NATURAL DEDUCTION RULES FOR QUANTFICATIONAL LOGIC

DIAGRAMMATIC SUMMARY OF RULES USED IN IFL2

Variables and parameters

In the syntax adopted in *IFL2*, the variables that can occur quantified, like the 'x' in '$\forall\mathsf{x}(\mathsf{Fx} \to \mathsf{Gx})$', cannot also appear free, not bound by a quantifier. So when we want to instantiate the universal quantification, we must either use a proper name, as in '$(\mathsf{Fm} \to \mathsf{Gm})$', or use a parameter or dummy name, as in '$(\mathsf{Fa} \to \mathsf{Ga})$'. Dummy names and proper names behave in just the the same way syntactically: the difference is in their semantic role. So for us, *terms* include proper names and dummy names, but not bound variables.

Rules for quantifiers (to be added to the rules for propositional logic)

(∀E)
$$\begin{array}{c} \forall\xi\alpha(\xi) \\ \vdots \\ \alpha(\tau) \end{array}$$

(∃I)
$$\begin{array}{c} \alpha(\tau) \\ \vdots \\ \exists\xi\alpha(\xi) \end{array}$$

(∀I)
$$\begin{array}{c} \alpha(\delta) \\ \vdots \\ \forall\xi\alpha(\xi) \end{array}$$

(∃E)
$$\begin{array}{l} \exists\xi\alpha(\xi) \\ \vdots \\ \quad\left|\begin{array}{l} \alpha(\delta) \\ \hline \vdots \\ \gamma \end{array}\right. \\ \gamma \end{array}$$

In all these rules, $\alpha(\tau)$ or $\alpha(\delta)$ is an instance of the corresponding quantified wff; τ can be any kind of term, but δ must be a dummy name.

The following restrictions must be observed on the dummy names δ:

For (∀I), δ must not appear in any live assumption for $\alpha(\delta)$ or in the conclusion $\forall\xi\alpha(\xi)$.

For (∃E), δ must be new to the proof and must not appear in the conclusion γ.

Rules for identity

(=I)
$$\begin{array}{c} \vdots \\ \tau = \tau \end{array}$$

(=E)
$$\begin{array}{c} \tau_1 = \tau_2 \;\; or \;\; \tau_2 = \tau_1 \\ \vdots \\ \alpha(\tau_1) \\ \vdots \\ \alpha(\tau_2) \end{array}$$

The τs can be any terms. $\alpha(\tau_2)$ is the result of replacing some or all occurrences of τ_1 in $\alpha(\tau_1)$ by τ_2.