Charles Parsons
Mathematical Thought and Its Objects

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This is the second, revised, version of a section-by-section discussion of Charles Parsons's eagerly awaited *Mathematical Thought and Its Objects* (CUP, 2008: pp. xx + 378)[1] – though not every section gets the same level of attention. For convenience, the divisions here correspond to the chapter/section divisions of the book.

Parsons himself says that his book has been a very long time in the writing. Its chapters extensively "draw on", "incorporate material from", "overlap considerably with", or "are expanded versions of" papers published over the last twenty-five or so years. So what we are reading is a multi-layered text with different passages added at different times.[2] This does make for a pretty bumpy read, with the to-and-fro of argument not always ideally well signalled. The prose style can make for hard going too. Consequently, I sometimes am not too confident that I am reading Parsons aright. But let's dive in ...

1 Objects and Logic

§§1–4 Logic and the notion of an object The key claim of the book's opening sections is that "Speaking of objects just is using the linguistic devices of singular terms, predication, identity and quantification to make serious statements".

Thus construed, the idea of objects in general is – in the first place – loosened from ties with any idea of 'actuality' (Kant's *Wirklichkeit*), where this has something to do, in Frege's words, with "act[ing] on our senses or at least producing effects which may cause sense-perceptions as near or remote consequences". Talk of objects is also – in the second place – being loosened from ties with ideas of intuitability (whatever that Kantian idea comes to: things are in fact left pretty murky at this stage, but then Parsons

[1] Some much abbreviated excerpts from the first version appeared as a Critical Notice in *Analysis Reviews*, in *Analysis* Vol. 69, 2009, pp. 549–556.

I am very grateful to fellow members of a reading group – especially Bob Hanna, Luca Incurvati, Steven Methven, Michael Potter, Tim Storer and Rob Trueman – for illuminating discussions, though they will probably think I've not been illuminated enough.

I am even more grateful to Charles Parsons for some very helpful emails, responding to the first version of these comments. Inevitably, there are quite a few disagreements remaining in what follows – this is philosophy, after all! The considerable length of these remarks bears witness, however, that I think Parsons's book is intriguing and important.

Finally, in putting together a revised versions of these notes, I've been spurred on and helped by another discussion group, this time at the University of Canterbury, Christchurh NZ.

[2] So there is another route Parsons could have taken: he could have reprinted the relevant papers with with postscripts, and then top-and-tailed the collection with an extensive preface and perhaps added concluding reflections. It must sound ungrateful, but I rather suspect that that route might, at least in some respects, have worked better.

is going to talk a lot about intuition later in the book). Consequently, Parsons thinks, endorsing the logical conception of an object will "defuse too-high expectations of what the existence of objects of some mathematical type such as numbers would entail." The suggestion is that those who are inclined to deny the existence of abstract objects – meaning, broadly, things outside the causal nexus (objects which we don't have a direct quasi-perceptual cognitive access to) – or who find abstract objects puzzling, are likely to be explicitly or implicitly imposing requirements on being an object that go beyond those captured in the core logical conception. (And certainly, he adds, it behoves philosophers to pause to think long and hard about the relevant concept of an object before rushing to any conclusion that "mathematics is engaged in mythology, in speaking of remarkable configurations of objects that just do not exist".)

Now, I am highly sympathetic to this line, which has its roots in Frege and whose "most important advocates in more recent times," according to Parsons,[3] "are Carnap and Quine". But let me take minor issue with Parsons when he speaks of "the view that the most general notion of object has its home in formal logic". That isn't really the happiest way of putting things. After all, suppose we translate back from first-order logical notation into a disciplined core fragment of English – the sort of regimented English which we use in giving determinate content to the wffs of the artificial language in the first place. Then here too, in regimented English, we will find the core devices of singular terms, predication, identity and quantification. And the Quinean should presumably say that our commitments to objects are revealed equally well by rendering our theory of the world into the idioms of this disciplined core of ordinary language: the virtue of using the notation of an artificial language of formal logic is that it keeps us disciplined not that it changes the subject.[4] Formal languages don't magically do what ordinary language can't do: they just do ordinary things like use singular terms and quantify but in entirely uniform ways. So turning to "formal logic" doesn't really give us a different take on the general notion of object. Surely Parsons spoke better when he expressed the position he is proposing as the view that "speaking of objects just is using the linguistic devices of singular terms, predication, identity and quantification to make serious," and indeed true, "statements".

But how are we to spell out this view in a bit more detail? Let's start with the following line of thought (my words, not Parsons!):

> Such is the mess and conversational plasticity in our various ordinary ways of talking that you can't just read off our ontological commitments from our unregimented talk. After all, to take some familiar trite examples, we speak of doing things for Mary's *sake*, or having *ideas* at the back of our *minds*. We have apparent singular terms here, but might well ask: are we really committed to sakes, or even ideas and minds as objects? Well, to be sure, when we regiment our story of the world into a logically well-behaved language (whether a fully formalized logical language, or a decently regimented fragment of ordinary language), we will paraphrase away talk of sakes and certainly won't end up quantifying over any such things. What about ideas or minds? Well, maybe with hard work and some careful tidying, we can also paraphrase away such talk, so we can again get by without quantifying over

[3]But see fn. 8.

[4]Well, perhaps – on second thoughts – English can't be disciplined rigorously enough without uncomfortable mutilation, turning it into a species of Loglish: but if so, this this is not, so to speak, a *deep* failing of the vernacular.

such things or making identity claims about them, etc. But let that turn out with those particular examples as it may. Our genuine commitments will only be revealed by our serious use of the quantifiers and singular terms once we've done some pre-processing, some paraphrasing, some regimentation.

However note that this core Quinean thought leaves it wide open what the constraints on the project of regimentation are.

In particular, the core thought – as it stands – is quite compatible with the suggestion that there are constraints on the project of regimentation that are provided by some a priori understanding of what kosher objects must be like (actual, intuitable, or whatever), and acceptable regimentations of our overall theory of the world are to be shaped by such a priori constraints. Of course, this suggestion does offend against another, but quite different, Quinean idea, namely that there is no first philosophy prior to regimented science. But note, *that* idea is independent of the claim that ontological commitment is revealed in the use of the devices of singular terms and quantification of our theory once regimented: it's a *further* thought. So it isn't so much the 'logical' conception of objects by itself that stands against e.g. the idea that objects must be 'actual': it is the 'logical' conception *plus* the further thought that the constraints on the project of regimentation are just the usual constraints on scientific theorizing, where we give weight to organization, simplicity, economy etc. but otherwise let things go where they will unconstrained by a priori metaphysical prejudice.

There's no real disagreement with Parsons here, of course; I just want to bring out that he is endorsing two separable strands of thought. But so far, all so Quinean. However, there's more to be said.[5]

There's a familiar kind of philosophical 'unmasking' project, which aims to explain why, although an area of discourse looks just like ordinary fact-stating discourse on the surface, there is a good sense in which it isn't really. For example, the Humean projectivist claims that, although moral talk has the look of fact-stating discourse, it really is in a different game – and part of his story[6] aims to tell us why, despite the deep underlying difference, it is still appropriate that moral talk takes on the surface structure of fact-stating discourse. Note, the sophisticated projectivist doesn't offer a story in which we can readily paraphrase away moral talk – indeed, he'll offer a story about why that *can't* be done. So his story is to be applied *after* the sort of initial regimentation which gets rid of momentary distractions like sakes and the like: he claims that, despite the appearances left in place after the more superficial initial work is done, further reflection reveals differences in the fact-stating (property-discerning, object-referring) statuses of different areas of discourse. Generalizing, it would seem possible to hold that, while indeed it is in the use of regimented singular terms and quantification to make true claims that we must discern reference to objects, still – for one reason or another – not *all* such use is fully committing in reference to kosher objects. It as yet remains open that we have to discriminate even among regimented singular terms, e.g. by reflecting on how some singular term might be canonically introduced into the regimented language.

Now, I mention the possibility of this kind of discriminatory position because it is suggested by Dummett's late position. The idea is that we can distinguish expressions which have, in Dummett's words, "genuine, full-blown reference" (hooking up to an independent reality) from those which have reference only in a thin sense (perhaps, as it

[5] Thanks to Douglas Campbell, for remarks that showed that my first attempt here was unclear and misleading.

[6] At least as told by Simon Blackburn.

were, by casting their shadows on reality). And we make the distinction by considering how expressions are canonically introduced into regimented language – so, for example, the idea might be that singular terms introduced by abstraction principles don't have reference in the robust sense.

Contrast this with the early-Dummett/Hale/Wright line which resists making such discriminations:[7] at a first pass, the truths are what the relevant discipline (fallibly) aims at, a singular term in such truths is whatever walks, quacks, and swims like a singular term in a disciplined way, and the objects are (without distinction) whatever are the referents of such singular terms in true sentences. So on this egalitarian line – also Quine's own line? – we don't discriminate among regimented singular terms by how they are introduced, picking out some as having robust reference, and taking the kosher objects to be what are referred to by *them*. Rather, in mathematics for example, we first identify true sentences by the appropriate criteria; we identify the singular terms in those sentences by their compositional behaviour: and then (the claim is) the relevant objects – mathematical objects in our example – are what those singular terms, functioning in the given truths, need to refer to make the truths indeed come out true.

My impression is that Parsons is of the latter egalitarian persuasion: so that's a third grade of Quinean involvement. But he doesn't actually say why he rejects the possibility of Dummettian discrimination, so there something of a gap in his defences here.[8]

§5 Is whatever is an object? Parsons has been proposing the view that "speaking of objects just is using the linguistic devices of singular terms, predication, identity and quantification". And the focus so far has been on first-order quantification. But what about generalizations about *properties*, the sort of generalization apparently involved in familiar mathematical statements like the full induction principle for arithmetic, or the separation axiom in set theory? Should we construe those as involving generalization over something like Frege's "unsaturated" concepts, entities which aren't objects? Or is the commitment here just to more objects?

One way of perhaps resisting the Fregean line arises from noting that we can easily parlay quantification into predicate position into just more quantification into subject position (or so it seems). Suppose, using Parsons's notation, we use '$(Ox)Fx$' to denote some object corresponding to the Fregean concept expressed by 'F'. And suppose we use 'η' for an appropriate copula ('has' if the object is a property/quality, 'is a member of' if the object is a set, etc.) Then we have Ft if and only if $t\ \eta\ (Ox)Fx$. And so, given a context when we are minded to quantify into the position held by 'F' we could instead first nominalize and then quantify into the position held by the singular term '$(Ox)Fx$' instead. It seems then that we can treat quantification over properties (as we might initially put it) as just more quantification over objects. This after all is a common mathematical practice, as e.g. when we familiarly regiment informal second-order arithmetic into a formal theory of numbers and *sets* of numbers.

Still, at least two objections to the nominalizing strategy as an across-the-board way of eliminating 'direct' quantification into predicate position readily suggest themselves (as Parsons notes). First, the claim that Ft if and only if $t\ \eta\ (Ox)Fx$ is, itself, intended

[7] Latterly, e.g. in a response to Rumfitt, Hale and Wright have seemingly wanted to qualify their previous line: so take the description with a pinch of salt.

[8] Indeed, from the other side of the Atlantic, I would regret Parsons's whole lack of engagement with what is surely the most sophisticated recent development of the Fregean line in the hands of Crispin Wright in his *Frege's Conception of Numbers as Objects*, in Bob Hale's *Abstract Objects*, and then in further debates e.g. between Dummett on the one hand and Hale and Wright on the other.

as a generalization, to express which we need to generalize into predicate position in a way that can't be nominalized away. And second, that generalization in any case has to be restricted or else or we could instantiate with the predicate '$\neg x \ \eta \ x$', and paradox ensues.

However, that's not yet game set and match to the Fregean. Can't the force of the first objection be turned by adding the device of semantic ascent to our armoury? We can, for example, generalize about the possibility of nominalization by saying that for any predicate 'F' (and term 't'), 'Ft' is true if and only if '$t \ \eta \ (Ox)F$' is true.

Ah, it will be protested, the device of semantic ascent still doesn't really allow us fully to capture what we want to say by means of quantifications over properties. Compare for example the familiar thought that the content of the full informal arithmetic induction axiom is not captured by semantically ascending and saying that all instances of the first-order schema are true. Reply: that familiar thought is true, *if* we require the instances to be drawn from a fixed language. But suppose we treat the schema in an open-ended way, available to be instantiated however we extend our language (as Parsons puts it, "In practice, in any language in which we talk about natural numbers, we are prepared to affirm induction for any predicate of that language"). Then, by treating the schema in this way, we arguably recapture the intended sweep of the informal axiom, still without taking on ontological commitments to Fregean concepts.

And as to the second objection against the nominalizing strategy, the threat of paradox only arises if we take the reference of '$(Ox)Fx$' as an object that is, so to speak, already in the original domain of objects (i.e. the original domain of subjects of predication). But we could take the moral here to be that objects segregate into different types, the references of nominalized predicates being of a different type to the references of common-or-garden singular terms.

So where does this to-and-fro take us? Parsons summarizes: "the present discussion does show that considerations about predication do not lead inevitably to our taking second-order logic as our canonical framework and admitting, as values of our second-order variables, entities that are not objects."

Three comments about all this. First, about semantic ascent and the open-ended nature of our commitment e.g. to the induction schema. Just *why* do we stand prepared to take on all-comers and instantiate the schema with any novel predicate we care to extend our language with? Kreisel suggested long since that we accept the instances of the induction schema because we *already* accept the full second-order induction axiom. I think there are issues about that claim: but the claim is a familiar one that many have found persuasive. And Parsons does indeed engage with it – but not at this point. The relevant discussions come much later, e.g. in §47. We'll have to return to the issue.

Second, about avoiding paradox on the nominalizing strategy. The Fregean might well riposte that claiming that the way to go is to segregate objects into different types sounds too much like theft of Frege's key insight rather than an alternative story. After all, speaking with the vulgar, the Fregean will say that what he is arguing for is precisely a distinction among "entities" between saturated and unsaturated types. So he has a principled story about type-distinctions to tell. Further, he will add, once the distinctions are made in the right way, the temptation to pursue the nominalizing strategy, putting all the work of unifying propositions into a copula, should evaporate. And what is the alternative principled story about types supposed to be?

Third, I'm left unclear exactly how Parsons thinks about the relationship between the two ways of avoiding second-order quantification that he discusses (i.e. the routes via nominalization and ascent). He does say that "The laws of logic have a certain dialectical character, in that the method of nominalization and the method of semantic ascent can both be used to state them, and neither can completely displace the other." But *that* is rather unclear, to say the least: and if it is true that there is no satisfactory stable way of explicitly stating the laws of logic (echoes of the *Tractatus*?), then this should surely get more discussion.

But finally, a comment before proceeding. Note that Parsons has proposed that (1) "speaking of objects just is using the linguistic devices of singular terms, predication, identity and quantification" to make serious, and indeed true, statements. And defending that view about, so to speak, the measure of what *objects* we are committed to falls well short of saying that (2) standard first-order logic is the universal measure of ontology in general. Resisting the more sweeping claim is quite consistent with accepting Parsons's initial Fregean claim about *objects*. Not that I'm suggesting that Parsons thinks otherwise. I'm just emphasizing that if you are not persuaded by Parsons's suggestions in this current section, and hold that we are committed to entities that are not objects – first-level Fregean functions and concepts, for a start – then you can accept formulation (1) without accepting (2).

§6 Being and existence At the outset of this section, Parsons writes that one point at which "reservations about standard first-order logic as the universal measure of ontology can affect the notion of mathematical object is the ancient question whether reference to objects is necessarily reference to objects that exist." Parsons discusses Meinongian views in some detail: indeed, this is one of the longest sections in the book, but not the clearest. Here's part of his final summary of the discussion:

> We are left with the question whether the "true" meaning of the existential quantifier is [i] the permissive Meinongian one [allowing quantification over objects that do not exist], [ii] existence that allows freely for abstract objects but that rules out impossibilia, or [iii] something like actuality. The logic based concept of object does not decide between these alternatives, although, once it has been set forth, the case for [iii] is weakened. But in order to understand the notions of object and existence in mathematics we have to put more flesh on the bare form given by formal logic. We need to fill out the logic-based conception by looking at cases. ... [C]onsiderations proper to mathematics will not lead us to favour [i] over [ii]. General as the notion of object in mathematics is, there is still a constraint of possibility, coherence, or consistency that objects postulated in Meinongian theories are allowed to violate.

The talk here of having to "fill out the logic-based conception" might initially seem surprising given what has gone before. But I take it that the thought is simply this. The Fregean thesis (read the Wright way, if not the right way) is that objects are whatever we have to construe singular terms in true sentences as referring to, if the sentences are indeed to come out true (where being a singular term is a matter of interacting with quantifiers and the identity predicate in the right way). Hence, to "fill out" this general template view about objects, we have to say what kinds of sentences we do in fact accept as being true.

If we, for example, take statements like "Sherlock Holmes is more famous than any living detective" and "There's a fictional detective who is more famous than any living detective" at face value – i.e. construe their logical form as given in their surface structure – while cleaving to the logical conception of objects, and take them moreover as literally true claims, then perhaps we are committed to the likes of Sherlock Holmes as objects (and on one natural but not compulsory elaboration of that view, (i) Meinongian non-existent objects). If we paraphrase away apparent talk of fictional objects and the like, but accept that there are true mathematical statements talking of numbers, sets, etc., then (ii) we are not committed to non-existent objects, but have to accept that there are abstract objects which aren't "actual". If we insist on also paraphrasing away even apparent straight talk of numbers (e.g. construing it as governed by an operator "in the arithmetical fiction ..."), then perhaps (iii) we may only be committed to actual objects.

Parsons is pretty sceptical about whether we have any need to take line (i) and "to admit into the range of our quantifiers such objects as the golden mountain, the round square, Pegasus and Sherlock Holmes", though it is not his concern to argue for the general point here. But he *does* argue that "considerations *proper to mathematics*" (my emphasis) don't give any impetus for preferring the Meinongian views (i) over (ii). For mathematics doesn't countenance impossibilia like the round square, or present itself as fictional discourse. And that's surely right. As to (iii), I assume Parsons's thought is that a fictionalist critic of our common-or-garden standards of mathematical truth on the basis of a metaphysical repudiation of abstract objects is (in danger of) getting things upside down, at least by the lights of the truth-first, "logic-based conception" of objects, according to which we don't have a handle on the notion of an object except via a prior grip on the notion of truth for the relevant object-referring statements.

If this reading of Parsons is right, then I agree with him.[9]

§7 Abstract objects and their concrete representations Back in §1, Parsons says "Roughly speaking, an object is abstract if it is not located in space and time and does not stand in causal relations." In the last section of the first chapter, he returns to the question of characterizing abstract objects, and suggests a distinction among them between pure abstract objects (e.g. pure sets) and those which "have an intrinsic relation to the concrete" – Parsons calls the latter quasi-concrete.

As a paradigm example of the quasi-concrete, Parsons takes the example of sentence types:[10] "what a sentence [type] is is a matter of what physical inscriptions are or would be its tokens". But how should we generalize from this case? Parsons writes "What makes an object quasi-concrete is that it is of a kind which goes with an intrinsic, concrete 'representation'". The scare quotes are there in Parsons – and you can see why. Should we really say, for example, that a sentence token is a *representation* of its type? Your first response might be: the token isn't *about* the type, so isn't a representation of it. But, reading on, it becomes clear that Parsons doesn't mean representation so much as *representative*. And then, yes, I suppose we might say that the token is a representative of the type. Parsons also writes "Although sets in general are not quasi-concrete, it does seem that sets of concrete objects should count as such; here the relation of representation would be just membership," (this time, no scare quotes!). Again, we might say the spoon

[9] A fictionalist, however, will of course think he goes to fast. Indeed fn. 12 of §4 is about the extent of Parsons's direct engagement with fictionalism.

[10] Just as an aside, I suppose we might then wonder for a moment whether sentence types might be a counter-example to the claim that abstract objects lack temporal location. We might ask: did the sentence type "the cat is on the mat" really exist in 2000 BC before anyone spoke English? Or what about that other quasi-concrete thing(?) the Earth's equator – does this only exist when the Earth is spinning?

in my coffee cup is a representative of the canteen of cutlery – though it surely can't be said to be a representation.

Now these cases might suggest that Parsons's paradigm quasi-concrete objects are those formed by abstraction from equivalence classes of concreta. The type-exemplified-by-token-S_1 is the same type as the type-exemplified-by-token-S_2 just if S_1 is equiform to S_2. The set-of-cutlery-of-which-s_1-is-a-representative-spoon is the same as the set-of-cutlery-of-which-s_2-is-a-representative-spoon just if s_1 belongs together with s_2. But if this is what Parsons has in mind in central cases, why not put it this way, highlighting abstraction operators, rather than in terms of representations/representatives?[11]

Let's agree though that it *is* plausible to say that some abstract objects are more directly tied to the concrete than others: and let's suppose that we have suitably tidied up Parsons's notion of such quasi-concrete abstracta. Then he raises the question, are *numbers* quasi-concrete in this sense? We might initially be tempted to say yes, suggesting that the number five, for example, has concrete representatives like: ||||| . Parsons, however, makes two good old Fregean points against this. First, to take that block as representative, we have already to take it *as* a group of strokes (rather than as a single grid, for example). So the representative here is *not* simply the concrete thing, independently of how it is conceptualized. We might say: the representative is already one step away from being purely concrete. But second, numbers don't merely have representatives like ||||| , which are bound up with the concrete. For numbers can number anything, including the purely abstract. So a number can't just be an abstraction from an equivalence class of concreta. (Parsons is going to return to talk about related matters in Chapter 6, so I'll say no more for the moment.)

[11] Though, as Philip Catton noted, the abstraction operator approach doesn't deal with the equator example.