Variables and parameters

In the syntax adopted in IFL2, the variables that can occur quantified, like the ‘x’ in ‘\(\forall x (Fx \rightarrow Gx)\)’, cannot also appear free, not bound by a quantifier. So when we want to instantiate the universal quantification, we must either use a proper name, as in ‘\((Fm \rightarrow Gm)\)’, or use a parameter or dummy name, as in ‘\((Fa \rightarrow Ga)\)’. Dummy names and proper names behave in just the same way syntactically: the difference is in their semantic role. So for us, terms include proper names and dummy names, but not bound variables.

Rules for quantifiers (to be added to the rules for propositional logic)

\[
\begin{align*}
\forall \xi \alpha(\xi) & \quad \vdash \alpha(\tau) \quad \forall E \quad \forall I \quad \exists I \quad \exists E \\
\exists \xi \alpha(\xi) & \quad \vdash \alpha(\tau) \quad \exists E \quad \exists I \quad \forall I \quad \forall E \\
\forall \xi \alpha(\xi) & \quad \vdash \alpha(\delta) \quad \forall \exists E \quad \exists \forall I \\
\exists \xi \alpha(\xi) & \quad \vdash \gamma \quad \exists \forall E \quad \forall \exists I
\end{align*}
\]

In all these rules, \(\alpha(\tau)\) or \(\alpha(\delta)\) is an instance of the corresponding quantified wff; \(\tau\) can be any kind of term, but \(\delta\) must be a dummy name.

The following restrictions must be observed on the dummy names \(\delta\):

For \((\forall I)\), \(\delta\) must not appear in any live assumption for \(\alpha(\delta)\) or in the conclusion \(\forall \xi \alpha(\xi)\).

For \((\exists E)\), \(\delta\) must be new to the proof and must not appear in the conclusion \(\gamma\).

Rules for identity

\[
\begin{align*}
\tau_1 = \tau_2 & \quad or \quad \tau_2 = \tau_1 \\
(=I) & \quad \vdash \tau = \tau \\
(=E) & \quad \vdash \alpha(\tau_1) \quad \vdash \alpha(\tau_2)
\end{align*}
\]

The \(\tau\)s can be any terms. \(\alpha(\tau_2)\) is the result of replacing some or all occurrences of \(\tau_1\) in \(\alpha(\tau_1)\) by \(\tau_2\).