Faculty of Philosophy

Formal Logic

Lecture 6

Peter Smith

- Testing arguments – the idea illustrated
- Developing PL: syntax
- Developing PL: semantics
  - Interpretations
  - Evaluations

How to test an argument for validity – 1

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We don't know the truth-value of P and Q, let's suppose. But the possibilities can be set out as follows:

P	Q	$(P \vee Q)$	$\neg P$	Q
T	T			
T	F			
F	T			
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F	T			
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We can then evaluate the premisses and conclusion in each case.

How to test an argument for validity – 2

Consider the argument 'Either Cameron or Clegg supports the policy. But Cameron doesn't. So Clegg does.'

Using 'P' for *Cameron supports the policy*, and 'Q' for *Clegg supports the policy*, we can render this argument like this:

$$(P \lor Q),\ \neg P\ \textit{So}\ Q.$$

We don't know the truth-value of P and Q, let's suppose. But the possibilities can be set out as follows:

P	Q	$(P \lor Q)$	$\neg P$	Q
T	T	T		
T	F	T		
F	T	T		
F	F	F		

That evaluates the first premiss for each way the world might be.

How to test an argument for validity – 3

Consider the argument 'Either Cameron or Clegg supports the policy. But Cameron doesn't. So Clegg does.'

Using 'P' for *Cameron supports the policy*, and 'Q' for *Clegg supports the policy*, we can render this argument like this:

$$(P \vee Q),\ \neg P\ So\ Q.$$

We don't know the truth-value of P and Q, let's suppose. But the possibilities can be set out as follows:

P	Q	$(P \vee Q)$	$\neg P$	Q
T	T	T	F	
T	F	T	F	
F	T	T	T	
F	F	F	T	

And that evaluates the second premiss.

How to test an argument for validity – 4

Consider the argument 'Either Cameron or Clegg supports the policy. But Cameron doesn't. So Clegg does.'

Using 'P' for *Cameron supports the policy*, and 'Q' for *Clegg supports the policy*, we can render this argument like this:

$(P \vee Q)$, $\neg P$ *So* Q.

We don't know the truth-value of P and Q, let's suppose. But the possibilities can be set out as follows:

P	Q	$(P \vee Q)$	$\neg P$	Q
T	T	T	F	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

We record the conclusion . . .

How to test an argument for validity – 4

Consider the argument 'Either Cameron or Clegg supports the policy. But Cameron doesn't. So Clegg does.'

Using 'P' for *Cameron supports the policy*, and 'Q' for *Clegg supports the policy*, we can render this argument like this:

$(P \lor Q)$, $\neg P$ *So* Q.

We don't know the truth-value of P and Q, let's suppose. But the possibilities can be set out as follows:

P	Q	$(P \lor Q)$	$\neg P$	Q
T	T	T	F	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

Now look through the possible ways things can be ...

Testing arguments

Here is the table again

P	Q	$(P \lor Q)$	$\neg P$	Q
T	T	T	F	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

Testing arguments

Here is the table again

P	Q	$(P \vee Q)$	$\neg P$	Q
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F	T	T	T	T
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Each row represents a type of way the world can be (ways which fixes the truth-values of P and Q, and hence fix the truth-value of the premisses and conclusion of our argument).

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Each row represents a type of way the world can be (ways which fixes the truth-values of P and Q, and hence fix the truth-value of the premisses and conclusion of our argument).

In this case, there is just one type of way the world can be that makes the premisses true . . .

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In this case, there is just one type of way the world can be that makes the premisses true . . . and that makes the conclusion true too.

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	P	Q	$(P \vee Q)$	$\neg P$	Q
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Each row represents a type of way the world can be (ways which fixes the truth-values of P and Q, and hence fix the truth-value of the premisses and conclusion of our argument).

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Each row represents a type of way the world can be (ways which fixes the truth-values of P and Q, and hence fix the truth-value of the premisses and conclusion of our argument).

In other words, every possible situation in which the premisses are true is a situation in which the conclusion is true – so the inference must be valid!

The underlying idea

- An inference is valid if there is no possible way for its premisses to be true and conclusion false. Or equivalently, in any situation in which the premisses are all true, the conclusion has to be true too.

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- If an argument's relevant logical materials are just 'and', 'or', and 'not' (used in the core senses that we represent with '$\wedge$', '$\vee$', '$\neg$') then the truth-values of the premisses and conclusion depends only on the truth-values of the relevant 'atomic' propositions.

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- If an argument's relevant logical materials are just 'and', 'or', and 'not' (used in the core senses that we represent with '$\land$', '$\lor$', '$\neg$') then the truth-values of the premisses and conclusion depends only on the truth-values of the relevant 'atomic' propositions.
- So by looking through all the possible assignments of truth-values to the relevant 'atomic' propositions, we can see whether there is indeed any way of making the premisses all true and conclusion false.

The underlying idea

So to evaluate the inference

$$(P \vee Q), \neg P \text{ So } Q.$$

we check across the table . . .

P	Q	$(P \vee Q)$	$\neg P$	Q
T	T	T	F	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

. . . to see whether there is a row in which the premisses are all true and conclusion false. But there isn't. So the inference is valid.

Another example

- Consider the argument

  Either Cameron or Clegg supports the policy. It isn't true that Clegg does while Miliband doesn't. Hence it isn't true that Miliband supports the policy and Cameron doesn't.

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  P = Cameron supports the policy.
  Q = Clegg supports the policy.
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  we can regiment the argument

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  $P =$ Cameron supports the policy.
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  we can regiment the argument

  $$(P \lor Q),\ \neg(Q \land \neg R),\ \mathit{So}\ \neg(R \land \neg P)$$

Another example

We want to evaluate the argument

$$(P \lor Q),\ \neg(Q \land \neg R),\ So\ \neg(R \land \neg P)$$

And the first step is to set out all the possible ways the world can be with respect to the truth or falsity of P, Q, R:

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P	Q	R	$(P \lor Q)$	$\neg(Q \land \neg R)$	$\neg(R \land \neg P)$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
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T	T	F			
T	F	T			
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F	T	T			
F	T	F			
F	F	T			
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And, next step, we evaluate the first premiss on every row.♦

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T	T	F	T		
T	F	T	T		
T	F	F	T		
F	T	T	T		
F	T	F	T		
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Next step: we evaluate the second premiss on every row.♦

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T	T	F	T	F	
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T	F	F	T	T	
F	T	T	T	T	
F	T	F	T	F	
F	F	T	F	T	
F	F	F	F	T	

Next step: we evaluate the conclusion on every row.♦

Another example

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►	F	T	T	T	T	F
	F	T	F	T	F	T
	F	F	T	F	T	F
	F	F	F	F	T	T

There is a row where the premisses are true and conclusion false!

Another example

So the following row

	P	Q	R	$(P \lor Q)$	$\neg(Q \land \neg R)$	$\neg(R \land \neg P)$
	⋮	⋮	⋮	⋮	⋮	⋮
▶	F	T	T	T	T	F
	⋮	⋮	⋮	⋮	⋮	⋮

reveals a way the world can be where the premisses of

$(P \lor Q)$, $\neg(Q \land \neg R)$, *So* $\neg(R \land \neg P)$

are true and conclusion false, i.e. when

P is False.
Q is True.
R is True.

Another example

So returning to our original argument

> *Either Cameron or Clegg supports the policy. It isn't true that Clegg does while Miliband doesn't. Hence it isn't true that Miliband supports the policy and Cameron doesn't.*

our truth-table reveals that the situation where

> Cameron supports the policy is False.
> Clegg supports the policy is True.
> Miliband supports the policy is True.

makes the premisses true and conclusion false – so the inference is indeed invalid.

- Testing arguments – the idea illustrated
- Developing PL: syntax
- Developing PL: semantics
  Interpretations
  Evaluations

Back to developing the language PL

- We've looked ahead to see how we can sometimes (1) regiment arguments using the propositional connectives '$\wedge$', '$\vee$' and '$\neg$' and capture their logical form that way, and then (2) use a truth-table to test whether the inference is valid.

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- That gives us a motivation to continue more carefully the development of the language **PL** for regimenting arguments involving these connectives. (We have – in effect – been translating the ordinary language arguments into this language).
- So our next tasks are to be more rigorous about the syntax and semantics of **PL**.

The design brief for PL

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3. Since we are interested (for the moment) in inferences whose validity depends *only* on the distribution of '$\wedge$', '$\vee$' and '$\neg$' in the premisses and conclusion, we don't need to expose any more structure inside propositions.
4. So we might as well continue represent the basic level of propositions – the 'atoms' which get build up into 'molecular' propositions using the connectives – by very simple expressions, e.g. single letters 'P', 'Q', 'R', 'S',. . . .

The symbolism of PL

- First shot at describing the basic vocabulary:
  1. We need letters for the atomic formulae:

     $$P, Q, R, S \ldots.$$

  2. The connectives:

     $$\land, \lor, \neg.$$

  3. The brackets:

     $$(,)$$

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- But the open-ended list in (1) is not entirely satisfactory. For example, it doesn't tell us whether Z is or isn't an atomic formula. (Or if it is, what comes next.)
- We want it to be effectively decidable what's an atomic formula (and later, what's an formula of **PL** more generally). We consider one way of ensuring this.

The symbolism of PL

- The basic vocabulary
  1. 1.1 There are letters used in making the atomic formulae:

     $$P, Q, R, S.$$

     1.2 We need the prime symbol

     $$\prime$$

     to help us build more atoms (P', Q', ..., P'', Q'',...).
  2. The connectives:

     $$\wedge, \vee, \neg.$$

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The symbolism of PL

- The basic vocabulary
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  2. The connectives:

     $$\wedge, \vee, \neg.$$

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- We'll now give the official definitions of the atomic formulae and the wider class of (atomic or molecular) well-formed formulae (the 'wffs' for short).

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   But that isn't enough – it leaves it open that , e.g. Julius Ceasar is an atomic formula!.

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So for example:

'R' is an atomic wff – by (1).
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NB it now is effectively decidable what is an atomic formula.

Defining the well-formed formulae of PL

1. Any atomic formula is a wff.
2. If ϕ and ψ are wffs, so is $(\phi \wedge \psi)$.
3. If ϕ and ψ are wffs, so is $(\phi \vee \psi)$.
4. If ϕ is a wff, so is $\neg\phi$.
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Defining the well-formed formulae of PL

1. Any atomic formula is a wff.
2. If ϕ and ψ are wffs, so is $(\phi \land \psi)$.
3. If ϕ and ψ are wffs, so is $(\phi \lor \psi)$.
4. If ϕ is a wff, so is $\neg\phi$.
5. Nothing else is a wff.

So for example:

i. 'Q' is a wff – by (1).
ii. 'R'' is a wff – by (1).
iii. '$(Q \land R')$' is a wff – from i, ii by (2).
iv. '$\neg(Q \land R')$' is a wff – from iii by (4).
v. 'S' is a wff – by (1).
vi. '$(\neg(Q \land R') \lor S)$' is a wff – from iv, v by (3).
vii. '$\neg(\neg(Q \land R') \lor S)$' is a wff – from vi by (4).
viii. '$(Q \lor \neg(\neg(Q \land R') \lor S))$' is a wff – from i, vii by (3).

Constructional histories

Note again the last clause:

1. Any atomic formula is a wff.
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3. If ϕ and ψ are wffs, so is $(\phi \vee \psi)$.
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This means that any wff has to be built up by a series of steps of adding connectives to atoms in accordance with rules (1) to (4). In other words, every wff has a constructional history.

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We can present constructional histories in various ways. The connective added at the last stage in a constructional history for a wff is the main connective of the wff. Wffs that appear in the constructional history of a wff are its sub-formulae.

Setting out a construction history

The example $(Q \lor \neg(\neg(Q \land R') \lor S))$ again:

$$\dfrac{Q \text{ is a wff} \qquad \dfrac{\dfrac{\dfrac{\dfrac{Q \text{ is a wff} \quad R' \text{ is a wff}}{(Q \land R') \text{ is a wff}}}{\neg(Q \land R') \text{ is a wff}} \quad S \text{ is a wff}}{(\neg(Q \land R') \lor S) \text{ is a wff}}}{\neg(\neg(Q \land R') \lor S) \text{ is a wff}}}{(Q \lor \neg(\neg(Q \land R') \lor S)) \text{ is a wff}}$$

Highlighting the main connectives

The example $(Q \lor \neg(\neg(Q \land R') \lor S))$ again:

$$\dfrac{Q \text{ is a wff} \qquad \dfrac{\dfrac{\dfrac{\dfrac{Q \text{ is a wff} \qquad R' \text{ is a wff}}{(Q \land R') \text{ is a wff}}}{\neg(Q \land R') \text{ is a wff}} \qquad S \text{ is a wff}}{(\neg(Q \land R') \lor S)) \text{ is a wff}}}{\neg(\neg(Q \land R') \lor S)) \text{ is a wff}}}{(Q \lor \neg(\neg(Q \land R') \lor S)) \text{ is a wff}}$$

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- Given a PL wff, what does it mean?
- To interpret a PL wff, we need to know what the atoms it contains mean. E.g.
  1. 'P' means: Blair supports the policy
  2. 'Q' means: Brown supports the policy
  3. 'R' means: ...

  On different occasions of use, PL atoms can be recruited to express different messages. (That's why the atoms are often called propositional variables, even though their local interpretation is fixed.)

Interpretations

- Given a PL wff, what does it mean?
- To interpret a PL wff, we need to know what the atoms it contains mean. E.g.
  1. 'P' means: Blair supports the policy
  2. 'Q' means: Brown supports the policy
  3. 'R' means: ...

  On different occasions of use, PL atoms can be recruited to express different messages. (That's why the atoms are often called propositional variables, even though their local interpretation is fixed.)
- The interpretation of the connectives, however, always stays fixed: we have – roughly –
  1. '$\ldots \wedge \ldots$' means *. . . and . . .*
  2. '$\ldots \vee \ldots$' means *either . . . or . . . (or both)*
  3. '$\neg \ldots$' means *it is not the case that . . .*

Interpretations vs evaluations

- However, we are concerned mostly not with interpretations (which give the message expressed by a wff) but with the evaluations of wffs (assignments of truth-values which are determined by what the wff says *and* how the world might be).
- We fix an evaluation by giving an assignment of values to the relevant atoms. E.g. $P \Rightarrow$ T, $Q \Rightarrow$ F, $R \Rightarrow \ldots$
- We then calculate the truth-values of molecular propositions in the way we've illustrated, using the truth-tables for the connectives.

ϕ	ψ	$(\phi \wedge \psi)$	$(\phi \vee \psi)$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	F

ϕ	$\neg\phi$
T	F
F	T

Calculating truth-values

Take $(Q \lor \neg(\neg(Q \land R') \lor S))$ again. And suppose

$Q \Rightarrow$ F, $R' \Rightarrow$ F, $S \Rightarrow$ T

Then we can evaluate the complex wff in effect by working down its construction tree assigning values as we go ...

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$$Q \Rightarrow \mathsf{F},\ R' \Rightarrow \mathsf{F},\ S \Rightarrow \mathsf{T}$$

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$$\dfrac{Q \Rightarrow \mathsf{F} \qquad \dfrac{\dfrac{\dfrac{\dfrac{Q \Rightarrow \mathsf{F} \qquad R' \Rightarrow \mathsf{F}}{(Q \wedge R') \Rightarrow \mathsf{F}}}{\neg(Q \wedge R') \Rightarrow \mathsf{T}} \qquad S \Rightarrow \mathsf{T}}{(\neg(Q \wedge R') \vee S)) \Rightarrow \mathsf{T}}}{\neg(\neg(Q \wedge R') \vee S)) \Rightarrow \mathsf{F}}}{(Q \vee \neg(\neg(Q \wedge R') \vee S)) \Rightarrow \mathsf{F}}$$

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