Faculty of Philosophy

Formal Logic

Lecture 11

Peter Smith

Outline

- Where next?
- Introducing PL trees
- Branching trees

Recap: The logician's 'divide and rule' strategy

- Assessing real life arguments for (classical) validity in the general case involves two steps

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  2. Evaluate the formalized argument.

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- Two questions arising:
  1. Can we test for tautological validity in more elegant ways than using a brute-force truth-table test?
  2. How do we extend our technique to other kinds of arguments (e.g. those involving quantifiers)?

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- What are truth-relevant semantic values for predicates like F, G and names like n?
- Something like objects for n, sets of objects ('extensions') for predicates like F.
- So there are indefinitely many different assignments of semantic values that we might make to F, G and n: so we can't do a brute-force search through all the possibilities.

Two answers – 2

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- And this 'tree method' can be carried over to the assessment of argument couched in a formal language QL for dealing with quantified arguments.
- So first we introduce PL trees, then the language QL, then QL trees.

- Where next?
- Introducing PL trees
- Branching trees

'Working backwards' – 1

- To take an utterly trivial example, consider the argument

  $$\neg P,\ \neg Q,\ \neg R,\ \neg S,\ \neg P' \text{ So } Q'$$

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- We can immediately see that there is a ‘counter-valuation’ (a ‘bad line’ on a truth-table) which makes the premisses true and conclusion false, without a brute-force search through the space of possibilities. To make the premisses true and conclusion false, i.e.

  $$\neg P \Rightarrow \mathsf{T},\ \neg Q \Rightarrow \mathsf{T},\ \neg R \Rightarrow \mathsf{T},\ \neg S \Rightarrow \mathsf{T},\ \neg P' \Rightarrow \mathsf{T},\ Q' \Rightarrow \mathsf{F},$$

  we just need

  $$P \Rightarrow \mathsf{F},\ Q \Rightarrow \mathsf{F},\ R \Rightarrow \mathsf{F},\ S \Rightarrow \mathsf{F},\ P' \Rightarrow \mathsf{F},\ Q' \Rightarrow \mathsf{F}$$

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- Another, only slightly less trivial example: consider the argument

  $$(P \land \neg Q),\ (\neg Q \land R),\ (P \land S)\ \text{So}\ \neg P'$$

  Again, we don’t need to do a 32 line truth-table. For we can argue like this:

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$$\begin{gathered}(P \land \neg Q) \Rightarrow \mathsf{T}\\ (\neg Q \land R) \Rightarrow \mathsf{T}\\ (P \land S) \Rightarrow \mathsf{T}\\ \neg P' \Rightarrow \mathsf{F}\end{gathered}$$

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- And evidently there is such a valuation:

  $$P \Rightarrow \mathsf{T},\ Q \Rightarrow \mathsf{F},\ R \Rightarrow \mathsf{T},\ S \Rightarrow \mathsf{T},\ P' \Rightarrow \mathsf{T}$$

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- But we've already hit a contradiction, between $Q \Rightarrow$ F and $Q \Rightarrow$ T. So there can't be countervaluation, so the original inference is valid.

‘Working backwards’ – 4

The basic idea again. Given an inference up for evaluation, assume that there is a countervaluation and see if we can work out this must look like if it exists.

1. If that assumption leads to contradiction, we know there can’t be a countervaluation and the inference is valid.
2. If we can work from that assumption to a countervaluation then the inference is invalid.

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“But where do ‘trees’ come into it?” The story continues . . .

- Where next?
- Introducing PL trees
- Branching trees

Branching trees – 1

Consider the inference

$$(P \vee Q), \text{ So } P.$$

This is evidently invalid. But how do we show this by the 'working backwards' method?

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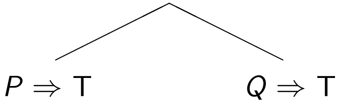
We now need to consider branching alternatives:



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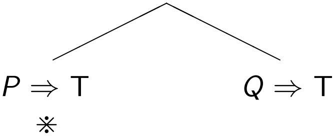

$P \Rightarrow \mathsf{T}$ ※

$Q \Rightarrow \mathsf{T}$

where on the left branch we hit a contradiction: but the right branch gives us a coherent counter-valuation.

Branching trees – 2

Consider next the inference

$$(P \vee Q), \neg P \text{ So } Q$$

Branching trees – 2

Consider next the inference

$$(P \lor Q),\ \neg P \text{ So } Q$$

Using the 'working backwards' method we suppose

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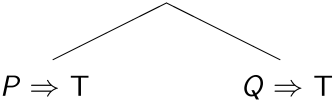

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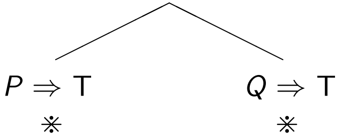

$P \Rightarrow \mathsf{T}$ $\qquad$ $Q \Rightarrow \mathsf{T}$

$\divideontimes$ $\qquad$ $\divideontimes$

where we hit a contradiction on both branches. So the inference is valid.

Branching trees – 3

Consider next the inference

$$\neg P,\ (P \lor Q),\ (\neg Q \lor R) \text{ So } R$$

Branching trees – 3

Consider next the inference

$$\neg P,\ (P \vee Q),\ (\neg Q \vee R) \text{ So } R$$

Here's a corresponding tree:

$$\begin{gathered} \neg P \Rightarrow \mathsf{T} \\ (P \vee Q) \Rightarrow \mathsf{T} \\ (\neg Q \vee R) \Rightarrow \mathsf{T} \\ R \Rightarrow \mathsf{F} \end{gathered}$$

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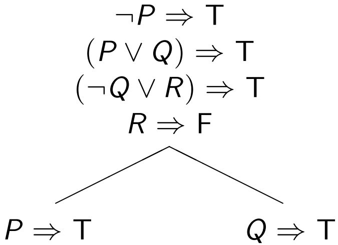

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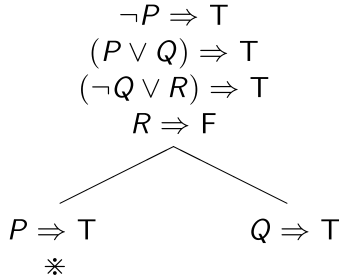

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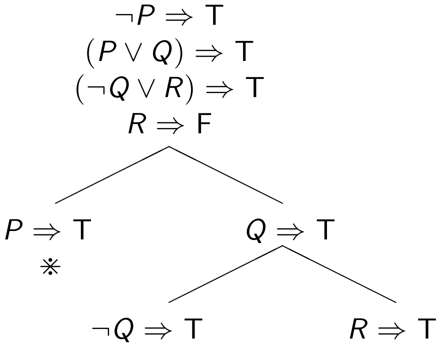

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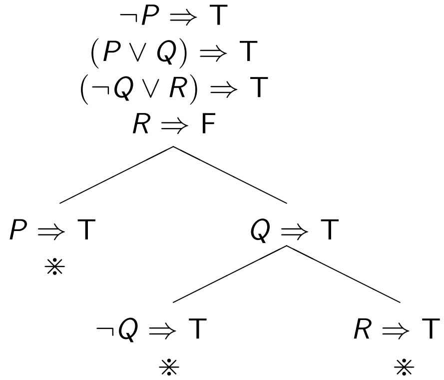

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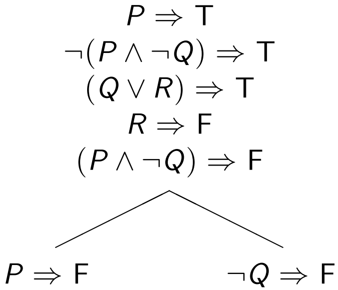

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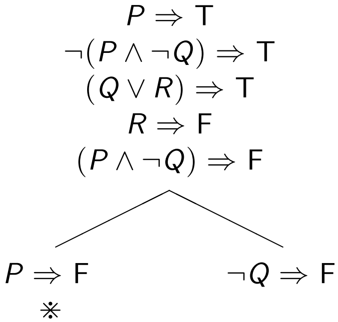

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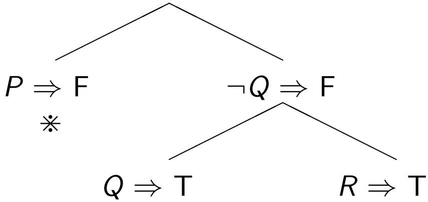

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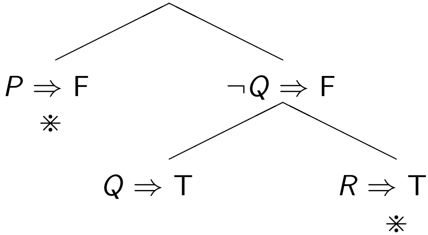

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4. If all branches eventually get closed off with an absurdity marker, then the assumption that the inference is invalid has lead to unavoidable absurdity and the inference is valid.

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To operate the 'working backwards' method for seeing whether an inference is tautologically valid we . . .

1. Start by assuming the inference is invalid, i.e. there is a valuation which makes the premisses true and conclusion false.
2. Then work out the implications of those assumptions: we may need to consider branching alternatives.
3. If a branch contains a contradictory assignment of values to wffs, close it off with an absurdity marker – this branch can't represent a possible way of making the premisses true and conclusion false.
4. If all branches eventually get closed off with an absurdity marker, then the assumption that the inference is invalid has lead to unavoidable absurdity and the inference is valid.
5. If a branch remains left open when we have unpacked the implications of every assignment of values to wffs other than atoms and their negations (so there is no more information to be used) then no contradiction has emerged, and the argument is indeed invalid.