

Exercises 31: Informal quantifier arguments

In Chapter 31 we introduced four principles for arguing in a disciplined but still informal way with quantified statements regimented with prefixed ‘every’ and ‘some’ quantifiers. These were versions of universal instantiation and universal generalization, existential generalization and the sort of generalized ‘arguing by cases’ which takes us from (i) (Something x is that that) Fx , and (ii) a proof from a representative case Fa to C , to (iii) the outright conclusion C (so long as a is suitably arbitrary and C doesn’t mention it).

Regiment the premisses and conclusions of the following arguments using informal prefixed quantifiers and variables. Using the four quantifier principles we have met plus propositional reasoning, give informal derivations in the style of this chapter to show that the arguments are valid:

- (1) If Jo can do the exercises, then everyone in the class can do the exercises. Mo is in the class, and can’t do the exercises. So Jo can’t do the exercises.
- (2) No whales are fish. So no fish are whales.
- (3) All leptons have half-integer spin. All electrons are leptons. So all electrons have half-integer spin.
- (4) Some chaotic attractors are not fractals. Every Cantor set is a fractal. Hence some chaotic attractors are not Cantor sets.
- (5) Some philosophers are logicians. All logicians are rational people. No rational person is a flat-earthier. Therefore some philosophers are not flat-earthiers.
- (6) All lions and tigers are dangerous animals. Dangerous animals should be avoided. Leo is a lion. So Leo should be avoided.

Give informal derivations warranting these arguments too (a little more difficult!):

- (7) There is someone who loves everyone. Hence everyone is loved by someone or other. [NB the difference between $(\exists x)(\forall y)$ and $(\forall y)(\exists x)$!]
- (8) Everyone loves logic. Hence it isn’t the case that someone doesn’t love logic.
- (9) Any philosopher who is not a fool likes logic. There is a philosopher who isn’t a fool. Therefore not every philosopher fails to like logic.