Faculty of Philosophy

Formal Logic

Lecture 3

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The first logic class!

- Worksheet 1 available from www.logicmatters.net
- Work to be handed in on Monday
- When? by noon p.m.
- How? Either given to Dr Clare Chambers at the end of her 11 a.m. lecture, or placed directly in the appropriate folder in the pigeon holes outside Faculty.
- Classes: 10 Tuesday. Take a copy of *IFL*

Logic is the beginning of wisdom.

St. Thomas Aquinas, 13th Century

- Last lecture . . .
- Proofs, direct and indirect
- Where next?
- Divide and rule

The counterexample method

To show an invalid argument *is* invalid . . .

- (1) Locate the pattern of inference step on which the target argument is relying.
- (2) Show this pattern of inference step is unreliable by finding an counterexample

A counterexample is an argument using the pattern of inference step in question which is such that it is evidently possible that the premisses could be true while its conclusion is false

This could be evident because the premisses actually *are* true while the conclusion is false. But that's not required – a clearly possible counterexample is enough to show that the inference step doesn't absolutely guarantee that, if the premisses are true, the conclusion is too.

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The basic idea of a proof

- The counterexample technique can only establish *invalidity*. (Failing to find a counterexample doesn't show there isn't one – only, perhaps, that we are not good at spotting them! So failing to find a counterexample doesn't establish validity.)
- So how then do we establish the validity of an unobviously valid argument?
- One familiar way: by breaking down the big, unobvious, leap from premisses to conclusion into smaller steps, each small step being obviously valid.
- That is, by giving a proof.

A silly example

Consider the following inference

> Babies cannot understand syllogisms.
> Nobody is despised who can wrestle a crocodile.
> Anyone who is logical can understand syllogisms.
> People who are not logical are despised.
> Hence, Babies cannot wrestle a crocodile.

This is in fact a valid inference from four premisses!

Blame Lewis Carroll!

Charles Lutwidge Dodgson (1832–98) – wrote a two volume work *Symbolic Logic*, and published a famous and much discussed short paper on logical inference in *Mind* 1895, 'What the Tortoise Said to Achilles'.

A silly example (continued)

Let's think how we can fill in between the premisses and the conclusion of the "crocodile argument".

1)	Babies cannot understand syllogisms	(Premiss)
2)	Nobody is despised who can wrestle a crocodile.	(Premiss)
3)	Anyone who is logical can understand syllogisms.	(Premiss)
4)	People who are not logical are despised.	(Premiss)
5)	Babies are not logical	(From 1,3)
6)	Babies are despised	(From 4,5)
7)	Babies cannot wrestle a crocodile	(From 2,6)

We can fill the gap between initial premisses and final conclusion by a couple of little steps that are obviously truth-preserving, so the big inferential leap must be valid.

An even sillier example.

Take the inference 'Everyone loves a lover; Romeo loves Juliet; so everyone loves Juliet'. (Assume a *lover* is a person who loves someone, and *Everyone loves a lover* means everyone loves any lover.)

Then this inference is valid. To show this, consider . . .

1)	Everyone loves a lover	(Premiss)
2)	Romeo loves Juliet	(Premiss)
3)	Romeo is a lover	(From 2)
4)	Everyone loves Romeo	(From 1, 3)
5)	Juliet loves Romeo	(From 4)
6)	Juliet is a lover	(From 5)
7)	Everyone loves Juliet	(From 1, 6)

Each small inference step is obviously valid = necessarily truth-preserving. So if the initial premisses are true, truth must percolate down to the final conclusion. So the big inference leap from premisses to final conclusion is valid.

Why do proofs work?

1. The simple case: If A entails B and B entails C, then A entails C.
2. In symbols, using '$\vDash$' for 'entails':

$$\frac{A \vDash B \qquad B \vDash C}{A \vDash C}$$

3. Why does this hold? Suppose the possible situations in which A holds are included in those in which B is true; and suppose the possible situations in which B holds are included in those in which C is true. Then it follows that the possible situations in which A are included in those in which C is true.
4. Generalized transisitivity

$$\frac{\Gamma \vDash B \qquad \Delta, B \vDash C}{\Gamma, \Delta \vDash C}$$

A Reductio proof

- Our two examples of proofs so far have been of *direct* proofs, where we march in a straight line from premisses to conclusion. Some useful kinds of proof are *indirect*.
- In our last example, we made the following inference, en route to our conclusion:

  1) Babies cannot understand syllogisms (Premiss)
  3) Anyone who is logical can understand syllogisms (Premiss)
  5) Babies are not logical (From 1,3)

  Suppose someone is being dim and doesn't see that this follows, how could we convince them?

A Reductio proof

- Our two examples of proofs so far have been of *direct* proofs, where we march in a straight line from premisses to conclusion. Some useful kinds of proof are *indirect*.
- In our last example, we made the following inference, en route to our conclusion:

1)	Babies cannot understand syllogisms	(Premiss)
3)	Anyone who is logical can understand syllogisms	(Premiss)
Suppose, for the sake of argument		
4a)	Babies are logical	(Suppos'n)
4b)	Babies can understand syllogisms	(From 3,4)
4c)	Contradiction!	(From 1,4a)
So the supposition must be false		
5)	Babies are not logical	(From 4–4b)

The last inference is by Reductio ad Absurdum

The pattern here

1	*Some premisses*	
2	$\quad A$	Temporary assumption
3	$\quad \vdots$	
4	$\quad \vdots$	
5	$\quad$ *Contradiction*	
6	$not - A$	

Another Reductio proof

- Consider the following argument: 'If Blair was telling the truth, then there were WMD in Iraq. If WMD haven't been found in Iraq, then there weren't any. But they haven't been found. So Blair wasn't telling the truth.'
- Here's a proof of the conclusion from the (tidied) premisses:

1)	If Blair was telling the truth, then there were WMD in Iraq	(Premiss)
2)	If WMD haven't been found in Iraq, then there weren't any WMD in Iraq.	(Premiss)
3)	WMD haven't been found in Iraq.	
4)	Blair was telling the truth	(Suppos'n)
5)	There were WMD in Iraq	(From 1, 4)
6)	There weren't any WMD in Iraq	(From 2,3)
7)	Contradiction!	(From 5,6)
8)	Blair wasn't telling the truth	(4, Reductio)

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Two techniques ...

- The counterexample technique for demonstrating argument A is invalid:

  Find an argument of the same form as A which has (or evidently can have) true premisses and a false conclusion. That shows that the argument form in question cannot be reliable.

- The proof technique for demonstrating argument B is valid:

  Show that you can get from the premisses of B to its conclusion via little steps (maybe making temporary assumption en route), each small step being obviously and uncontroversially valid.

... what our techniques share

- Note: both of these techniques – at least in their initial, informal versions – are rather hit-or-miss.
  1. Failing to find a counterexample doesn't show that an argument is valid (except in some special cases where we can do systematic searches 'through the space of possible counterexamples').
  2. Failing to find a deduction doesn't show that the argument isn't valid.
- So ... Can we find more methodical ways (ideally, *mechanical* ways) of testing various arguments for validity?

Getting methodical

- NB, Our more methodical methods of logical evaluation are based on the idea of looking for counterexamples or constructing proofs.
- For example, we will be soon looking at the so-called 'truth-table test'. This (we'll find) is a systematic way of looking for counterexamples for certain classes of inferences.
- For another example, we will be looking at 'tree proofs'. These are systematized proofs for certain inferences expressible in certain limited languages.

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Issues of interpretation – 1

Recall some propositions from our examples:

1. If WMD haven't been found in Iraq, then there weren't any. In evaluating the argument, we had to read that as '...there weren't any WMD in Iraq.' (Compare 'Blair says that there were good reasons for invading. But if WMD haven't been found in Iraq, then there weren't any.' Here the last clause is naturally read '...there weren't any good reasons for invading.')
2. Everyone loves a lover. We read that as 'Everyone loves anyone who is a lover'. (Similarly 'I rather dislike Wagner, but I just love a good Verdi opera' is naturally understood as '...I love any good Verdi opera'. But compare 'Every boy loves a girl' which in many contexts is probably more naturally read as 'Every boy loves some girl or other'.)

Issues of interpretation – 2

A philosophical example

1. Consider All the tickets might be winners. What does that mean? It could mean
   1.1 Any particular ticket, taken by itself, might be a winner. [It is a fair draw]
   1.2 It might be that the whole lot are winners together. [The tickets are for shares in an entry to a high-price lottery.]
2. Compare All our perceptual experiences might be delusory What does that mean? It could mean
   2.1 Any particular experience, taken by itself, might be a delusory one.
   2.2 It might be that the whole lot are delusory together.

Plainly, to argue using the proposition All our perceptual experiences might be delusory, we need to disambiguate.

Issues of interpretation – 3: Contextual variation

- Further complications come from the contextual variation of the message conveyed by given words. Consider e.g. ‘Mary had a little lamb’
- . . . as a reply to ‘What pet did Mary have as a five year old?’
- . . . as a reply to ‘What did Mary have for lunch?’
- ‘Oh horror! The alien mad scientists are bringing about monstrous births . . . ’

Issues of interpretation – 4

- In sum, to evaluate arguments we often first need to do interpretative work (perhaps with hints from the context) to disambiguate and/or to fill out the exact message being conveyed by premisses and conclusions.
- Only after any such interpretative work is done can we then evaluate the inference between premisses and conclusions. But notice the kind of interpretative step isn't the sort of thing that we can readily give any sort of systematic theory for.
- So overall, the evaluation of real-life arguments for validity is a two step process –
  1. preliminary interpretative clarification
  2. logical evaluation
- Step 1 is all important – but it isn't especially the business of logicians (and it isn't the sort of thing that we can readily give any sort of systematic theory for).

The vagaries of English – 1

- When we try to deal systematically with arguments couched in a natural language like English, even once we have done any required disambiguation, there are problems.
- For example consider the arguments

  Every play by Shakespeare is worth seeing;
  Hamlet is a play by Shakespeare;
  So, Hamlet is worth seeing.

  Any play by Shakespeare is worth seeing;
  Hamlet is a play by Shakespeare;
  So, Hamlet is worth seeing.

  Each play by Shakespeare is worth seeing;
  Hamlet is a play by Shakespeare;
  So, Hamlet is worth seeing.

- Despite the different uses of 'every', 'each', 'any' , these arguments surely in good sense involve the same logical move.

The vagaries of English – 2

- But it isn't that e.g. 'Every play by Shakespeare is worth seeing' and 'Any play by Shakespeare is worth seeing' are just trivial stylistic variants either.
- E.g. contrast

  I doubt that every play by Shakespeare is worth seeing.
  I doubt that any play by Shakespeare is worth seeing.

- E.g. contrast

  If Mary has read every play by Shakespeare, I'll eat my hat.
  If Mary has read any play by Shakespeare, I'll eat my hat.

- So English quantifiers like 'every' and 'any' behave in complex ways.

How to cope

- Add in the complexities of the behaviour of 'each' and 'all' as well, and we have four different ways of expressing generality in English, with subtly different logical behaviours.
- We'll be seeing more examples of untidy logical behaviour in English in a moment.
- Logicians, then, will naturally be interested in moving from the messy vernacular to some clean, uniform, unambiguous ways of representing premisses and conclusions, before assessing arguments.
- That's why logicians construct artificial languages of logic for perspicuously representing the logically relevant content of premisses and conclusions.
- A key part of this course will introduce a couple of basic languages of logic.

Divide and rule

- In sum: How are we going to separate out the business of interpretative clarification from the core business of logic, i.e. the evaluation of arguments once their premisses and conclusions are clarified?
- The standard strategy (even prefigured in Aristotle): Introduce some special symbolical languages that by stipulation are clear and unambiguous (and have clear content independently of context). Then the overall evaluation of arguments can proceed in two steps:
    1. We regiment an ordinary-language argument into some appropriate symbolic language.
    2. Assess the regimented argument in its regimented symbolic form.

Implementing the divide and rule strategy

- Our main target in this course will be to consider the logic of arguments that can be regimented into a very powerful standard language of logic often called simply **QL** (the language of quantificational logic).
- As the name suggests, this language has the resources to express the sorts of claims we express in English using quantifiers like all, some, none, etc. (Some philosophers think **QL** is a rich enough type of language to regiment any scientific theory, for example.)
- But before turning to **QL** later in the term, we'll first spend quite a bit of time on a fragment of this language called **PL** (the language of propositional logic). It will be a lot easier to introduce a range of key logical ideas in the context of this simpler language – so that's our excuse for spending a lot of time on it at the outset.