Faculty of Philosophy

Formal Logic

Lecture 15

Peter Smith

- QL so far
- How to disambiguate
- Introducing quantifiers into QL
- Some examples of QL in action
- Existential commitment

Names, predicates

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- NB: QL predicates all have a fixed adicity (compare ordinary language multigrade predicates like 'work well together', 'conspired to commit murder').

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- The order here – predicate followed by name(s) – is purely conventional but utterly standard.
- The interpretation of an atomic sentence – a predicate followed by name(s) – is as you'd expect. The sentence says that the individuals named have the property/stand in the relation expressed by the predicate (order of names matters!).

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- The key point to emphasize is that the connectives remain propositional connectives, joining whole propositional clauses. So $(Fa \wedge Fb)$ and $(Fa \wedge Ga)$ are permitted, but NOT $F(a \wedge b)$ or $(F \wedge G)a$.

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    2. semantics: "$\varphi(\mathcal{E})$" says of everyone what "$\varphi(a)$" says of what "a" names.
- Trouble: this generates ambiguity! Consider "$\neg F\mathcal{E}$". Does this assert everyone is not-F? Or deny that everyone is F?

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- We need symbols for pronouns. We'll borrow a device from the mathematicians, use 'variables' $x, y, z, \ldots$. And to ensure that it is clear which quantifiers is tied to which pronoun, we'll tag quantifiers with variables like this:
  $(Everyone\ x\ is\ such\ that)\neg Ax$
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- Thus we'll have the following as wffs of **QL**:

$$\neg\forall xLxb, \quad \forall x\neg Lxb$$
$$\forall x\exists yLxy, \quad \exists y\forall xLxy$$

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'n denotes Plato	'G' means ① is a philosopher
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Domain is people	'R' means ① prefers ② to ③

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- Socrates teaches no one ⇒ $\forall x \neg Kmx$ or $\neg \exists x Kmx$

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