Faculty of Philosophy

# Formal Logic

## Lecture 16

Peter Smith

- QL so far
- QL in action
- Existential commitment

# The ingredients of QL – 1

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- Atomic wffs: an $n$-place predicate followed by $n$ names (says the named individual(s) have the property/stand in the relation expressed by the predicate).
- Now add the usual propositional connectives.

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- $\ldots$ and two quantifier-formers: $\forall$, $\exists$
- The basic syntactic rule: if $\varphi(n)$ is a wff, so are $\forall v\varphi(v)$ and $\exists v\varphi(v)$. [Here, $\varphi(n)$ represents any wff containing one or more occurrences of some name $n$, and $\varphi(v)$ is the result of replacing those occurrences of the name $n$ by the variable $v$.]

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- Semantics: $\forall v \varphi(v)$ says that everything [in some antecedently specified domain] satisfies the condition expressed by $\varphi$ – i.e. says that what $\varphi(n)$ claims about $n$ is in fact true of everything.
- $\exists v \varphi(v)$ says that at least one thing [in the relevant domain] satisfies the condition expressed by $\varphi$.

# Scope distinctions

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- The point of the quantifier-variable idea is that it marks the scope of a quantifier. [Recall our troubles over e.g. "every experience might be delusory" where the relative scope of the quantifier and the modality is unclear.]
- Compare: $\neg\forall xFx$ and $\forall x\neg Fx$.
- Compare: $\forall x\exists yLxy$ and $\exists y\forall xLxy$.
- The quantifier/variable way of marking scope was discovered by Frege (though he used a more cumbersome notation).

# Gottlob Frege 1848–1925

Frege developed the quantifier/variable idea in his *Begriffsschrift* ["Concept script"] (1879).

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# Examples – 1

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| ‘$m$’ denotes Socrates | ‘$F$’ means ① is wise |
| ‘$n$ denotes Plato | ‘$G$’ means ① is a philosopher |
| ‘$o$’ denotes Aristotle | ‘$K$’ means ① teaches ② |
| | ‘$L$’ means ① loves ② |
| Domain is people | ‘$R$’ means ① prefers ② to ③ |

Translate

- $\forall xLxx$
- $\forall y(Lym \supset Rymn)$
- $\exists x(Kxm \wedge Lmx)$
- $\forall y((Fy \wedge Gy) \supset Lyo)$
- $\forall x(Fx \supset \exists y(Gy \wedge Lxy))$

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- $\exists x(Kxm \wedge Lmx) \Rightarrow$ Socrates loves at least one of his teachers.
- $\forall y((Fy \wedge Gy) \supset Lyo) \Rightarrow$ Every wise philosopher loves Aristotle.
- $\forall x(Fx \supset \exists y(Gy \wedge Lxy)) \Rightarrow$ Anyone wise loves some philosopher.

# Examples – 2

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Translate

- If everyone loves Socrates, then Plato loves him.
- If anyone loves Aristotle, then Plato does.
- Aristotle loves anyone.
- Aristotle loves anyone who is wise.

# Examples – 2

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- If everyone loves Socrates, then Plato loves him

$$\Rightarrow (\forall x Lxm \supset Lnm)$$

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- If everyone loves Socrates, then Plato loves him

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- If anyone loves Aristotle, then Plato does

  $$\Rightarrow (\exists x Lxo \supset Lnm)$$

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Translate

- If everyone loves Socrates, then Plato loves him
  $\Rightarrow (\forall x Lxm \supset Lnm)$
- If anyone loves Aristotle, then Plato does
  $\Rightarrow (\exists x Lxo \supset Lnm)$
- Aristotle loves anyone $\Rightarrow \forall x Lox$

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Translate

- If everyone loves Socrates, then Plato loves him
  $\Rightarrow (\forall x Lxm \supset Lnm)$
- If anyone loves Aristotle, then Plato does
  $\Rightarrow (\exists x Lxo \supset Lnm)$
- Aristotle loves anyone $\Rightarrow \forall x Lox$
- Aristotle loves anyone who is wise $\Rightarrow \forall x(Fx \supset Lox)$

# Comments

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  - Aristotle loves anyone who is wise $\Rightarrow \forall x(Fx \supset Lox)$

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- NB behaviour of 'anyone': sometimes gets translated by $\forall$, sometimes by $\exists$. (Our formal language avoids the semantic variability of English.)
- NB our translation of the restricted quantifier:
  - Aristotle loves anyone $\Rightarrow \forall x Lox$
  - Aristotle loves anyone who is wise $\Rightarrow \forall x(Fx \supset Lox)$
- As well as restricting quantifications by relative clauses like 'who is wise'. English also restricts quantifications by using kind terms, as in 'All philosophers are wise', 'Some students love logic' (or by both 'All even numbers which are greater than two are the sum of two primes).

# Translating restricted 'all' and 'some'

All philosophers are wise $\Rightarrow \forall x(Gx \supset Fx)$
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- Remember the equivalence $(P \supset \neg Q) \Leftrightarrow \neg(P \wedge Q)$.

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- Note the shift from ‘$\supset$’ to ‘$\exists$’ as we move from restricted Universal to restricted Existential. Why is this?
- Remember the equivalence $(P \supset \neg Q) \Leftrightarrow \neg(P \wedge Q)$.
- Some philosophers are wise
  $\Leftrightarrow$ It isn’t the case that (all philosophers are unwise)
  $\Leftrightarrow$ It isn’t the case that $\forall x(Gx \supset \neg Fx)$
  $\Leftrightarrow$ It isnt the case that $\forall x \neg(Gx \wedge Fx)$
  $\Leftrightarrow \neg\forall x \neg(Gx \wedge Fx)$
  $\Leftrightarrow \exists x(Gx \wedge Fx)$

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Translate

- Every philosopher loves Socrates
- Every philosopher loves someone
- Socrates loves someone wise
- Every philosopher loves someone wise
- Every philosopher loves someone who loves Socrates
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Translate

- Every philosopher loves Socrates $\Rightarrow \forall x(Gx \supset Lxm)$

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Translate

- Every philosopher loves Socrates ⇒ $\forall x(Gx \supset Lxm)$
- Every philosopher loves someone ⇒ $\forall x(Gx \supset \exists yLxy)$
- Socrates loves someone wise ⇒ $\exists y(Fy \wedge Lmy)$

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- Every philosopher loves someone wise
  $\Rightarrow \forall x(Gx \supset \exists y(Fy \wedge Lxy))$
- Every philosopher loves someone who loves Socrates
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## Simple valuations

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| Domain is animals | '$F$' means ① is a unicorn<br>'$G$' means ① is white |

What are the truth-values of the following?

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- $\forall x(Fx \supset Gx)$

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- $\forall x Gx$ is False
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What are the truth-values of the following?

- $\forall xGx$ is False
- $\exists x(Fx \wedge Gx)$ is False
- $\forall x(Fx \supset Gx)$ is True

## An invalid inference

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- Since $\forall x(Fx \supset Gx)$ is true and $\exists x(Fx \wedge Gx)$ is false on this interpretation, that means $\forall x(Fx \supset Gx)$ doesn't logically entail $\exists x(Fx \wedge Gx)$.

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  2. All trespassers will be prosecuted.

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- But do they really? Consider
  1. All objects subject to zero net force move in a straight line with constant velocity.
  2. All trespassers will be prosecuted.
- Maybe *All Fs* is more natural when we think there are some *F*s, *Any F* when we are neutral/doubtful about that.

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- It is standardly assumed that, in any application, the domain of interpretation is non-empty.
- $\forall xFx$ can only be true if everything in domain satisfies the condition expressed by $F$; so there will be at least one thing that satisfies $F$; so $\exists xFx$ is true too. So unrestricted universals DO have existential commitment.

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- $\forall xFx$ can only be true if everything in domain satisfies the condition expressed by $F$; so there will be at least one thing that satisfies $F$; so $\exists xFx$ is true too. So unrestricted universals DO have existential commitment.
- Why this asymmetry of treatment between the cases? Nothing deep.

# Two conventions, two trade-offs

- We standardly take domains to be non-empty. We could allow empty domains (some modern books do: e.g. Hodges *Logic*). You might say: that's the more principled line – it isn't a matter of logic what, if anything, there is.

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- The standard choice has historical roots: it is also in some ways makes life a bit easier.
- We take quantifiers to be single-sorted. Iin a particular context we take all quantifiers as running over the *same* domain, when necessary restricting generalizations to the $F$s by using the likes of '$Fx \supset \ldots$' or '$Fx \wedge \ldots$'. Ordinary language quantification is many sorted. (Be more natural to regiment 'All Fs are Gs' by something of the form $\forall x[Fx; Gx]$??)

# Two conventions, two trade-offs

- We standardly take domains to be non-empty. We could allow empty domains (some modern books do: e.g. Hodges *Logic*). You might say: that's the more principled line – it isn't a matter of logic what, if anything, there is.
- The standard choice has historical roots: it is also in some ways makes life a bit easier.
- We take quantifiers to be single-sorted. Iin a particular context we take all quantifiers as running over the *same* domain, when necessary restricting generalizations to the $F$s by using the likes of '$Fx \ \supset \ \ldots$' or '$Fx \ \wedge \ \ldots$'. Ordinary language quantification is many sorted. (Be more natural to regiment 'All Fs are Gs' by something of the form $\forall x[Fx; Gx]$??)
- Keeping things single-sorted buys a lot of simplicity at the cost of some departure from the logical form of ordinary arguments. A price mostly worth paying.

# And now read on . . .

- For many more worked examples and exercises read *IFL* Chs. 21–24 . . .

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- . . . and do the Vacation Worksheet.