Revisit summer ... go to the Fitzwilliam Museum!

Faculty of Philosophy

Formal Logic

Lecture 5

Peter Smith

- Propositional connectives, and the assumption of bivalence
- Propositions and worlds
- Complex propositions
- Two kinds of symbols?

Introducing the language **PL**

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- Vernacular 'and', 'or', and 'not' are subject to various (lexical) semantic complexities and ambiguities and their use can create (structural) scope ambiguities.
- So following the 'divide and rule' strategy, we first need to characterize a formal language **PL** for regimenting arguments clearly and without ambiguities. Then we discuss how to assess arguments once regimented.

We introduced the three basic connectives

'$\land$' and '$\lor$' are called propositional connectives, for obvious reasons. '$\neg$' is also treated as an honorary connective.

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  1. vague propositions (?? it is neither true nor false that the borderline bald man is bald).
  2. liar propositions (?? "This proposition is not true" is neither true nor false).
  3. propositions involving denotationless names (?? "Mr Sviatolak Brintangle is married" is neither true nor false if there is no such person as Mr B.)

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- So given a proposition ϕ let's write $\mathbf{\Phi}$ for the corresponding set containing all possible worlds where ϕ is true.
- Then ϕ is equivalent to the proposition that the actual world is in $\mathbf{\Phi}$, i.e. the actual world is one of the worlds where ϕ is true, i.e. ϕ is true.

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- (This happy alignment of notation is not an accident!)

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Using more than one connective

- We can build up complex propositions by using more than one connective, as in:

$$(P \land \neg Q)$$
$$\neg(P \land \neg Q)$$
$$(R \lor \neg(P \land \neg Q))$$
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- NB, to reiterate:
  1. Every occurrence of the connectives '$\land$' and '$\lor$' always comes with an accompanying pair of brackets, which make the scope of the connectives entirely clear.
  2. The connective '$\neg$' never introduces extra brackets – the rule (in rough terms) is that 'a negation governs what immediately follows it'.

Interpreting examples

Suppose:

$P =$ Felix is on the mat.
$Q =$ Fido is on the mat.

Then:

1. $(P \wedge \neg Q) =$ Felix is on the mat and Fido isn't.

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   = Neither Felix nor Fido are on the mat.
3. $(\neg P \vee Q)$ = Either Felix isn't on the mat or Fido is.

Calculating truth values

Suppose:

> P is false.
> Q is true.

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2. $\neg(P \lor Q)$ is false.
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Then:

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So:

1. $(P \land \neg Q)$ is false.
2. $\neg(P \lor Q)$ is false.
3. $(\neg P \lor Q)$ is true.

NB, to calculate the truth-values of the complex ('molecular') propositions we only need to know the truth-values of the simple ('atomic') ones. You don't need to remember what they mean!

Some more examples

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Suppose that P is True, Q is False, R is True.
Then, what are the truth-values of

1. $(Q \wedge R)$
2. $((Q \wedge R) \vee P)$
3. $(Q \wedge (R \vee P))$
4. $\neg(P \wedge \neg Q)$

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1. $(Q \wedge R)$ is F.
2. $((Q \wedge R) \vee P)$ is T.
3. $(Q \wedge (R \vee P))$ is F.
   Note the importance of bracketing here: $Q \wedge R \vee P$ would be scope-ambiguous (and have different truth-values depending on how we disambiguate it): hence the need to insist on the brackets with the connectives '$\wedge$' and '$\vee$'.

Some more examples

P is T, Q is F, R is T.

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1. $(Q \land R)$ is F.
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4. $\neg(P \land \neg Q)$ is F – because $(P \land \neg Q)$ is T.

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- Roughly: φ, ψ are devices for *generalization* about propositions, as in: whatever propositions φ, ψ are in play, the result of writing '(' followed by φ followed by '$\vee$' followed by ')' is true just when one of φ, ψ is true.

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- More on this in due course (*IFL*, Ch. 10).