Peter Smith, *Introduction to Formal Logic* (CUP, 2nd edition)

Exercises 31: Informal quantifier arguments

Regiment the premisses and conclusions of the following arguments using informal prefixed quantifiers and variables. Using the four quantifier principles we have met plus propositional reasoning, give informal derivations in the style of this chapter to show that the arguments are valid:

Our informal apparatus in this chapter is *not* carefully defined (that's the business for Chapter 32) The intention for now is just to highlight some key ideas about arguing with quantifiers when regimented as quantifier prefixes. These exercises and their answers should help further elucidate these ideas in an informal way before getting down to formal work.

(1) *If Jo can do the exercises, then everyone in the class can do the exercises. Mo is in the class, and can't do the exercises. So Jo can't do the exercises.*

An easy warm-up exercise. We evidently need a simple reductio argument:

(1) If Jo can do the exercises, then (everyone x is such that)(if x is in the class, x can do the exercises). (premiss)
(2) Mo is in the class, and Mo can't do the exercises. (premiss)
(3) Jo can do the exercises. (supposition)
(4) (everyone x is such that)(if x is in the class, x can do the exercises) (MP from 3 and 1)
What applies to everyone in the relevant domain applies to Mo in particular, so
(5) If Mo is in the class, Mo can do the exercises (from 4)
(6) Contradiction! (from 2, 4)
(7) Jo can't do the exercises (RAA from 3–6)

(2) *No whales are fish. So no fish are whales.*

Regiment the 'no' propositions as we did in Chapter 31, using prefixed universal quantifiers and negation, we can argue as follows:

(1) (Everything x is such that) if x is a whale, x is not a fish. (premiss)
Now pick any thing in the domain as an arbitrary representative, temporarily dub it 'Arb'. Then:
(2) If Arb is a whale, Arb is not a fish. (from 1)
(3) If Arb is a fish, Arb is not a whale. (from 2)
But Arb was arbitrarily chosen, and we have appealed to no special facts about it; so what we can deduce about it applies equally to anything:
(4) (Everything x is such that) if x is a fish, x is not a whale. (from 3)

The inference from (2) to (3) is of course just a propositional inference: we contrapose (2) and then drop a double negation!

(3) *All leptons have half-integer spin. All electrons are leptons. So all electrons have half-integer spin.*

(1) (Everything x is such that) if x is a lepton, x has half-integer spin. (premiss)
(2) (Everything x is such that) if x is an electron, x is a lepton. (premiss)
Now pick any thing in the domain as an arbitrary representative, temporarily dub it 'Arb':
(3) If Arb is a lepton, Arb has half-integer spin. (from 1)
(4) If Arb is an electron, Arb is a lepton (from 2)
(5) If Arb is an electron, Arb has half-integer spin (from 1, 2)
But Arb was arbitrarily chosen, and we have appealed to no special facts about it; so what we can deduce about it applies equally to anything:
(6) (Everything x is such that) if x is an electron, x has half-integer spin. (from 3)

(4) *Some chaotic attractors are not fractals. Every Cantor set is a fractal. Hence some chaotic attractors are not Cantor sets.*

(1) (Something x is such that) x is a chaotic attractor and x is not fractal. (premiss)
(2) (Everything x is such that) if x is a Cantor set, then x is fractal (premiss)
Now pick something in the domain as an arbitrary representative, and temporarily dub it 'Arb'. Let's suppose
(3) Arb is a chaotic attractor and Arb is not fractal. (supposition)
(4) if Arb is a Cantor set, then Arb is fractal. (from 2)
(5) Arb is a chaotic attractor and Arb is not a Cantor set. (from 3, 4)
(6) (Something x is such that) x is a chaotic attractor and x is not a Cantor set. (from 5)
But our premiss (1) tells us that there is something *who is like Arb in being, as supposed, a chaotic attractor and not fractal. Our interim conclusion (6) doesn't depend on who Alex is. So we can argue using (1) and the inference from (3) to (6) to infer outright:*
(7) (Something x is such that) x is a chaotic attractor and x is not a Cantor set. (from 1, 3–6)

A general comment, by the way, on the last two examples. You may well not have the foggiest idea what leptons are, what it is to have half-integer spin, what it is to be a Cantor set, and so on (though these are in fact all perfectly good concepts from physics and maths). Still, you were able to do the examples – I hope! – because you understood the general structure or form of the arguments. And the arguments are, in a good sense, valid in virtue of that form, something you can appreciate without understanding all the predicates they involve. But note: it *doesn't* follow from the fact that you needn't understand the content of the predicates to appreciate that these arguments are valid, that the predicates needn't *have* any content for the arguments to be valid! – No content for the predicates would imply we aren't dealing with contentful propositions; and if there aren't contentful propositions we don't have a real argument, and so (a fortiori) don't have a *valid* argument.

(5) *Some philosophers are logicians. All logicians are rational people. No rational person is a flat-earther. Therefore some philosophers are not flat-earthers.*

(1) (Someone x is such that) x is a philosopher and x is a logician. (premiss)
(2) (Everyone x is such that) if x is a logician, then x is rational. (premiss)
(3) (Everyone x is such that) if x is rational, then x is not a flat-earther. (premiss)
Now pick someone in the domain as an arbitrary representative, and temporarily dub them 'Alex'. Let's suppose
(4) Alex is a philosopher and Alex is a logician. (supposition)
(5) If Alex is a logician, then Alex is rational. (from 2)
(6) If Alex is rational, then Alex is not a flat-earther. (from 3)
(7) If Alex is a philosopher then Alex is not a flat-earther. (from 4, 5, 6)
(8) (Someone x is such that) x is a philosopher and x is not a flat-earther. (from 7)
But our premiss (1) tells us that there is someone *who is like Alex in being, as supposed, a philosopher and logician. Our interim conclusion (8) doesn't depend on who Alex is. So we can infer outright:*
(9) (Someone x is such that) x is a philosopher and x is not a flat-earther. (from 1, 4–8)

(6) *All lions and tigers are dangerous animals. Dangerous animals should be avoided. Leo is a lion. So Leo should be avoided.*

'All lions are dangerous' gets regimented as '(Everything x is such that)(if x is a lion, x is dangerous)'. Now, on that model, you might be momentarily tempted to regiment 'All lions and tigers are dangerous' as '(Everything x is such that)(if x is a lion and x is a tiger, x is dangerous)'. But you've only got to write that down to see that it won't do – after all, nothing is a lion *and* a tiger!

In fact, we have three equivalent options for regimenting the first premiss here, along the lines of either '(Everything x is such that)(if x is a lion, x is dangerous) and (everything x is such that)(if x is a tiger, x is dangerous) '; or '(Everything x is such that)((if x is a lion, x is dangerous) and (if x is a tiger, x is dangerous))'; or '(Everything x is such that)(if x is a lion or x is a tiger, then x is dangerous)'. Let's choose the third of these.

(1) (Everything x is such that)(if x is a lion or x is a tiger, then x is a dangerous animal) (premiss)
(2) (Everything x is such that)(if x is a dangerous animal, then x should be avoided) (premiss)
(3) Leo is a lion. (premiss)
What is true of everything is true of Leo, so
(4) If Leo is a lion or Leo is a tiger, then Leo is a dangerous animal. (from 1)
(5) If Leo is a dangerous animal, then Leo should be avoided. (from 2)
(6) Leo is a lion or Leo is a tiger. (from 3)
(7) Leo is a dangerous animal. (MP 6, 4)
(8) Leo should be avoided. (MP 7, 5)

Give informal derivations warranting these arguments too (a little more difficult!):

(7) *There is someone who loves everyone. Hence everyone is loved by someone or other. [NB the difference between* $(\exists x)(\forall y)$ *and* $(\forall y)(\exists x)$*!]*

An example that involves one quantifier nested inside the scope of another. But not too difficult if you keep your wits about you!

The natural reading of the premiss gives it the Loglish form $(\exists x)(\forall y)$ x loves y; while the natural reading of the conclusion gives it the form $(\forall y)(\exists x)$ x loves y. We've in effect swapped the order of the quantifiers, then, something we can't always do. Why is it OK in this case?

Well, the first premiss tells us that there is someone (Alex, let's pretend) who loves everyone. But then, take anyone you like, there is indeed at least one person who loves them (namely Alex!).

Regimenting that line of thought, we have

(1) (There is someone x such that)(everyone y is such that) x loves y. (premiss)
Now pick someone in the domain as an arbitrary representative, and temporarily dub them 'Alex'. And let's make the supposition
(2) (Everyone y is such that) Alex loves y. (supposition)
And now again pick someone from the domain as an arbitrary representative, and temporarily dub them 'Bobbie'. Then what is true of everyone is true of Bobbie. So
(3) Alex loves Bobbie (from 2)
(4) (Someone x is such that) x loves Bobbie (from 3)
But Bobbie was arbitrarily chosen, and we have appealed to no special facts about them; so what we can deduce about them applies equally to everyone:
(5) (Everyone y is such that)(someone x is such that) x loves y (from 4)
Now, our premiss (1) tells us that there is someone *who is like Alex who, as supposed, loves everyone. Our interim conclusion (5) doesn't depend on who Alex is. So we can (1) and our inference from (2) to (5) to conclude outright that*
(6) (Everyone y is such that)(someone x is such that) x loves y. (from 1 and 2–5)

(8) *Everyone loves logic. Hence it isn't the case that someone doesn't love logic.*

We want to prove the negation of something. So (as usual!) the obvious idea is to try a reductio argument. Then we can argue

(1) (Everyone x is such that) x loves logic (premiss)
(2) (Someone x is such that) x doesn't like logic. (supposition)
Now pick someone in the domain as an arbitrary representative, and temporarily dub them 'Alex'. And let's make the additional assumption
(3) Alex doesn't like logic (supposition)
But what is true of everyone is true of Alex in particular, so
(4) Alex likes logic. (from 1)
(5) Contradiction!! (from 3 and 4)
Now, our supposition (2) tells us that there is someone *who is like Alex in, as supposed, not loving logic. Our interim conclusion (5) doesn't depend on who Alex is. So we can use (2) and our subproof from the supposition (3) to conclude*
(6) Contradiction!!. (from 2, 3–5)
(7) It isn't the case that (Someone x is such that) x doesn't like logic. (RAA 2–6)

(9) *Any philosopher who is not a fool likes logic. There is a philosopher who isn't a fool. Therefore not every philosopher fails to like logic.*

Again we set off on a reductio argument by supposing that every philosopher fails to like logic!

(1) (Everyone x is such that) if x is a philosopher and x is not a fool, then x likes logic (premiss)
(2) (Someone x is such that) x is a philosopher and x is not a fool. (premiss)
(3) (Everyone x is such that) if x is a philosopher, x does not like logic. (supp)
Now pick someone in the domain as an arbitrary representative, and temporarily dub them 'Alex'. And let's make the additional supposition
(4) Alex is a philosopher and Alex is not a fool (supp)
But what is true of everyone is true of Alex in particular, so
(5) If Alex is a philosopher and Alex is not a fool, Alex likes logic. (from 1)
(6) If Alex is a philosopher then Alex does not like logic. (from 3)
(7) Contradiction!! (from 4, 5, 6)
Now, our premiss (2) tells us that there is someone *who is like Alex in being a philosopher and not a fool. Our interim conclusion (5) doesn't depend on who Alex is. So we (2) and the subproof starting with (4) and ending in contradiction to conclude outright*
(8) Contradiction!!. (from 2, 4–7)
(9) It isn't the case that (everyone x is such that)(if x is a philosopher, x does not like logic). (RAA 3–8)

Comment on these last two examples: if it still isn't clear what is going on in these cases, don't panic! We'll be looking at formal versions of the informal proofs in the Exercises for the next chapter: and the crisp clarity of the formal proofs should make everything more obvious!