Faculty of Philosophy

Formal Logic

Lecture 9

Peter Smith

Logic is the hygiene that keeps ideas healthy and strong

Hermann Weyl, 1885–1955

- Some very quick reminders
- The only conditional-like truth-function
- The parallels between ‘if’ and ‘$\supset$’
- Contrasting ‘if’ and ‘$\supset$’

The idea of tautological entailment

- A PL argument ϕ_1, ϕ_1, $\ldots \phi_n$, so ψ is tautologically valid if ψ is true on every possible valuation of the relevant atoms where all of ϕ_1, ϕ_1, $\ldots \phi_n$ are true.

The idea of tautological entailment

- A PL argument ϕ_1, ϕ_1, $\ldots\phi_n$, so ψ is tautologically valid if ψ is true on every possible valuation of the relevant atoms where all of ϕ_1, ϕ_1, $\ldots\phi_n$ are true.
- We can determine whether an argument is tautologically valid by a brute-force search through the space of possibilities, which can be set out as a truth-table test.

The idea of tautological entailment

- A PL argument ϕ_1, ϕ_1, $\ldots \phi_n$, so ψ is tautologically valid if ψ is true on every possible valuation of the relevant atoms where all of ϕ_1, ϕ_1, $\ldots \phi_n$ are true.
- We can determine whether an argument is tautologically valid by a brute-force search through the space of possibilities, which can be set out as a truth-table test.
- If a PL argument is tautologically valid, it is valid in virtue of the distribution of the connectives '$\wedge$', '$\vee$' and '$\neg$' in premisses and conclusion. Fixing the sense of those connectives suffices to ensure that, necessarily, if the premisses are true then the conclusion is too.

The idea of tautological entailment

- A PL argument ϕ_1, ϕ_1, $\ldots\phi_n$, so ψ is tautologically valid if ψ is true on every possible valuation of the relevant atoms where all of ϕ_1, ϕ_1, $\ldots\phi_n$ are true.
- We can determine whether an argument is tautologically valid by a brute-force search through the space of possibilities, which can be set out as a truth-table test.
- If a PL argument is tautologically valid, it is valid in virtue of the distribution of the connectives '$\wedge$', '$\vee$' and '$\neg$' in premisses and conclusion. Fixing the sense of those connectives suffices to ensure that, necessarily, if the premisses are true then the conclusion is too.
- If an argument is tautologically valid, it is plain valid.

The idea of tautological entailment

- A PL argument ϕ_1, ϕ_1, $\ldots \phi_n$, so ψ is tautologically valid if ψ is true on every possible valuation of the relevant atoms where all of ϕ_1, ϕ_1, $\ldots \phi_n$ are true.
- We can determine whether an argument is tautologically valid by a brute-force search through the space of possibilities, which can be set out as a truth-table test.
- If a PL argument is tautologically valid, it is valid in virtue of the distribution of the connectives '$\land$', '$\lor$' and '$\neg$' in premisses and conclusion. Fixing the sense of those connectives suffices to ensure that, necessarily, if the premisses are true then the conclusion is too.
- If an argument is tautologically valid, it is plain valid.
- But not conversely. A PL argument can be valid without being tautologically valid.

Why does the truth-table test work?

- The crucial stage of the test is where we work out for each possible assignment the truth-values of the premisses and conclusion of the argument.

Why does the truth-table test work?

- The crucial stage of the test is where we work out for each possible assignment the truth-values of the premisses and conclusion of the argument.
- This requires that fixing the truth-values of atoms in a PL wff fixes the truth-value of the wff.

Why does the truth-table test work?

- The crucial stage of the test is where we work out for each possible assignment the truth-values of the premisses and conclusion of the argument.
- This requires that fixing the truth-values of atoms in a PL wff fixes the truth-value of the wff.
- That requires the wff-building connectives to be truth-functional, each connective maps the truth-values of the wffs it operates on to a determinate value.

Why does the truth-table test work?

- The crucial stage of the test is where we work out for each possible assignment the truth-values of the premisses and conclusion of the argument.
- This requires that fixing the truth-values of atoms in a PL wff fixes the truth-value of the wff.
- That requires the wff-building connectives to be truth-functional, each connective maps the truth-values of the wffs it operates on to a determinate value.
- Equivalently, the wff-building connectives must be definable by truth-tables.

- Some very quick reminders
- The only conditional-like truth-function
- The parallels between ‘if’ and ‘$\supset$’
- Contrasting ‘if’ and ‘$\supset$’

Conditionals

- It would evidently be good if we could extend our truth-functional logic to cover arguments involving conditionals, i.e. propositions of the kind *if P then Q*.

Conditionals

- It would evidently be good if we could extend our truth-functional logic to cover arguments involving conditionals, i.e. propositions of the kind *if P then Q*.
- So we will try to construct a suitable truth-table for a conditional connective, so we can run a truth-table test on arguments involving this connective.

Conditionals

- It would evidently be good if we could extend our truth-functional logic to cover arguments involving conditionals, i.e. propositions of the kind *if P then Q*.
- So we will try to construct a suitable truth-table for a conditional connective, so we can run a truth-table test on arguments involving this connective.
- We will use '$\supset$' as our symbol for this truth-functional connective (some books use '$\to$').

Conditionals

- It would evidently be good if we could extend our truth-functional logic to cover arguments involving conditionals, i.e. propositions of the kind *if P then Q*.
- So we will try to construct a suitable truth-table for a conditional connective, so we can run a truth-table test on arguments involving this connective.
- We will use '$\supset$' as our symbol for this truth-functional connective (some books use '$\rightarrow$').
- So we need to complete the following table *without gaps*.

ϕ	ψ	$(\phi \supset \psi)$
T	T	
T	F	
F	T	
F	F	

The only possible truth-table for '$\supset$' – 1

Jargon alert! In the conditional 'if A then C', A is the antecedent, C is the consequent

The only possible truth-table for '$\supset$' – 1

Jargon alert! In the conditional 'if A then C', A is the antecedent, C is the consequent

We'll build the only possible truth-table for '$\supset$' by appeal to a series of obvious facts about the conditional it is supposed to capture.

The only possible truth-table for '$\supset$' – 1

Jargon alert! In the conditional 'if A then C', A is the antecedent, C is the consequent

We'll build the only possible truth-table for '$\supset$' by appeal to a series of obvious facts about the conditional it is supposed to capture.

If ϕ is true and ψ is false then it can't be true that *if* ϕ *then* ψ.

The only possible truth-table for '$\supset$' – 1

Jargon alert! In the conditional 'if A then C', A is the antecedent, C is the consequent

We'll build the only possible truth-table for '$\supset$' by appeal to a series of obvious facts about the conditional it is supposed to capture.

If ϕ is true and ψ is false then it can't be true that *if* ϕ *then* ψ.

So that fixes

ϕ	ψ	$(\phi \supset \psi)$
T	T	
T	F	F
F	T	
F	F	

The only possible truth-table for '$\supset$' – 2

The table for an if-like connective can't start

ϕ	ψ	$(\phi \supset \psi)$
T	T	F
T	F	F
F	T	
F	F	

For if it did, we could never have P and $(P \supset Q)$ true together. But of course we can have P and *if P then Q* true together.

Indeed, arguably the whole point of the conditional construction is to set ourselves up for the modus ponens inference P, *if P then Q*, so Q.

The only possible truth-table for '$\supset$' – 3

So the table for an if-like connective must start

ϕ	ψ	$(\phi \supset \psi)$
T	T	T
T	F	F
F	T	
F	F	

The only possible truth-table for '⊃' – 3

So the table for an if-like connective must be one of

ϕ	ψ	$(\phi \supset \psi)$			
T	T	T			
T	F	F			
F	T	T	T	F	F
F	F	T	F	T	F
		a	*b*	*c*	*d*

The only possible truth-table for '⊃' – 3

So the table for an if-like connective must be one of

ϕ	ψ	$(\phi \supset \psi)$			
T	T	T			
T	F	F			
F	T	T	T	F	F
F	F	T	F	T	F
		a	*b*	*c*	*d*

- (d) gives '⊃' the table for '∧': but 'if's aren't 'and's.

The only possible truth-table for '⊃' – 3

So the table for an if-like connective must be one of

ϕ	ψ	$(\phi \supset \psi)$			
T	T	T			
T	F	F			
F	T	T	T	F	F
F	F	T	F	T	F
		a	*b*	*c*	*d*

- (d) gives '⊃' the table for '∧': but 'if's aren't 'and's.
- (c) gives $(\phi \supset \psi)$ the same table as $(\psi \supset \phi)$: but 'if's aren't symmetric.

The only possible truth-table for '$\supset$' – 3

So the table for an if-like connective must be one of

ϕ	ψ	$(\phi \supset \psi)$			
T	T	T			
T	F	F			
F	T	T	T	F	F
F	F	T	F	T	F
		a	*b*	*c*	*d*

- (d) gives '$\supset$' the table for '$\wedge$': but 'if's aren't 'and's.
- (c) gives $(\phi \supset \psi)$ the same table as $(\psi \supset \phi)$: but 'if's aren't symmetric.
- (b) gives $(\phi \supset \psi)$ the same table as ψ: but there is a difference between a conditional and its bare consequent.

The only possible truth-table for '$\supset$' – 3

So the table for an if-like connective must be one of

ϕ	ψ	$(\phi \supset \psi)$			
T	T	T			
T	F	F			
F	T	T	T	F	F
F	F	T	F	T	F
		a	*b*	*c*	*d*

- (d) gives '$\supset$' the table for '$\wedge$': but 'if's aren't 'and's.
- (c) gives $(\phi \supset \psi)$ the same table as $(\psi \supset \phi)$: but 'if's aren't symmetric.
- (b) gives $(\phi \supset \psi)$ the same table as ψ: but there is a difference between a conditional and its bare consequent.
- Which leaves (a) as the only possible truth-table for '$\supset$' as an if-like connective.

The material conditional

So, we've arrived at the following definition:

ϕ	ψ	$(\phi \supset \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

This is standardly known as the material conditional or the truth-functional conditional.

The material conditional

So, we've arrived at the following definition:

ϕ	ψ	$(\phi \supset \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

This is standardly known as the material conditional or the truth-functional conditional.

How close is '$\supset$' to the ordinary language 'if . . . then . . . '? (Does it capture the core sense of 'if' as '$\wedge$' captures the core of 'and'?)

Two kinds of conditional

1. Distinguish indicative from subjunctive conditionals. Examples of subjunctive conditionals:
   1.1 If Gareth hadn't scored, Wales would have lost.
   1.2 If I had drunk a bottle of champagne for breakfast, this lecture wouldn't have been very coherent
   1.3 If kangaroos had no tails, they would topple over.

Two kinds of conditional

1. Distinguish indicative from subjunctive conditionals. Examples of subjunctive conditionals:
   1.1 If Gareth hadn't scored, Wales would have lost.
   1.2 If I had drunk a bottle of champagne for breakfast, this lecture wouldn't have been very coherent
   1.3 If kangaroos had no tails, they would topple over.
2. These are 'possible world' conditionals. 'If P were true, Q would be true' = 'In the possible worlds which are most similar to the actual worlds except that P holds, Q holds'.

Two kinds of conditional

1. Distinguish indicative from subjunctive conditionals. Examples of subjunctive conditionals:
   1.1 If Gareth hadn't scored, Wales would have lost.
   1.2 If I had drunk a bottle of champagne for breakfast, this lecture wouldn't have been very coherent
   1.3 If kangaroos had no tails, they would topple over.
2. These are 'possible world' conditionals. 'If P were true, Q would be true' = 'In the possible worlds which are most similar to the actual worlds except that P holds, Q holds'.
3. Possible world conditionals aren't truth-functional conditionals. So '$\supset$' is at most a candidate for regimenting indicative conditionals.

Some parallels between ‘if’ and ‘$\supset$’ – 1

Modus ponens is valid, both for ordinary (indicative) conditionals and for the material conditional.

If Smith is a university lecturer then he is underpaid; Smith is a university lecturer; so Smith is underpaid!

$(P \supset Q)$, P, *so* Q

Some parallels between 'if' and '$\supset$' – 1

Modus ponens is valid, both for ordinary (indicative) conditionals and for the material conditional.

If Smith is a university lecturer then he is underpaid; Smith is a university lecturer; so Smith is underpaid!

$(P \supset Q)$, P, so Q

To show that is tautologically valid, consider

P	Q	P	$(P \supset Q)$	Q
T	T			
T	F			
F	T			
F	F			

Some parallels between 'if' and '⊃' – 1

Modus ponens is valid, both for ordinary (indicative) conditionals and for the material conditional.

If Smith is a university lecturer then he is underpaid;
Smith is a university lecturer; so Smith is underpaid!

$(P \supset Q)$, P, so Q

To show that is tautologically valid, consider

P	Q	P	$(P \supset Q)$	Q
T	T			T
T	F			F
F	T			T
F	F			F

Some parallels between ‘if’ and ‘⊃’ – 1

Modus ponens is valid, both for ordinary (indicative) conditionals and for the material conditional.

If Smith is a university lecturer then he is underpaid; Smith is a university lecturer; so Smith is underpaid!

$(P \supset Q)$, P, so Q

To show that is tautologically valid, consider

P	Q	P	$(P \supset Q)$	Q
T	T			T
T	F	T		F
F	T			T
F	F	F		F

Some parallels between 'if' and '$\supset$' – 1

Modus ponens is valid, both for ordinary (indicative) conditionals and for the material conditional.

If Smith is a university lecturer then he is underpaid; Smith is a university lecturer; so Smith is underpaid!

$(P \supset Q)$, P, *so* Q

To show that is tautologically valid, consider

P	Q	P	$(P \supset Q)$	Q
T	T			T
T	F	T	F	F
F	T			T
F	F	F		F

Some parallels between ‘if’ and ‘⊃’ – 2

Modus tollens is valid, both for ordinary (indicative) conditionals and for the material conditional.

> *If I have inherited lots of money, then my solicitor is a crook. But my solicitor is not a crook. So I haven't inherited lots of money.*
>
> $(P \supset Q)$, $\neg Q$ *so* $\neg P$

Some parallels between 'if' and '⊃' – 2

Modus tollens is valid, both for ordinary (indicative) conditionals and for the material conditional.

If I have inherited lots of money, then my solicitor is a crook. But my solicitor is not a crook. So I haven't inherited lots of money.

$(P \supset Q)$, $\neg Q$ *so* $\neg P$

To show that is tautologically valid, consider

P	Q	$(P \supset Q)$	$\neg Q$	$\neg P$
T	T			
T	F			
F	T			
F	F			

Some parallels between ‘if’ and ‘⊃’ – 2

Modus tollens is valid, both for ordinary (indicative) conditionals and for the material conditional.

If I have inherited lots of money, then my solicitor is a crook. But my solicitor is not a crook. So I haven't inherited lots of money.

$(P \supset Q)$, $\neg Q$ *so* $\neg P$

To show that is tautologically valid, consider

P	Q	$(P \supset Q)$	$\neg Q$	$\neg P$
T	T			F
T	F			F
F	T			T
F	F			T

Some parallels between ‘if’ and ‘$\supset$’ – 2

Modus tollens is valid, both for ordinary (indicative) conditionals and for the material conditional.

If I have inherited lots of money, then my solicitor is a crook. But my solicitor is not a crook. So I haven’t inherited lots of money.

$(P \supset Q)$, $\neg Q$ *so* $\neg P$

To show that is tautologically valid, consider

P	Q	$(P \supset Q)$	$\neg Q$	$\neg P$
T	T		F	F
T	F		T	F
F	T			T
F	F			T

Some parallels between 'if' and '$\supset$' – 2

Modus tollens is valid, both for ordinary (indicative) conditionals and for the material conditional.

If I have inherited lots of money, then my solicitor is a crook. But my solicitor is not a crook. So I haven't inherited lots of money.

$(P \supset Q)$, $\neg Q$ *so* $\neg P$

To show that is tautologically valid, consider

P	Q	$(P \supset Q)$	$\neg Q$	$\neg P$
T	T		F	F
T	F	F	T	F
F	T			T
F	F			T

Some parallels between 'if' and '$\supset$' – 3

Contraposition is valid for both 'if' and '$\supset$':

If Clegg loses, Cameron wins. So if Cameron doesn't win, Clegg doesn't lose.

$(P \supset Q)$, *so* $(\neg Q \supset \neg P)$

Some parallels between ‘if’ and ‘⊃’ – 3

Contraposition is valid for both ‘if’ and ‘⊃’:

If Clegg loses, Cameron wins. So if Cameron doesn’t win, Clegg doesn’t lose.

$(P \supset Q)$, *so* $(\neg Q \supset \neg P)$

To show that is tautologically valid, consider

P	Q	$(P \supset Q)$	$(\neg Q \supset \neg P)$
T	T		
T	F		
F	T		
F	F		

Some parallels between 'if' and '⊃' – 3

Contraposition is valid for both 'if' and '⊃':

If Clegg loses, Cameron wins. So if Cameron doesn't win, Clegg doesn't lose.

$(P \supset Q)$, *so* $(\neg Q \supset \neg P)$

To show that is tautologically valid, consider

P	Q	$(P \supset Q)$	$(\neg Q \supset \neg P)$
T	T		T
T	F		F
F	T		T
F	F		T

Some parallels between ‘if’ and ‘⊃’ – 3

Contraposition is valid for both ‘if’ and ‘⊃’:

If Clegg loses, Cameron wins. So if Cameron doesn’t win, Clegg doesn’t lose.

$(P \supset Q)$, *so* $(\neg Q \supset \neg P)$

To show that is tautologically valid, consider

P	Q	$(P \supset Q)$	$(\neg Q \supset \neg P)$
T	T		T
T	F	F	F
F	T		T
F	F		T

Some parallels between ‘if’ and ‘$\supset$’ – 3

Contraposition is valid for both ‘if’ and ‘$\supset$’:

If Clegg loses, Cameron wins. So if Cameron doesn’t win, Clegg doesn’t lose.

$(P \supset Q)$, *so* $(\neg Q \supset \neg P)$

NB! Distinguish contraposition from the fallacious principle

$(P \supset Q)$, *so* $(\neg P \supset \neg Q)$

From ‘if Jo is a woman, Jo is human’ you can’t infer ‘if Jo is not a woman, Jo is not human’.

Some parallels between ‘if’ and ‘⊃’ – 4

Affirming the consequent is a fallacy for inferences using both ‘if’ and ‘⊃’:

If Clegg supports the policy, Cameron does too. Cameron supports the policy. So Clegg does.

$(P \supset Q)$, Q *so* P

To show that isn’t tautologically valid, consider

P	Q	$(P \supset Q)$	Q	P
T	T			
T	F			
F	T			
F	F			

Some parallels between ‘if’ and ‘⊃’ – 4

Affirming the consequent is a fallacy for inferences using both ‘if’ and ‘⊃’:

> *If Clegg supports the policy, Cameron does too. Cameron supports the policy. So Clegg does.*
>
> $(P \supset Q)$, Q *so* P

To show that isn't tautologically valid, consider

P	Q	$(P \supset Q)$	Q	P
T	T			T
T	F			T
F	T			F
F	F			F

Some parallels between ‘if’ and ‘⊃’ – 4

Affirming the consequent is a fallacy for inferences using both ‘if’ and ‘⊃’:

If Clegg supports the policy, Cameron does too. Cameron supports the policy. So Clegg does.

$(P \supset Q)$, Q *so* P

To show that isn't tautologically valid, consider

P	Q	$(P \supset Q)$	Q	P
T	T			T
T	F			T
F	T		T	F
F	F		F	F

Some parallels between 'if' and '⊃' – 4

Affirming the consequent is a fallacy for inferences using both 'if' and '⊃':

If Clegg supports the policy, Cameron does too. Cameron supports the policy. So Clegg does.

$(P \supset Q)$, Q *so* P

To show that isn't tautologically valid, consider

P	Q	$(P \supset Q)$	Q	P
T	T			T
T	F			T
F	T	T	T	F
F	F		F	F

Some parallels between ‘if’ and ‘⊃’ – 4

Affirming the consequent is a fallacy for inferences using both ‘if’ and ‘⊃’:

If Clegg supports the policy, Cameron does too. Cameron supports the policy. So Clegg does.

$(P \supset Q)$, Q *so* P

To show that isn't tautologically valid, consider

P	Q	$(P \supset Q)$	Q	P
T	T			T
T	F			T
F	T	T	T	F
F	F		F	F

NB Tautological invalidity doesn't always entail plain invalidity – but there is nothing else to make this argument valid other than truth-functional structure, so it isn't.

Some parallels between ‘if’ and ‘$\supset$’ – 5

Transitivity holds for inferences using both ‘if’ and ‘$\supset$’:

If the alarm doesn’t go off, I’ll oversleep. If I oversleep, I’ll be late for the lecture. So if the alarm doesn’t go off, I’ll be late for the lecture.

$(P \supset Q)$, $(Q \supset R)$ *so* $(P \supset R)$

Some parallels between ‘if’ and ‘$\supset$’ – 5

Transitivity holds for inferences using both ‘if’ and ‘$\supset$’:

If the alarm doesn’t go off, I’ll oversleep. If I oversleep, I’ll be late for the lecture. So if the alarm doesn’t go off, I’ll be late for the lecture.

$(P \supset Q)$, $(Q \supset R)$ *so* $(P \supset R)$

P	Q	R	$(P \supset Q)$	$(Q \supset R)$	$(P \supset R)$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

Some parallels between 'if' and '$\supset$' – 5

Transitivity holds for inferences using both 'if' and '$\supset$':

If the alarm doesn't go off, I'll oversleep. If I oversleep, I'll be late for the lecture. So if the alarm doesn't go off, I'll be late for the lecture.

$(P \supset Q)$, $(Q \supset R)$ *so* $(P \supset R)$

P	Q	R	$(P \supset Q)$	$(Q \supset R)$	$(P \supset R)$
T	T	T			T
T	T	F			F
T	F	T			T
T	F	F			F
F	T	T			T
F	T	F			T
F	F	T			T
F	F	F			T

Some parallels between 'if' and '⊃' – 5

Transitivity holds for inferences using both 'if' and '⊃':

If the alarm doesn't go off, I'll oversleep. If I oversleep, I'll be late for the lecture. So if the alarm doesn't go off, I'll be late for the lecture.

$(P \supset Q)$, $(Q \supset R)$ *so* $(P \supset R)$

P	Q	R	$(P \supset Q)$	$(Q \supset R)$	$(P \supset R)$
T	T	T			T
T	T	F	T		F
T	F	T			T
T	F	F	F		F
F	T	T			T
F	T	F			T
F	F	T			T
F	F	F			T

Some parallels between ‘if’ and ‘⊃’ – 5

Transitivity holds for inferences using both ‘if’ and ‘⊃’:

If the alarm doesn’t go off, I’ll oversleep. If I oversleep, I’ll be late for the lecture. So if the alarm doesn’t go off, I’ll be late for the lecture.

$(P \supset Q)$, $(Q \supset R)$ *so* $(P \supset R)$

P	Q	R	$(P \supset Q)$	$(Q \supset R)$	$(P \supset R)$
T	T	T			T
T	T	F	T	F	F
T	F	T			T
T	F	F	F		F
F	T	T			T
F	T	F			T
F	F	T			T
F	F	F			T

- Some very quick reminders
- The only conditional-like truth-function
- The parallels between ‘if’ and ‘$\supset$’
- Contrasting ‘if’ and ‘$\supset$’

Some apparent differences between ‘if’ and ‘$\supset$’

- We say that two wffs ϕ and ψ are tautologically equivalent if they have the same truth-tables.

Some apparent differences between ‘if’ and ‘$\supset$’

- We say that two wffs ϕ and ψ are tautologically equivalent if they have the same truth-tables.
- More carefully, ϕ and ψ are tautologically equivalent if for each valuation of the atoms in either or both wffs, the wffs take the same values.

Some apparent differences between ‘if’ and ‘$\supset$’

- We say that two wffs ϕ and ψ are tautologically equivalent if they have the same truth-tables.
- More carefully, ϕ and ψ are tautologically equivalent if for each valuation of the atoms in either or both wffs, the wffs take the same values.
- Now note the following equivalences:

P	Q	$(P \supset Q)$	$(\neg P \lor Q)$	$\neg(P \land \neg Q)$
T	T	T		
T	F	F		
F	T	T		
F	F	T		

Some apparent differences between 'if' and '⊃'

- We say that two wffs ϕ and ψ are tautologically equivalent if they have the same truth-tables.
- More carefully, ϕ and ψ are tautologically equivalent if for each valuation of the atoms in either or both wffs, the wffs take the same values.
- Now note the following equivalences:

P	Q	$(P \supset Q)$	$(\neg P \lor Q)$	$\neg(P \land \neg Q)$
T	T	T	T	
T	F	F	F	
F	T	T	T	
F	F	T	T	

Some apparent differences between 'if' and '$\supset$'

- We say that two wffs ϕ and ψ are tautologically equivalent if they have the same truth-tables.
- More carefully, ϕ and ψ are tautologically equivalent if for each valuation of the atoms in either or both wffs, the wffs take the same values.
- Now note the following equivalences:

P	Q	$(P \supset Q)$	$(\neg P \vee Q)$	$\neg(P \wedge \neg Q)$
T	T	T	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Some apparent differences between 'if' and '$\supset$'

- We say that two wffs ϕ and ψ are tautologically equivalent if they have the same truth-tables.
- More carefully, ϕ and ψ are tautologically equivalent if for each valuation of the atoms in either or both wffs, the wffs take the same values.
- Now note the following equivalences:

P	Q	$(P \supset Q)$	$(\neg P \lor Q)$	$\neg(P \land \neg Q)$
T	T	T	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

- This emphasizes again that $(P \supset Q)$ holds just when either $\neg P$ holds or Q holds. No connection at all between the truth of P and the truth of Q is required.

Some apparent differences between ‘if’ and ‘⊃’

So – assuming Shakespeare wrote Hamlet – compare:

1. (Shakespeare didn’t write Hamlet ⊃ Bacon wrote Hamlet)
2. (Shakespeare didn’t write Hamlet ⊃ Wordsworth wrote Hamlet)

both come out are straightforwardly true.

Some apparent differences between ‘if’ and ‘⊃’

So – assuming Shakespeare wrote Hamlet – compare:

1. (Shakespeare didn’t write Hamlet ⊃ Bacon wrote Hamlet)
2. (Shakespeare didn’t write Hamlet ⊃ Wordsworth wrote Hamlet)

both come out are straightforwardly true. But

$1'$. If Shakespeare didn’t write Hamlet, then Bacon wrote Hamlet

$2'$. If Shakespeare didn’t write Hamlet, then Wordsworth wrote Hamlet

are surely not both straightforwardly true (maybe $1'$ is true, but $2'$ is surely false).

Some more apparent differences between ‘if’ and ‘⊃’

Compare:

1. (Cameron is Prime Minister ⊃ today is Thursday)
2. (Today is Tuesday ⊃ today is Thursday)

also both come out are straightforwardly true.

Some more apparent differences between ‘if’ and ‘$\supset$’

Compare:

1. (Cameron is Prime Minister $\supset$ today is Thursday)
2. (Today is Tuesday $\supset$ today is Thursday)

also both come out are straightforwardly true. But

$1'$. If Cameron is Prime Minister then today is Thursday

$2'$. If today is Tuesday then today is Thursday

are surely not both straightforwardly true ($1'$ seems odd as the antecedent and consequent seemingly have no connection, and $2'$ looks surely false).