11 ‘P’s, ‘Q’s, ‘α’s, ‘β’s – and form again

We should pause to make it clear how our ‘P’s and ‘Q’s, and ‘α’s and ‘β’s, are being used, and why it is crucial to differentiate between them.

We also need to explain the standard convention governing the use of quotation marks. We will then add some more remarks about the idea of a wff’s form.

11.1 Styles of variable: object languages and metalanguages

(a) Recall the use of e.g. ‘n’ and ‘F’ which we introduced in a rough and ready way to stand in for everyday names and general terms right back in §1.5. These are handy augmentations of logicians’ English, particularly useful when we want to talk perspicuously about general patterns of inference.

We have similarly used ‘A’, ‘B’ in a rough and ready way to stand in for whole sentences; and again the role of these symbols is to help us speak snappily about forms of sentences, patterns of inference, etc. For example, in §8.1, we used them to enable us to display forms of ordinary-language arguments involving conjunction and disjunction.

Then in §8.7, we started using ‘α’ and ‘β’ in order to talk briskly and clearly – but still in augmented English – about wffs in a PL language.

The use of symbolic letters as schematic variables in these ways is always dispensable. At the expense of longwindedness, we could use plain unadorned English prose instead. Thus, rather than using the likes ‘A’ and ‘B’ or ‘α’ and ‘β’ in *displaying* patterns of inference, we could *describe* the same patterns using expressions such as ‘the first proposition’ and ‘the second wff’. And note too that, just as ‘the first proposition’ and ‘the second wff’ don’t themselves express propositional messages, neither do English-augmenting schematic variables like ‘A’ or ‘α’.

(b) By contrast, our recently introduced ‘P’, ‘Q’, etc. do not belong to English, augmented or otherwise. Instead they are atomic wffs (in effect, basic sentences) of one of our new artificial PL languages. Unlike the schematic variables, these new symbols *do* potentially express complete propositional claims; they can encode whole messages, as it might be about Alun’s loving Bethan, or Jack’s taking Jill to the party, or whatever.

‘P’, ‘Q’, etc. are available to express different messages in different PL languages (what stays constant across languages is the meaning of the connectives). In this sense, the atomic letters can perhaps also be thought of as another kind of ‘variable’, i.e. they are potentially open to various interpretations in different languages. But within a fixed language, we suppose they are assigned a fixed meaning. And the crucial contrast remains: Greek-letter variables like 'α', 'β', etc. are used in augmented English to *talk about* PL wffs and arguments; 'P', 'Q', etc. belong to various PL languages and can themselves *express* messages on all kinds of subject matter, depending on the glossary or interpretation-manual in force.

(c) Here is some standard terminology:

> The *object language* is the language which is the object of logical investigation at a particular point. The *metalanguage* is the language in which we conduct our investigation and discuss what is going on in the current object language.

In the last few chapters, PL languages have been the object of investigation. Our metalanguage has been English augmented with variables like 'α', 'β', etc. (hence those are often also called *metalinguistic variables*). While, in an Italian translation of this book, the object language might at some point be the same PL language, but the metalanguage in which we would be discussing it would now be augmented Italian.

(Of course, we might well want to discuss facts about logical features of English sentences in English, as we ourselves did earlier in the book; and then the metalanguage subsumes the object language. But in general, in logical investigations, the object language and metalanguage are distinct.)

(d) To avoid confusion, it is more or less essential to mark typographically, one way or another, the distinction between (i) elements of the formalized languages like PL languages which are (from now on) the primary objects of our investigation and (ii) the informal metalinguistic apparatus which we use 'from outside' in discussing these object languages. Here are our conventions:

> In this book, the constituents of formalized languages such as PL languages will always be printed in sans-serif type, like 'P', 'Q'.
>
> Symbols printed in *serif italics*, like 'A', 'n', 'F', etc., or Greek letters like 'α', 'β', etc., are always augmentations of our English metalanguage. We reserve Greek letters for when we are talking about PL wffs and arguments, or about wffs and arguments in other artificial languages.

11.2 Quotation marks, use and mention

(a) We now turn to talk about one key use of quotation marks.

If we want to refer to a certain *man*, we may of course use his name, as in the sentence

(1) Socrates was snub-nosed.

However, sometimes we want to discuss not that philosopher himself but his (English) *name*. There are various ways of referring to this name, e.g. laboriously by

> The name consisting of the nineteenth letter of the alphabet, followed by the fifteenth letter, followed by the third letter, ...

Or more indirectly, we could use

The name which begins the sentence (1).

But, much more economically, we can use quotation marks, thus:

'Socrates'

This whole expression refers to our philosopher's name, by the default logicians' convention governing the use of quotation marks:

> Given an expression beginning with an opening quotation mark and ending with a matching closing quotation mark, the whole expression *including* the quotation marks is to be construed as referring to the word, sentence or other expression displayed *inside* the quotes.

There are other uses of quotation marks, for example as so-called 'scare quotes' – as now in this very sentence. Such quotes highlight an expression perhaps as novel, or perhaps as being used in a contentious way. But this section is about the primary use of quotes round an expression as a device for *mentioning* that expression.

(b) Compare, then,

(1) Socrates was snub-nosed.
(2) Socrates contains the first letter of the alphabet.
(3) 'Socrates' is snub-nosed.
(4) 'Socrates' contains the first letter of the alphabet.
(5) 'Socrates was snub-nosed' is an English sentence.
(6) 'Snub Socrates nosed was' is not an English sentence.

The first is famously true; the second false (people don't contain letters of the alphabet). The third is false too (names don't have noses to be snub or otherwise); the rest are true.

Again, consider the following:

(7) Charles is called Chuck by his friends and Dad by his children.

If we strictly observe the convention of using quotation marks when we are mentioning or talking about an expression, this needs to be marked up as follows:

(7′) Charles is called 'Chuck' by his friends and 'Dad' by his children.

And since Socrates is a man, not an expression, how should we insert quotes in the following in order to yield a truth?

(8) The initial referring expression in example (3) is Socrates.

We need the following:

(8′) The initial referring expression in example (3) is ' 'Socrates' '.

That's because, in order to denote the referring expression at the beginning of (3), which already involves quotation marks, we need to put that whole expression in another pair of quotation marks – hence the double ration. Or for clarity's sake, to make it plain that there *are* two pairs of quotation marks here, we could/should use two different styles of quotation marks as we have done a few times before, like this:

(8″) The first referring expression in example (3) is " 'Socrates' ".

(c) Let's have another series of examples. Consider *this* wff:

$(\mathsf{P} \lor \mathsf{Q})$

(and here we refer to a type of expression by another common device which we have also often used before; i.e. we display a sample token and use a demonstrative to 'point through' to the type instantiated by the token). In one PL language we've met, that wff says that either Jack took Jill to the party or he took Jo. And in an argument deploying that wff, the PL disjunction sign is *used* to express this disjunctive claim.

But sometimes we want to talk about the disjunction sign itself, i.e. to *mention* it in English. We can do this in various ways. We could use, as we have just done, the description 'the disjunction sign'. Or we can call it, as some do, simply 'vel' or 'the vel sign'. The vel sign itself is no part of English – though the word 'vel' *is* used as part of logician's English: we use it to refer to the PL connective.

We could also use quotation marks again, as we have often done, and refer to the disjunction sign via *this* expression:

'$\lor$'

This quotation expression is then available for use *in our extended English*, as when we report that '$\lor$' is a binary connective in PL languages. (Compare: the word 'soleil' is no part of English. However we can use its quotation in an English sentence – as we have just done, or as when we say that the French word 'soleil' is used to refer to the sun.)

Similarly for the atomic wffs of PL languages. Given their interpretations in a particular language, we can *use* them to report Bethan's love affairs or Jack's party-going or whatever. But we will also want to *mention* the atoms, i.e. talk about them in English, our metalanguage. Again the easy way of referring to them is to do what we have been doing all along – i.e. to use quotation marks to form expressions (augmenting English!) to refer to wffs. Thus consider:

(9) 'P' is an atomic wff of some PL languages.
(10) 'P' is a sentence of English.
(11) " 'P' " is an expression which can occur in PL wffs.
(12) " 'P' " is an expression which can occur in English sentences.
(13) The atomic wff 'P' is a subformula of the molecular wff '$\neg(\mathsf{P} \lor \neg\mathsf{Q})$'.

(9) is true, but (10) is false; (11) is false too (since PL languages don't contain quotation marks). (12), however, is true; for example, (9) and (13) provide instances of true English claims where the quotation of 'P' occurs.

11.3 To Quine-quote or not to Quine-quote?

(a) The distinction between directly *using* an expression to say something and *mentioning* that expression (i.e. referring to the words or symbols) is evidently crucial. We therefore need some convention for marking the distinction between use and mention. The logicians' default convention is to use quotation marks explicitly to signal when expressions are being mentioned.

However, strictly abiding by this convention does lead to unsightly rashes of quotation marks, and most logicians adopt a more relaxed policy when no confusion will result.

And we have in fact already done so, forgivably cutting ourselves some slack in various cases. For example, in giving a glossary for a formalized language, instead of writing something like

(1) 'P' means *Jack loves Jill*,

we simply put

(2) P: Jack loves Jill.

Again, when making assignments of truth-values to wffs, instead of writing

(3) 'P' := T,

(i.e, 'P' takes the value T), we simply wrote

(4) P := T,

allowing ':=' to, so to speak, generate its own invisible quotation marks on the left.

(b) Let's also note another – rather more subtle – sort of case where we have already been relaxed about quotation. Recall that in §9.1(c), we briskly stated one of the wff-building rules using schematic variables as follows:

(W2) If α and β are wffs, so is $(\alpha \wedge \beta)$.

It would have been plain how to read this rule, even if we hadn't immediately explained it. Yet W2 does mix augmented English (the metalinguistic variables 'α' and 'β') with PL expressions (the conjunction sign, of course, but also the two brackets as well). So we seem to have an unholy mixture of languages here, and no quotation marks to keep things well-behaved. And note that we can't untangle things by writing

If α and β are wffs, so is '$(\alpha \wedge \beta)$',

for what is inside the quotation marks there is *not* a wff of a PL language (remember, the Greek letters 'α' and 'β' are *not* symbols of that type of language).

What to do? As we explained before, W2 is to be understood as saying

(W2′) If α and β are wffs, so is the expression formed from '(' followed by α followed by '$\wedge$' followed by β followed by ')'.

So, if we want to be punctilious with quotation marks, we will have to introduce a new species – corner quotes, often called *Quine quotes* after their inventor – and write

(W2″) If α and β are wffs, so is $\ulcorner(\alpha \wedge \beta)\urcorner$,

where the corner quotes are defined so as to make this equivalent to W2′. And we will see in an Appendix how to define these Quine quotes so they work as advertised.

However, there is no real risk of muddle in using W2 as it stands, undecorated by any kind of quotation mark. Insisting on using Quine quotes would be a fussy step too far.

11.4 How strict about quotation shall we be?

To repeat, as examples (2), (4) and W2 in the last section show, we have already taken a somewhat relaxed line on the use of quotation marks, omitting them in some cases. And once we have emphasized the use/mention distinction, it is a judgement call how much *more* relaxed about quotation marks we want to be in the rest of this book.

Consider the following:

(1) The wff $(\mathsf{P} \lor \neg\mathsf{P})$ is an instance of the schema $(\alpha \lor \neg\alpha)$.

It is obvious from the phrases 'the wff $(\mathsf{P} \lor \neg\mathsf{P})$' and 'the schema $(\alpha \lor \neg\alpha)$' that we are here *mentioning* a wff and a schema. We are not *using* the wff to express a message, and not using the schema to do some generalizing work. If we were following the logicians' default quotation policy, we would write

(2) The wff '$(\mathsf{P} \lor \neg\mathsf{P})$' is an instance of the schema '$(\alpha \lor \neg\alpha)$';

But this begins to look like unnecessary overkill. We can certainly drop explicit quotation marks in such cases, after 'the wff' or 'the schema', without any risk of confusion. Let's do this in future!

Another example. Consider

(3) $(\mathsf{P} \lor \mathsf{Q})$ is a subformula of $((\mathsf{P} \lor \mathsf{Q}) \land \mathsf{R})$.

What are these sans-serif expressions from some PL language doing in an English sentence? Well, even though we haven't written 'the wff $(\mathsf{P} \lor \mathsf{Q})$' or 'the wff $((\mathsf{P} \lor \mathsf{Q}) \land \mathsf{R})$', we again know that the formal expressions in (3) must be being mentioned, and are not being used to express messages. We could therefore write, as we did in §9.3,

(4) '$(\mathsf{P} \lor \mathsf{Q})$' is a subformula of '$((\mathsf{P} \lor \mathsf{Q}) \land \mathsf{R})$'.

But this does seem excessively fastidious, given that (3) is already entirely clear as it stands and there is no possibility of confusion.

Some introductory logic texts take a very stern line about quotation marks, and insist on surrounding all mentioned expressions with inverted commas (unless displayed on a separate line), even when omitting the quotation marks would cause no confusion. But this is surely unnecessary. From now on, then, we are going to incline mostly to the more general mathematical practice of minimizing the use of quotation marks. In particular, with the obvious exception of names for types of language like 'PL' and 'QL',

> When sans serif expressions appear in an English sentence, we will take it as understood that they are formal expressions which are being mentioned (and so can be thought of as accompanied by invisible quotation marks).

However, if in doubt, we will err on the side of notational caution. We will continue to use explicit quotation marks rather more than mathematicians do – particularly around single symbols, or when we want to highlight that we are mentioning an expression. Better to be locally clear than to stick religiously either to a global maximalist or global minimalist policy about quotes!

11.5 Why Greek-letter variables?

Let's now return to comment on the use of Greek-letter variables when talking about PL wffs. Why are we using these, instead of continuing with the familiar 'A', 'B', etc., which we had used earlier when talking about informal propositions? The danger is that sprinklings of Greek letters can begin to make a logic text look dauntingly mathematical (even when it isn't). But here are three reasons that, on balance, tip the scales in favour of their use – though this is another judgement call.

(1) It is a *widely adopted* convention to use Greek letters as metalinguistic variables like this. So, like it or not, you might as well get accustomed to it!

(2) It is a *helpful* convention. Once formalized languages are in play, we need to be able to keep track of the distinction between symbols from these languages and any metalanguage symbols we might use. And yes, in print, we could easily enough distinguish e.g. italic letters (in our metalanguage) from upright sans serif letters (in formal object languages). But in handwriting on the blackboard or in notebooks, that distinction can get lost, and it is much easier to distinguish metalinguistic Greek letters from object-language Roman letters.

(3) But most importantly, our convention *marks a difference*. Our earlier use of 'A', 'B', etc. was indeed rough and ready, and the rules for the use of such metalinguistic variables were intentionally left unclear. By contrast, there is a very clean and crisp story to be told about our use of Greek-letter metalinguistic variables.

To expand the third point, our informal practice of using schemas and schematic variables like 'n', 'F', 'A' etc. can be pretty casual. Look again how free and easy we were in allowing different sorts of English expression as substitutions for the likes of '(is) F' and then grammatically tidying up the results – we cheerfully allowed 'is wise' but also 'is a philosopher' or even (without an 'is') 'understands quantum mechanics' to count as instances. Or consider our relaxed use of '*not-A*' in §2.1(d), §4.5(c), etc., where we really didn't pin down what would count as an allowable instance. Informal schemas are often treated like this, as sitting *very* loosely to the details of surface form. Now, this lack of precision, the need for more than a bit of charity in applying the symbolism, was no real problem given our earlier introductory purposes: as they say, 'Sufficient unto the day is the rigour thereof.' But it does leave us with some messy questions about just how relaxed and flexible we want to be in using schemas with italic variables.

By contrast, when it comes to the use of Greek-letter variables in talking about formal wffs and formal arguments, everything will be crystal clear and rigorous (as we announced in §7.6). For example, when we say that a wff has the form $(\alpha \wedge \beta)$, we are talking about nothing other than its surface shape. We of course just mean that, for some wffs α, β of the relevant PL-language, the wff in question has the form of a left-hand bracket followed by α followed by the conjunction sign followed by β followed by a right-hand bracket. No charitable tidying-up is required!

11.6 The idea of form, again

That last point is simple but important. The idea is that a formal Greek-letter schema gives a kind of template or pattern for constructing PL expressions; and a wff counts as having the form displayed by some schema when the wff is built exactly to the required pattern. Let's spell this out, in two steps.

(a) Start, then, with the following definition:

A *substitution instance* of a Greek-letter schema is the result of systematically replacing the metalinguistic variables in the schema with wffs.

Here, 'systematic' replacement means replacing the same Greek-letter variable with the same wff throughout; remember, the whole point of using recurring metalinguistic symbols is to indicate patterns of recurrence! And note, the replacement wff may be atomic or molecular.

For example, then, the wff $((\mathsf{P} \land \mathsf{Q}) \lor \neg(\mathsf{P} \land \mathsf{Q}))$ is a substitution instance of the schema $((\alpha \land \beta) \lor \neg(\alpha \land \beta))$. But the same wff is *also* a substitution instance of the different schema $(\alpha \lor \neg\alpha)$. It is also, of course, a substitution instance of the minimally informative schema α!

(b) We can then say:

A PL wff has the form displayed by a schema using Greek-letter variables if and only if it is a substitution instance of that schema.

The wff $((\mathsf{P} \land \mathsf{Q}) \lor \neg(\mathsf{P} \land \mathsf{Q}))$ has the form displayed by the schema $((\alpha \land \beta) \lor \neg(\alpha \land \beta))$ – or for brevity, we will more simply say: it has the form $((\alpha \land \beta) \lor \neg(\alpha \land \beta))$. But the wff *also* has the form $(\alpha \lor \neg\alpha)$, and has the trivial form α.

So when we speak of a wff as having a certain form, we are making use of a notion that can be characterized purely syntactically. It is a notion having to do with the straightforward surface shape of a wff. Of course, this notion of form is of interest principally because, in PL and other formalized languages, syntactic form reflects relevant semantic structure. However, it remains the case that we can define form quite unproblematically in terms of surface syntax.

(c) We have just seen that we can no more talk about *the* form of a wff than we can talk about *the* form of an argument – see §3.3.

However, continuing with our last example, there is a sense in which the schema $((\alpha \land \beta) \lor \neg(\alpha \land \beta))$ is *basic*: it captures all the structure of the wff $((\mathsf{P} \land \mathsf{Q}) \lor \neg(\mathsf{P} \land \mathsf{Q}))$, right the way down to its atomic level. So:

Suppose we systematically replace the atoms in a wff with schematic variables – atoms of the same type to be replaced by the same schematic variable, atoms of different types to be replaced by different schematic variables. Then we can call the resulting schema a *basic* schema for the wff.

Basic schemas aren't unique, because we will have a choice of which schematic variables to use to indicate places in the pattern. Both $(\alpha \lor \beta)$ and $(\gamma \lor \delta)$ will do as basic schemas for the wff $(\mathsf{P} \lor \mathsf{Q})$. But basic schemas for a given wff will just be trivial alphabetical variants.

In due course, we will occasionally have cause to put the notion of a basic schema to work.

11.7 Summary

We need in general to distinguish the *object language* under discussion (whether it is a PL language, some later formal language, or some natural language) from the *metalanguage* that it is being discussed in (in this book, our metalanguage is slightly augmented English).

A key convention: we use sans-serif type as in 'P', 'Q', etc., for expressions belonging to some artificial formalized object language like a PL language.

Italicized letters like 'A', 'n', 'F' belong to our metalanguage of augmented English, as do Greek-letter variables like 'α' and 'β'. They are in principle dispensable devices; but they help us to speak briskly about e.g. general patterns of argument, or formation rules for wffs, etc.

So there is a key distinction between the sans-serif symbols of PL languages and the Greek-letter variables which we add to English specifically to help us talk about and generalize over formal expressions of PL languages (and of other formalized languages).

When we want to *mention* a particular expression, rather than *use* it in the ordinary way, quotation marks give us a standard way of doing this. But, from now on, we will be fairly relaxed about the use of quotes in contexts where it is transparently clear that we are indeed mentioning an expression rather than using it.

Also see the Appendix, 'More on quotation'.

Exercises 11

Where necessary, insert quotation marks into the following to make them accord to the strict convention for quotation marks and to come out true.

(1) The first word in this sentence is the.
(2) This is not a verb, but is is.
(3) George Orwell is the same person as Eric Blair.
(4) George Orwell was Eric Blair's pen-name.
(5) The Evening Star and The Morning Star denote the same planet.
(6) If we want to refer not to Sappho but her name, we need to use the expression Sappho.
(7) $\land$ means much the same as and.
(8) P can be interpreted as meaning that grass is green.
(9) P is a subformula of (Q $\land$ ¬P).
(10) If (Q $\land$ ¬P) is a subformula of α so is P.
(11) If α and β are PL wffs, so is their conjunction.
(12) The schema $(\alpha \land \beta)$ is formed from Greek letters, the connective $\land$, and brackets (and).