Peter Smith, *Introduction to Formal Logic* (CUP, 2nd edition)

Exercises 4

Which of the following arguments are valid? Where an argument is valid, sketch an informal proof. Some of the examples are enthymemes that need repair.

(1) *Only logicians are good philosophers. No existentialists are logicians. Some existentialists are French philosophers. So, some French philosophers are not good philosophers.*

Valid, as is shown by the following simple proof:

(1)	Only logicians are good philosophers.	premiss
(2)	No existentialists are logicians.	premiss
(3)	Some existentialists are French philosophers.	premiss
(4)	Some French philosophers are existentialists.	(from 3)
(5)	Some French philosophers are not logicians.	(from 2, 4)
(6)	Some French philosophers are not good philosophers.	(from 1, 5)

(2) *No philosopher is illogical. Jones keeps making argumentative mistakes. No logical person keeps making argumentative mistakes. All existentialists are philosophers. So, Jones is not an existentialist.*

This is valid. We can laboriously argue as follows:

(1)	No philosopher is illogical.	(premiss)
(2)	Jones keeps making argumentative mistakes.	(premiss)
(3)	No logical person keeps making argumentative mistakes.	(premiss)
(4)	All existentialists are philosophers.	(premiss)
(5)	Jones is illogical	(from 2, 3)
(6)	Jones is not a philosopher	(from 1, 5)
(7)	Jones is not an existentialist	(from 4, 6)

Or alternatively, from (4) onwards, we could reason starting with a new temporary supposition for the sake of argument:

(5)	Jones is an existentialist.	(supposition)
(6)	Jones is a philosopher.	(from 4, 5)
(7)	Jones is logical.	(from 1, 6)
(8)	Jones does not keep making argumentative mistakes.	(from 3, 7)
(9)	Contradiction!	(from 2, 8)
(10)	Jones is not an existentialist	(by RAA from 5–9)

Here, the temporary assumption at (6) leads quickly to something contradicting a premiss, so it has to be false, by a reductio ad absurdum argument.

A quick moral: we can validly reason from given premisses to a given conclusion in more than one way!

(3) *No experienced person is incompetent. Jenkins is always blundering. No competent person is always blundering. So, Jenkins is inexperienced.*

Valid.

(1)	No experienced person is incompetent.	(premiss)
(2)	Jenkins is always blundering.	(premiss)
(3)	No competent person is always blundering.	(premiss)
(4)	Jenkins is incompetent.	(from 2, 3)
(5)	Jenkins is inexperienced.	(from 1, 4)

Alternatively, after the initial premisses we could make a further temporary assumption for the sake of argument, and then show it is false by a reductio ad absurdum inference.

(4)	Jenkins is experienced.	(supposition)
(5)	Jenkins is competent.	(from 1, 4)
(6)	Jenkins is not always blundering.	(from 3, 5)
(7)	Contradiction!	(from 2, 6)
(8)	Jenkins is inexperienced.	(from 4 –7, by RAA)

(4) *Jane has a first cousin. Jane's father is an only child. So, if Jane's mother hasn't a sister, she has a brother.*

This at first sight looks valid. For consider the reasoning:

(1)	Jane has a first cousin.	(premiss)
(2)	Jane's father is an only child.	(premiss)
(3)	Someone is the child of a sibling of one of Jane's parents.	(from 1)
(4)	One of Jane's parents has a sibling.	(from 3)
(5)	Jane's mother has a sibling	(from 2 and 4)
(6)	if Jane's mother hasn't a sister, she has a brother.	(from 5)

However, two points about this. Jane can have a first cousin without her mother or father being alive. So we'll have to take 'is an only child' and 'has a sibling' as belonging to the so-called historical or narrative present, used when talking about a states of affairs which may not be current.

And is it right to assume that siblings are always either brothers or sisters? (It might be suggested, perhaps, that the notion of a sibling is a biological one, while the notion of a brother or sister belongs to a more contended language of gender.) But we don't want to tangle with such troublesome issues here!

(5) *Every event is causally determined. No action should be punished if the agent isn't responsible for it. Agents are only responsible for actions they can avoid doing. Hence no action should be punished.*

This isn't deductively valid as it stands. It requires the additional premiss 'If every event is causally determined, then no agent can avoid acting as they do'. Adding this, we would then have the following evidently valid argument:

(1)	Every event is causally determined.	(premiss)
(2)	No action should be punished if the agent isn't responsible for it.	(premiss)
(3)	Agents are only responsible for actions they can avoid doing.	(premiss)
(4)	If every is causally determined, then no agent can avoid acting as they do.	(added premiss)
(5)	No agent can avoid acting as they do.	(from 1, 4)
(6)	Agents are never responsible for what they do.	(from 3, 5)
(7)	No action should be punished.	(from 2, 6)

We do need to add the premiss (4)! And some would argue that, in the sense of 'avoid' in which (3) is true, (4) is false.

(6) *Some chaotic attractors are not fractals. All Cantor sets are fractals. Hence some chaotic attractors are not Cantor sets.*

Unlike the 'slithy toves' example in Exercises 1, this is contentful; and it is a valid syllogism. How can we show it is valid? In traditional syllogistic logic, we can in effect reason like this, relying on two fundamental principles:

I. Arguments of the form *All A are B; All B are C; so All A are C* are valid.
II. Propositions of the form *All A are B* and *Some A are non-B* contradict each other.

(1)	Some chaotic attractors are not fractals.	(premiss)
(2)	All Cantor sets are a fractal.	(premiss)
(3)	All chaotic attractors are Cantor sets.	(supposition)
(4)	All chaotic attractors are fractals	(from 2, 3 by I)
(5)	Contradiction!	(1, 4 by II)
(6)	It isn't the case that all chaotic attractors are Cantor sets.	(from 3–5 by RAA)
(7)	Some chaotic attractors are not Cantor sets.	(from 6 by I)

(7) *Something is an elementary particle only if it has no parts. Nothing which has no parts can disintegrate. An object that cannot be destroyed must continue to exist. So an elementary particle cannot cease to exist.*

As it stands, this is invalid. To get a valid inference we need to add the premiss 'If an object cannot disintegrate, then it cannot be destroyed' (why should that be true? can't something be destroyed in some other way than disintegration?). With the extra premiss, though, we have the following simple argument.

(1)	Something is an elementary particle only if it has no parts.	(premiss)
(2)	Nothing which has no parts can disintegrate.	(premiss)
(3)	An object that cannot be destroyed must continue to exist.	(premiss)
(4)	If an object cannot disintegrate, then it cannot be destroyed	(added premiss)
(5)	An elementary particle cannot disintegrate.	(from 1, 2)
(6)	An elementary particle cannot be destroyed.	(from 4, 5)
(7)	An elementary particle cannot cease to exist.	(from 6, 3)

Note here that 'Something is an elementary particle ...', and 'An elementary particle ...' make *general* claims about all elementary particles. One of the many vagaries of English expressions of generality, which we learn to negotiate as we acquire the language, is we can make claims about everything of a given kind by using words which in most contexts signal more particular, non-general, claims.

(8) *Either the butler or the cook committed the murder. The victim died from poison if the cook was the murderer. The butler carried out the murder only if the victim was stabbed. The victim didn't die from poison. So, the victim was stabbed.*

Another valid argument. We could argue like this (careful with the 'if's and 'only if's):

(1)	Either the butler or the cook committed the murder.	premiss
(2)	The victim died from poison if the cook did the murder.	premiss
(3)	The butler did the murder only if the victim was stabbed.	premiss
(4)	The victim didn't die from poison.	premiss
(5)	The cook committed the murder.	supposition
(6)	The victim died from poison.	(from 2, 5)
(7)	Contradiction	(from 4, 6)
(8)	The cook didn't commit the murder.	(from 5–7, by RAA)
(9)	The butler committed the murder.	(from 1, 8)
(10)	The victim was stabbed.	(from 3, 9)

(9) *Superman is none other than Clark Kent. The Superhero from Krypton is Superman. The Superhero from Krypton can fly. Hence Clark Kent can fly.*

Valid. Two key logical principles are involved here:

I. If a is none other than b, and b is none other than c, then a is none other than c.
II. If a is none other than c, then whatever properties are had by a are had by c.

So, using (I) we can infer from the first two premisses that the Superhero from Krypton is none other than Clark Kent. Then from that together with the third premiss we can infer our conclusion using (II).

(10) *Jack is useless at logic or he simply isn't ready for the exam. Either Jack will fail the exam or he is not useless at logic. Either it's wrong that he won't fail the exam or he is ready for it. So Jack will fail.*

This is valid. We need *an argument by cases* – a structure of argument we haven't highlighted before.

Suppose we are given A or B. Suppose we can then show in the first case, where A holds, that C follows. And suppose we can also show in the second case, where B holds, that C again follows. Then we can conclude that C holds either way.

So we are given Jack is useless at logic or he simply isn't ready for the exam. Take the first case, that Jack is useless. Then, given the second premiss, Jack will fail. Take the second case, Jack isn't ready. Then given the third premiss, it's wrong that he won't fail the exam, i.e. Jack again will fail. So indeed, either way, whether useless or not ready, he fails.

(11) *Any elephant weighs more than any horse. Some horses weigh more than any donkey. Hence any elephant weighs more than any donkey.*

Valid. Think of it like this. Pick some horse that weighs more than any donkey – we know there is at least one, dub it 'Eclipse'. Then any elephant weighs more than Eclipse (by the first premiss). But Eclipse weighs more than any arbitrarily selected donkey (by the choice of Eclipse). So any any elephant weighs more than any arbitrarily selected donkey.

Later, when discussing the logic of 'quantified' arguments like this, we will see how to give a tidier version of this sketched proof.

(12) *When I do an example without grumbling, it is one that I can understand. No easy logic example ever makes my head ache. This logic example is not arranged in regular order, like the examples I am used to. I can't understand these examples that are not arranged in regular order, like the examples I am used to. I never grumble at an example, unless it gives me a headache. So, this logic example is difficult.*

Blame Lewis Carroll again! Let's see how we can proceed (setting things out carefully will show the problem points):

(1)	When I do an example without grumbling, it is one that I can understand.	(premiss)
(2)	No easy logic example ever makes my head ache.	(premiss)
(3)	This logic example is not arranged in regular order, like the examples I am used to.	(premiss)
(4)	I can't understand these examples that are not arranged in regular order, like the examples I am used to.	(premiss)
(5)	I never grumble at an example, unless it gives me a headache.	(premiss)
(6)	I can't understand this logic example	(from 3, 4)
(7)	I'm not doing this example without grumbling	(from 1, 6)

But now to move on we need to assume the grumbling is at the example, but also that

(8)	I am doing this logic example	(extra premiss!)
(9)	I'm grumbling at this logic example.	(from 7, 8)
(10)	This logic example is giving me a headache.	(from 5, 9)
(11)	This is not an easy logic example.	(from 2, 10)

Even adding our extra premiss, however, we haven't quite got where Carroll wants us to get. For 'X is not easy' doesn't entail 'X is difficult': something could be in-between, neither easy nor difficult but so-so. So his conclusion doesn't actually follow.

Finally, an example that requires a minimal amount of arithmetical knowledge:

(13) *$\sqrt{2}$ cannot be a rational number, i.e. a fraction. We can show this as follows. Suppose $\sqrt{2} = m/n$, where this fraction is in lowest terms. Then (i) $m^2 = 2n^2$, so m is even, and hence $m = 2k$. (ii) Then $n^2 = 2k^2$, so n is even, and hence m isn't (or else m/n wouldn't be in lowest term). Hence (iii) our supposition leads to contradiction.*

Set this out as a line-by-line reductio proof, annotating the justification of each line.

If $\sqrt{2}$ is a fraction at all, then we can express it as a fraction 'in lowest terms', i.e. as a fraction m/n, where m and n are integers having no common factors. So we'll start our derivation making the following assumption for the sake of argument:

(1)	$\sqrt{2} = m/n$, where integers m and n have no common factors.	(supposition)
(2)	$2 = m^2/n^2$	(squaring 1)
(3)	$m^2 = 2n^2$	(rearranging 2)
(4)	m^2 is even	(from 3)
(5)	m is even	(from 4)

That last move depends on the easily checked arithmetical fact that odd integers always have odd squares: so if m^2 is even, m cannot be odd. Continuing:

(6)	$m = 2k$ for some integer k	(from 5)
(7)	$(2k)^2 = 2n^2$	(from 3, 6)
(8)	$n^2 = 2k^2$	(rearranging 7)
(9)	n^2 is even.	(from 8)
(10)	n is even.	(from 9)
(11)	m and n are both even.	(from 5, 10)
(12)	m and n have a common factor (i.e. 2).	(from 11)
(13)	Contradiction!	(from 1, 12)
(14)	$\sqrt{2} \neq m/n$ for integers m and n with no common factors.	(RAA 1–13)

Despite the relative simplicity of the argument, the result is deep. It shows that there can be incommensurable quantities, i.e. quantities which lack a common measure in the following sense. Take a right-angled isosceles triangle, for example. Then there can't be a unit length that goes into one of the equal sides exactly n times and into the hypotenuse exactly m times.