Faculty of Philosophy

# Formal Logic

## Lecture 7

Peter Smith

- PL syntax and semantics: some quick reminders
- An historical aside
- Tautologies
- A Philosophical Aside about Ideas of Necessity

# Syntax: defining the well-formed formulae of PL

First, the atomic formulae:

1. '$P$', '$Q$', '$R$', '$S$' are atomic formulae.
2. If $\phi$ is an atomic formula, so is $\phi'$.
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Next, the class of wffs, atomic and molecular:

1. Any atomic formula is a wff.
2. If $\phi$ and $\psi$ are wffs, so is $(\phi \wedge \psi)$.
3. If $\phi$ and $\psi$ are wffs, so is $(\phi \vee \psi)$.
4. If $\phi$ is a wff, so is $\neg\phi$.
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NB Wffs have a constructional history (unique up to trivial variation).

## Interpretations vs evaluations

- We are concerned mostly not with interpretations (which give the message expressed by a wff) but with the evaluations of wffs (assignments of truth-values).
- We fix an evaluation by given an assignment of values to the relevant atoms. E.g. $P \Rightarrow$ T, $Q \Rightarrow$ F, $R \Rightarrow \ldots$
- We then calculate the truth-values of molecular propositions on that assignment using the now familiar truth-tables for the connectives.

| $\phi$ | $\psi$ | $(\phi \wedge \psi)$ | $(\phi \vee \psi)$ |
|---|---|---|---|
| T | T | T | T |
| T | F | F | T |
| F | T | F | T |
| F | F | F | F |

| $\phi$ | $\neg\phi$ |
|---|---|
| T | F |
| F | T |

# Setting out a construction history

The example $(Q \vee \neg(\neg(Q \wedge R') \vee S))$ we saw last week:

$$\dfrac{Q \text{ is a wff} \qquad \dfrac{\dfrac{\dfrac{\dfrac{Q \text{ is a wff} \qquad R' \text{ is a wff}}{(Q \wedge R') \text{ is a wff}}}{\neg(Q \wedge R') \text{ is a wff}} \qquad S \text{ is a wff}}{(\neg(Q \wedge R') \vee S)) \text{ is a wff}}}{\neg(\neg(Q \wedge R') \vee S)) \text{ is a wff}}}{(Q \vee \neg(\neg(Q \wedge R') \vee S)) \text{ is a wff}}$$

## Calculating truth-values

Take $(Q \lor \neg(\neg(Q \land R') \lor S))$ again. And suppose

$$Q \Rightarrow \mathsf{F},\ R' \Rightarrow \mathsf{F},\ S \Rightarrow \mathsf{T}$$

Then we can evaluate the complex wff in effect by working down its construction tree assigning values as we go ...

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$$\dfrac{Q \Rightarrow \mathsf{F} \qquad \dfrac{\dfrac{\dfrac{\dfrac{Q \Rightarrow \mathsf{F} \qquad R' \Rightarrow \mathsf{F}}{(Q \land R') \Rightarrow \mathsf{F}}}{\neg(Q \land R') \Rightarrow \mathsf{T}} \qquad S \Rightarrow \mathsf{T}}{(\neg(Q \land R') \lor S)) \Rightarrow \mathsf{T}}}{\neg(\neg(Q \land R') \lor S)) \Rightarrow \mathsf{F}}}{(Q \lor \neg(\neg(Q \land R') \lor S)) \Rightarrow \mathsf{F}}$$

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# Cutting down the working even more

Here's three obvious rules for speeding things up!

1. If a conjunction has a false conjunct, its overall value is false (irrespective of the value of the other conjunct).
2. If a disjunction has a true disjunct, its overall value is true (irrespective of the value of the other disjunct).
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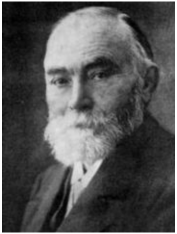
T    T T

# Where did PL originate?

1. Many principles of propositional logic were already discussed by the Stoic philosopher Chrysippus (c.280–207 BC). E.g.
   - Not both the first and the second; but the first; therefore, not the second. $\neg(\phi \wedge \psi)$, $\phi$, so $\neg\psi$.
   - Either the first or the second; but not the second; therefore the first. $(\phi \vee \psi)$, $\neg\psi$, so $\phi$.
2. Further developments run through e.g. Galen (c. 129–210 AD), Peter Abelard (1079–1142) and William of Ockham (1288–1347), and many others.
3. Later, two important Victorian figures were Augustus De Morgan (1806–1871) who was an undergraduate at Trinity, and George Boole (1815–1864).
4. De Morgan and Boole mathematized propositional logic, but didn't introduce a fully formalized dedicated language like PL. We owe that to . . .

# Gottlob Frege 1848–1925



Very arguably the greatest philosopher of the nineteenth century, Frege developed a formal notation for regimenting thought and reasoning – his system was first outlined in his *Begriffsschrift* ["Concept script"] (1879).

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## A first example of a tautology – 1

- Given a PL wff $\phi$, we can calculate its truth-value for any assignment of truth-values to the atoms in $\phi$. So ...

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- Given a PL wff $\phi$, we can calculate its truth-value for every assignment of truth-values to the atoms in $\phi$, and we can set out the calculation in a full truth-table.
- So consider, for example, the wff $\neg((\neg P \lor Q) \land (P \land \neg Q))$. The truth-table is as follows:

# A first example of a tautology – 2

- The table is as follows ...

| $P$ | $Q$ | $\neg((\neg P \lor Q) \land (P \land \neg Q))$ |
|---|---|---|
| T | T | |
| T | F | |
| F | T | |
| F | F | |

# A first example of a tautology – 2

- The table is as follows ...

| $P$ | $Q$ | $\neg((\neg P \vee Q) \wedge (P \wedge \neg Q))$ |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | F |
| F | F | F |

# A first example of a tautology – 2

- The table is as follows ...

| $P$ | $Q$ | $\neg(($ | $\neg P \lor Q)$ | $\land$ | $(P$ | $\land$ | $\neg Q))$ |
|---|---|---|---|---|---|---|---|
| T | T | | | F | | F | |
| T | F | | | | | T | |
| F | T | | | F | | F | |
| F | F | | | F | | F | |

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- The table is as follows ...

| $P$ | $Q$ | $\neg$ | $((\neg P$ | $\lor$ | $Q)$ | $\land$ | $(P$ | $\land$ | $\neg Q))$ |
|---|---|---|---|---|---|---|---|---|---|
| T | T | | | | | F | | F | |
| T | F | | | F | | | | T | |
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| T | T | | | | | F | | F | |
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| $P$ | $Q$ | $\neg$ | $((\neg P$ | $\vee$ | $Q)$ | $\wedge$ | $(P$ | $\wedge$ | $\neg Q))$ |
|---|---|---|---|---|---|---|---|---|---|
| T | T | T | | | | F | | F | |
| T | F | T | | F | | F | | T | |
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|---|---|---|---|---|---|
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| T | F | T | F | F | T |
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| T | T | T | | F | F |
| T | F | T | F | F | T |
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- So the wff $\neg((\neg P \lor Q) \land (P \land \neg Q))$ is true on every assignment of values to its atoms. In the standard jargon, it is a tautology.
- And intuitively that wff ought to be always true (it is intuitively a necessary truth – think about it!).

# More quick examples of tautologies

A PL tautology is a wff which is true on every assignment of values to its constituent atoms.

A PL contradiction [broad sense] is a wff which is false on every assignment of values to its constituent atoms.

- Some simple tautologies are

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  $(P \lor \neg P)$ – *law of excluded middle*
  $\neg(P \land \neg P)$ – *law of non-contradiction [$(P \land \neg P)$ is a contradiction in the narrow sense of 'contradiction', a wff conjoined with its negation]*

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- Any negation of a tautology is a contradition – e.g. $\neg(P \vee \neg P)$ and $\neg\neg(P \wedge \neg P)$
- There can be arbitrarily complex tautologies (involving any number of different atoms). For example

$$\neg([\neg(P \wedge Q) \wedge (Q \vee R)] \wedge [P \wedge \neg R])$$

# A three-variable tautology

| $P$ | $Q$ | $R$ | $\neg([\neg(P \wedge Q) \wedge (Q \vee R)] \wedge [P \wedge \neg R])$ |
|---|---|---|---|
| T | T | T | |
| T | T | F | |
| T | F | T | |
| T | F | F | |
| F | T | T | |
| F | T | F | |
| F | F | T | |
| F | F | F | |

# A three-variable tautology

| $P$ | $Q$ | $R$ | $\neg([\neg(P \land Q) \land (Q \lor R)] \land [P \land \neg R])$ |
|---|---|---|---|
| T | T | T | F |
| T | T | F | T |
| T | F | T | F |
| T | F | F | T |
| F | T | T | F |
| F | T | F | F |
| F | F | T | F |
| F | F | F | F |

# A three-variable tautology

| $P$ | $Q$ | $R$ | $\neg([\neg(P \land Q) \land (Q \lor R)]$ | $\land$ | $[P$ | $\land$ | $\neg R])$ |
|---|---|---|---|---|---|---|---|
| T | T | T | | F | | F | |
| T | T | F | | | | T | |
| T | F | T | | F | | F | |
| T | F | F | | | | T | |
| F | T | T | | F | | F | |
| F | T | F | | F | | F | |
| F | F | T | | F | | F | |
| F | F | F | | F | | F | |

# A three-variable tautology

| $P$ | $Q$ | $R$ | $\neg([$ | $\neg$ | $(P \land Q) \land (Q \lor R)]$ | $\land$ | $[P$ | $\land$ | $\neg R])$ |
|---|---|---|---|---|---|---|---|---|---|
| T | T | T | | | | F | | F | |
| T | T | F | | F | | | | T | |
| T | F | T | | | | F | | F | |
| T | F | F | | | | | | T | |
| F | T | T | | | | F | | F | |
| F | T | F | | | | F | | F | |
| F | F | T | | | | F | | F | |
| F | F | F | | | | F | | F | |

# A three-variable tautology

| $P$ | $Q$ | $R$ | $\neg($ | $[\neg($ | $P$ | $\land$ | $Q)$ | $\land$ | $(Q$ | $\lor$ | $R)]$ | $\land$ | $[P$ | $\land$ | $\neg R])$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | T | T | | | | | | | | | | F | | F | |
| T | T | F | | F | | | | F | | | | | | T | |
| T | F | T | | | | | | | | | | F | | F | |
| T | F | F | | | | | | | | | | | | T | |
| F | T | T | | | | | | | | | | F | | F | |
| F | T | F | | | | | | | | | | F | | F | |
| F | F | T | | | | | | | | | | F | | F | |
| F | F | F | | | | | | | | | | F | | F | |

# A three-variable tautology

| $P$ | $Q$ | $R$ | $\neg([$ | $\neg$ | $(P \land Q)$ | $\land$ | $(Q \lor R)]$ | $\land$ | $[P$ | $\land$ | $\neg R])$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| T | T | T | | | | | | F | | F | |
| T | T | F | | F | | F | | F | | T | |
| T | F | T | | | | | | F | | F | |
| T | F | F | | | | | | | | T | |
| F | T | T | | | | | | F | | F | |
| F | T | F | | | | | | F | | F | |
| F | F | T | | | | | | F | | F | |
| F | F | F | | | | | | F | | F | |

# A three-variable tautology

| $P$ | $Q$ | $R$ | $\neg([$ | $\neg$ | $(P \land Q)$ | $\land$ | $(Q$ | $\lor$ | $R)]$ | $\land$ | $[P$ | $\land$ | $\neg R])$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | T | T | | | | | | | | F | | F | |
| T | T | F | | F | | F | | | | F | | T | |
| T | F | T | | | | | | | | F | | F | |
| T | F | F | | | | | | F | | | | T | |
| F | T | T | | | | | | | | F | | F | |
| F | T | F | | | | | | | | F | | F | |
| F | F | T | | | | | | | | F | | F | |
| F | F | F | | | | | | | | F | | F | |

# A three-variable tautology

| $P$ | $Q$ | $R$ | $\neg$ | $([$ | $\neg$ | $(P$ | $\wedge$ | $Q)$ | $\wedge$ | $(Q$ | $\vee$ | $R)]$ | $\wedge$ | $[P$ | $\wedge$ | $\neg R])$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | T | T | | | | | | | | | | | F | | F | |
| T | T | F | | | F | | | | F | | | | F | | T | |
| T | F | T | | | | | | | | | | | F | | F | |
| T | F | F | | | | | | | F | | F | | | | T | |
| F | T | T | | | | | | | | | | | F | | F | |
| F | T | F | | | | | | | | | | | F | | F | |
| F | F | T | | | | | | | | | | | F | | F | |
| F | F | F | | | | | | | | | | | F | | F | |

# A three-variable tautology

| $P$ | $Q$ | $R$ | $\neg($ | $[\neg$ | $(P$ | $\land$ | $Q)$ | $\land$ | $(Q$ | $\lor$ | $R)]$ | $\land$ | $[P$ | $\land$ | $\neg R])$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T | T | T | | | | | | | | | | F | | F | |
| T | T | F | | F | | | | F | | | | F | | T | |
| T | F | T | | | | | | | | | | F | | F | |
| T | F | F | | | | | | F | | F | | F | | T | |
| F | T | T | | | | | | | | | | F | | F | |
| F | T | F | | | | | | | | | | F | | F | |
| F | F | T | | | | | | | | | | F | | F | |
| F | F | F | | | | | | | | | | F | | F | |

# A three-variable tautology

| $P$ | $Q$ | $R$ | $\neg($ | $[\neg$ | $(P \land Q)$ | $\land$ | $(Q \lor R)]$ | $\land$ | $[P \land \neg R])$ |
|---|---|---|---|---|---|---|---|---|---|
| T | T | T | T | | | | | F | F |
| T | T | F | T | F | | F | | F | T |
| T | F | T | T | | | | | F | F |
| T | F | F | T | | | F | F | F | T |
| F | T | T | T | | | | | F | F |
| F | T | F | T | | | | | F | F |
| F | F | T | T | | | | | F | F |
| F | F | F | T | | | | | F | F |

- PL syntax and semantics: some quick reminders
- An historical aside
- Tautologies
- A Philosophical Aside about Ideas of Necessity

## Three ideas

1. A proposition $\phi$ is necessarily true if there is no possible way the world could be in which $\phi$ is false – it just could not be false – there is no possible world in which it is false. Contrast being merely contingently true. [This is a metaphysical notion.]

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How can there be necessary truths, or truths known a priori?

3. Because some truths are 'trifling', 'merely verbal', 'true in virtue of the meanings of the worlds involved', analytic. Contrast being a synthetic truth. [The idea that the necessary truths are the a priori truths are the analytic truths is a theoretical – typically 'empiricist' – account of what makes for necessity and a prioricity.]

## Tautologies, necessity and analyticity

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3. Plausibly, some ordinary-language propositions – the vernacular equivalents of tautologies – are analytic for the same reason.
4. Evidently not all necessary propositions are tautologies (e.g. 'All brothers are male', 'Nothing is red and green all over').