Peter Smith, *Introduction to Formal Logic* (CUP, 2nd edition)

Exercises 33: More QL proofs

Do the unstarred examples in both (a) and (b) and check your answers before returning to the starred ones.

(a) As a warm-up exercise, consider which of the following QL arguments ought to be valid (assume the wffs are interpreted). Give proofs warranting the valid inferences.

(1) $\exists x\,Sxxx \;\therefore\; \exists x\exists y\exists z\,Sxyz$
(2) $\exists x\forall y\forall y\,Sxyy \;\therefore\; \exists x\,Sxxx$
(3) $\forall x\exists y\forall z\,Sxyz \;\therefore\; \exists x\forall y\exists z\,Sxzy$
(4*) $\neg\exists x\forall y\forall z\,Sxyz \;\therefore\; \forall x\exists z\exists y\neg Sxyz$
(5*) $\neg\exists x(Fx \land \exists y(Gy \land Lxy)) \;\therefore\; \forall x\forall y(Fx \to (Gy \to \neg Lxy))$

(b) Render the following inferences into suitable QL languages and provide derivations of the conclusions from the premisses in each case:

(1) Some people are boastful. No one likes anyone boastful. Therefore some people aren't liked by anyone.

(2) There's someone such that if *they* admire some philosopher, then I'm a Dutchman. So if *everyone* admires some philosopher, then I'm a Dutchman.

(3) Some good philosophers admire Frank; all wise people admire any good philosopher; Frank is wise; hence there is someone who both admires and is admired by Frank.

(4) Everyone loves themself if there's someone who loves them or whom they love. There's someone who is loved. Therefore someone loves themself.

(5*) Only rational people with good judgement are logicians. Those who take some creationist myth literally lack good judgement. So logicians do not take any creationist myth literally.

(6*) Given any two people, if the first admires Gödel and Gödel admires the second, then the first admires the second. Gödel admires anyone who has understood *Principia*. There's someone who has understood *Principia* who admires Gödel. Therefore there's someone who has understood *Principia* who admires everyone who has understood *Principia*!

(7*) Any adult elephant weighs more than any horse. Some horse weighs more than any donkey. If a first thing weighs more than a second thing, and the second thing weighs more than a third, then the first weighs more than the third. Hence any adult elephant weighs more than any donkey.

(c) Why should the following QL wffs be logically true (assuming the wffs are interpreted)?

(1) $\exists x(Fx \to \forall yFy)$
(2) $\forall x\exists y(\exists zLxz \to Lxy)$
(3) $\exists x\forall y(\neg Fy \lor Fx)$

Show that those wffs are QL theorems – feel free now to use the derived rules ($\neg\forall$) and ($\neg\exists$) from §32.5 and to skip PL reasoning. Also give derivations to warrant the following inferences:

(4) $(\forall xFx \to \exists yGy) \;\therefore\; \exists x\exists y(Fx \to Gy)$
(5) $(\exists zFz \to \exists zGz) \;\therefore\; \forall x\exists y(Fx \to Gy)$
(6) $\forall x\exists y(Fy \to Gx) \;\therefore\; \exists y\forall x(Fy \to Gx)$

(d*) Although all our examples of QL proofs so far start from zero or more sentences and end with a sentence, we won't build that into our official characterization of QL proofs – they can go from wffs involving dummy names to a conclusion which may involve a dummy name.

Say that the wffs Γ (not necessarily all sentences) are QL-*consistent* if there is no QL proof using wffs Γ as premisses and ending with '$\perp$'; otherwise Γ are QL-*inconsistent* – compare Exercises 22(d*).

Assuming that the terms mentioned belong to the relevant language, show

(1) If the wffs $\Gamma, \alpha(\tau)$ are QL-inconsistent and the wffs Γ include $\forall\xi\alpha(\xi)$ then those wffs Γ are already QL-inconsistent.

and then conclude that

(2) If the wffs Γ are QL-consistent and $\forall\xi\alpha(\xi)$ is one of them, then $\Gamma, \alpha(\tau)$ are also QL-consistent.

Show further that

(2*) If the wffs Γ are QL-consistent and $\forall\xi\alpha(\xi)$ is one of them, then $\Gamma, \alpha(\tau_1), \alpha(\tau_2), \ldots, \alpha(\tau_k)$ all together are also QL-consistent (for any terms $\tau_1, \tau_2, \ldots, \tau_k$ of the relevant language).

Show similarly that

(3) If the wffs Γ are QL-consistent and $\exists\xi\alpha(\xi)$ is one of them, then $\Gamma, \alpha(\delta)$ are also QL-consistent if δ is a dummy name that doesn't appear in Γ.

Also show that:

(4) If the wffs Γ are QL-consistent and $\neg\forall\xi\alpha(\xi)$ is one of them, then $\Gamma, \exists\xi\neg\alpha(\xi)$ are QL-consistent.

(5) If the wffs Γ are QL-consistent and $\neg\exists\xi\alpha(\xi)$ is one of them, then $\Gamma, \forall\xi\neg\alpha(\xi)$ are QL-consistent.