Faculty of Philosophy

Formal Logic

Lecture 4

Peter Smith

- Divide and rule
- ‘And’, ‘or’ and ‘not’
- The three basic propositional connectives
    - Conjunction
    - Disjunction
    - Negation
- Bivalent propositions and sets of possible worlds
- Complex propositions

Issues of interpretation – 1

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- E.g. we'll need to work out what is being said, in context, by 'Mary went to the bank' (lexical ambiguity), 'She kissed him' (referential ambiguity), "Mary had a little lamb'.
- Even if words are clear, we may have to deal with structural ambiguities: e.g. 'Every experience might be delusory'.

Issues of interpretation – 2

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- But notice the kind of interpretative step isn't the sort of thing that we can readily give any sort of systematic theory for.
- The possibility of a systematic logical theory kicks in at the next stage, where we evaluate inferences between sufficiently clearly regimented propositions.
- But natural languages like English aren't ideally suited for doing the regimentation. (Consider e.g. the vagaries of quantifiers in English.)

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- Logicians, then, will naturally be interested in moving from the messy vernacular to some clean, uniform, unambiguous ways of representing premisses and conclusions, before assessing arguments.
- That's why logicians construct artificial languages of logic for perspicuously representing the logically relevant content of premisses and conclusions.
- Then the overall evaluation of arguments can proceed in two steps:
    1. We regiment an ordinary-language argument into some appropriate symbolic language.
    2. Assess the regimented argument in its regimented symbolic form.

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- This will introduce us to the language called **PL** (the language of propositional logic). It will be easy to introduce a range of key logical ideas in the context of this toy language – so that's our excuse for spending a lot of time on it at the outset.

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   We are not out of petrol.
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1. Either we are out of petrol or the carburettor is blocked.
   We are not out of petrol.
   So, the carburettor is blocked.
2. Either Jill is in the library or she is in the bar.
   She is not in the library.
   So Jill is in the bar.

These inferences are both valid, as is any of the type

Either P or Q
Not-P
So: Q

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These inferences are both invalid, as is any that relies on an inference of the type

Not-(P and Q)
So: Not-P

A further example

1. It's not the case that Alice is clever and also that Bertie is clever too.
   Alice is clever
   So Bertie is not clever.
2. It's not true that Brown and Blair both supported the policy
   Brown supported the policy
   So Blair did not support it.

These inferences are this time both valid, as is any of the type

Not-(P and Q)
P
So: Not-Q

Scope matters!

Note the use of bracketing to make it clear what the 'not' applies to (what its scope is). So compare

1. Not-(P and Q)
2. (Not-P) and Q

If

P = Obama will order an attact
Q = There will be devastation

then

Not-(P and Q) = it isn't true that Obama will order an attack and there will be devastation;
(Not-P) and Q = Obama will not order an attack, but there will still be devastation.

The scope of the 'not' matters.

Introducing the language **PL**

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- For example, consider the party invitation 'You can bring your partner or come alone and have a good time'. (How does that 'group together', how should we 'bracket' it?)
- So following the 'divide and rule' strategy, we first need to characterize a formal language **PL** for regimenting arguments clearly and without ambiguities (whether word ambiguities or structural scope ambiguities).

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Bare Conjunction – 1

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- NB ‘$\wedge$’ always comes with a pair of brackets.

Bare Conjunction – 2

- Contrast

  Mary became pregnant and she got married.
  Mary got married and she became pregnant.

  with

  (Mary became pregnant $\wedge$ *she got married).*
  (Mary got married $\wedge$ *she became pregnant).*

  So does ordinary language 'and' mean more than '$\wedge$'?

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- Perhaps not. For compare

  Mary became pregnant. She got married.
  Mary got married. She became pregnant.

  Standard narrative conventions still generate implications of temporal order, so we don’t need to say that ‘and’ is responsible for such implications.

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- We are assuming ϕ and ψ are determinate propositions (not vague or subject to other failings – more on this later).
- And each way the world can turn out fixes whether $(\phi \wedge \psi)$ is true or false – as follows ...

The truth table for conjunction – 2

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- Given two propositions, ϕ and ψ, there are four ways that the world way may turn out, and they fix whether $(\phi \wedge \psi)$ is true or false:

ϕ	ψ	$(\phi \wedge \psi)$
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False	True	False
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- And we can now naturally abbreviate this – as follows ...

The truth table for conjunction – 2

- $(\phi \wedge \psi)$ is true just when ϕ and ψ are both true.
- Given two propositions, ϕ and ψ, there are four ways that the world way may turn out, and they fix whether $(\phi \wedge \psi)$ is true or false:

ϕ	ψ	$(\phi \wedge \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

- This is the truth-table for '$\wedge$'.

The truth table for conjunction – 3

Note that the statement

$(\phi \wedge \psi)$ is true just when ϕ and ψ are both true.

and the table

ϕ	ψ	$(\phi \wedge \psi)$
T	T	T
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F	T	F
F	F	F

are two ways of presenting the same basic characterization of bare conjunction.

Disjunction

- Compare
  1. Osborne will either increase taxes or reduce government spending.
  2. Either the peace initiative will be succeed or the war will carry on at least another year.

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- Inclusive 'or' is standardly symbolized by '$\lor$' [for Latin 'vel'].
- $(\phi \lor \psi)$ is true just when at least one of ϕ and ψ is true.

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- ‘It is not the case that ϕ’ almost always expresses the negation of plain ϕ.
- Inserted ‘not’ often expresses negation more simply – e.g. compare

  (a) Jill is married.
  (b) Jill is not married.

  (c) Thatcher was a KGB spy.
  (d) Thatcher was not a KGB spy.

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- Two ways of expressing the negation of (a):

  (c) It is not the case that some Welshmen are rugby fanatics.

  (d) No Welshman is a rugby fanatic.

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- Unlike '$\wedge$' and '$\vee$', the negation-sign '$\neg$' does not need brackets.

The three basic connectives

'$\wedge$' and '$\vee$' are called propositional connectives, for obvious reasons. '$\neg$' is also treated as an honorary connective.

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \vee \psi)$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

ϕ	$\neg\phi$
T	F
F	T