured theory lives to fight another day. If not, some conjecture has to be revised.

Deduction in science – 2

- On this picture, deductive reasoning comes into the story in deriving predictions from theoretical conjectures (plus initial conditions).
- Deductive logic tells us what else has to be true, if our theoretical conjectures are true. If the predictions are falsified, then – as a matter of deductive logic – the theory is in trouble.
- So deduction is, after all, essential to science. (The hypothetico-deductive model of science: Karl Popper's falsificationism.)

Without deductive logic, science would be entirely useless

Alfred North Whitehead, 1861–1947.

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Patterns of argument

Consider the following arguments:

> No woman is a great poet.
> Homer is a woman.
> So Homer is not a great poet.

> No elephant ever forgets.
> Jumbo is an elephant.
> So Jumbo never forgets.

> No Prime Minister leaves office willingly.
> Gordon Brown is a Prime Minister.
> So Gordon Brown didn't leave office willingly.

Evidently these all involve the SAME type of inferential move, and are valid for the same reason.

We can describe the pattern of inference here, laboriously in words. But ... It is a LOT easier to say:

> *Any inference of the following form*
>
> > *No F are G*
> > *n is F*
> > *So, n is not G.*
>
> *is valid.*

Intuitively, our examples illustrate this basic form, despite the slight differences in surface form (e.g. 'never' for 'not ever'; the tensed 'didn't').

As soon as we want to study deduction generally and systematically, we'll want to talk about forms of argument, using some symbolic apparatus: and we'll want to regiment away from insignificant differences in surface form.

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What kind of possibility?

- Notions like necessity, possibility, impossibility are said to be modal notions.
- The notion of validity is another modal notion: an inference is valid if it is not possible for the premisses to be true and conclusion false.
- But what kind of possibility is in question here?
- Not e.g. practical possibility, or physical possibility, . . .
- . . . but the weakest kind of possibility, logical possibility. (Related to conceivability, what can coherently be described.)
- So correspondingly, if an argument is valid, it is impossible in the strongest sense for the premisses to be true and conclusion false.

The invalidity principle

- If something is actually the case, then it is certainly possible.
- So an inference ab esse ad posse is valid.
- Hence: If an inference actually has true premisses and a false conclusion then it is certainly possible that it has true premisses and a false conclusion.
- In other words: If an inference actually has true premisses and a false conclusion then it is invalid.
- Call that the invalidity principle.
- An invalid argument can have any combination at all of true/false premisses and a true/false conclusion. (Invalidity imposes no constraints.) But a valid argument cannot have all true premisses and a false conclusion.

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Questions of validity aren't always obvious

We've taken a very few examples so far where it is obvious that the argument is valid/invalid as the case may be. But it is not always obvious what follows from what. And even very simple inferences may need a moment's thought. For example, what about . . .

> Most Italians are Catholics.
> Most Catholics oppose abortion.
> So, at least some Italians oppose abortion.

> There are more people than there are hairs on anyone's head.
> So, at least two people have the same numbers of hairs on their head.

Two methods

- We are now going to introduce two methods for evaluating arguments for validity:
  1. The counterexample method for showing that an invalid inference is invalid.
  2. The proof method for showing that a valid inference is valid.
- There is nothing at all puzzling/mysterious/difficult about these methods – in fact they are Sunday-best versions of methods we use in the everyday assessments of arguments.

Applying the invalidity principle

Consider:

Most grandmasters are men
Most men are no good at chess
So: At least some grandmasters are no good at chess

True premisses, false conclusion, so the inference is invalid by the invalidity principle. That shows that this argument pattern is unreliable:

Most F are G
Most G are H
So: At least some F are H

Compare:

Most Italians are Catholics.
Most Catholics oppose abortion.
So, at least some Italians oppose abortion.

Same pattern of reasoning, so this argument isn't valid either.

The 'you might as well argue' gambit

Put the point another way. Faced with the argument

Most Italians are Catholics.
Most Catholics oppose abortion.
So, at least some Italians oppose abortion.

we can challenge this: "But you might as well argue . . .

Most grandmasters are men
Most men are no good at chess
So: At least some grandmasters are no good at chess

But the SECOND argument is obviously hopeless (the inference is obviously is not truth-preserving), and the FIRST argument (which relies on the same type of inference) is no better.

Another example

Consider:

Most philosophy lecturers have read Spinoza's *Ethics*.
Few rugby fans have read Spinoza's *Ethics*.
So: Few philosophy lecturers are rugby fans.

This is relying on the pattern of argument:

Most F are H
Few G are H
So: Few F are G

Compare

Most philosophy lecturers have a philosophy PhD.
Few owners of televisions have a philosophy PhD.
So: Few philosophy lecturers own televisions.

True premisses, false conclusion, so the inference is invalid. The pattern of reasoning is not guaranteed to preserve truth, so the original argument which relies on it can't be a valid one.

The counterexample method

- The basic idea: we can bring together the invalidity principle with the point that arguments can share patterns of inference step on which they rely to give us a powerful technique for showing arguments to be invalid (when they are!).
- Step 1: Locate the type of inference step on which the target argument is relying.
- (This can require careful interpretation of an informally presented argument.)
- Step 2: Show this inference step is unreliable by producing an actual *counterexample* (an argument using that type of inference step which has actually true premisses and a false conclusion).

The counterexample method extended

- Step 1: Locate the type of inference step on which the target argument is relying.
- Step 2* Show this inference step is unreliable by producing an *evidently possible counterexample* (i.e. an argument using that type of inference step which is such that it is obviously possible that its premisses are true while its conclusion is false, so is invalid).

Possible counterexamples are bad enough! For if it is *possible* for a type of inference to lead from truth to falsehood, then an argument which relies on that type of inference cannot be absolutely guaranteed to preserve truth.

An example from Aristotle

Every art and every enquiry, and similarly every action and pursuit, is thought to aim at some good; and for this reason the good has rightly been declared to be that at which all things aim.

This looks to be the argument:

Every practice aims at some good
So there is a good ('the good') at which all practices aim

This exemplifies the pattern:

Every F has relation R to some G
So, there is some G such that every F has R to it

Compare:

Every road leads to some destination
So there is some destination (Rome?) where every road leads

Every boy loves some girl
So there is some girl loved by every boy

These parallel arguments (could) have true premisses, and false conclusion, so the pattern of reasoning isn't guaranteed to preserve truth.

Quantifier shift fallacy

- Logicians call *every* and *some* quantifiers (others are ‘no’, ‘all’, ‘at least two’, ...).
- So the fallacious pattern

  Every F is R to some G
  So, there is some G such that every F has R to it

  is a *quantifier shift fallacy*
- Compare also

  Every causal chain has an uncaused first link.
  So there is some uncaused first link (God?) at the start of all casual chains.
- Every ecological system has some designer.
  So there is some designer (God??) responsible for every ecological system.