

## Exercises 41: QL<sup>=</sup> proofs

(a) Use QL<sup>=</sup> derivations to show the following inferences are valid:

- (1) Mrs Jones isn't Kate. So Kate isn't Mrs Jones.
- (2) No one who isn't Bryn loves Angharad. At least one person loves Angharad. So Bryn loves Angharad.
- (3) If Clark Kent isn't Superman, then Clark isn't even himself. Superman can fly. So Clark can fly.
- (4) The goods were stolen by someone. Whoever stole the goods knew the safe combination. Only Jack knew the safe combination. Hence Jack stole the goods.
- (5) Take two people (perhaps the same): if the first is taller than the second, the second is not taller than the first. Therefore, if Kurt is taller than Gerhard, they are different people.
- (6) There is a wise philosopher. There is a philosopher who isn't wise. So there are at least two philosophers.
- (7) Anyone who loves Jo is a logician. Why? Because only one person loves Jo. And some logician loves Jo.
- (8) For any number, there's a larger one. There is no number which is larger than itself. So for any number, there's a distinct number which is larger than it.
- (9) Exactly one person admires Frank. All and only those who admire Frank love him. Hence exactly one person loves Frank.
- (10) The present King of France is bald. Bald men are sexy. Hence whoever is a present King of France is sexy.
- (11) Someone is a logician. But no one is the only logician. Therefore there at least two logicians.

(b) The following wffs are alternative renderings of a claim of the form *The F is G*:

$$(R) \quad \exists x((Fx \wedge \forall y(Fy \rightarrow y = x)) \wedge Gx)$$

$$(R') \quad \exists x\forall y((Fy \leftrightarrow y = x) \wedge Gx)$$

$$(R'') \quad (\{\exists xFx \wedge \forall x\forall y((Fx \wedge Fy) \rightarrow x = y)\} \wedge \forall x(Fx \rightarrow Gx)).$$

We claimed that from each wff we can derive the other two using a QL<sup>=</sup> proof. Give at least three of the six required proofs. Remember, for us an expression of the form  $(\alpha \leftrightarrow \beta)$  simply abbreviates the corresponding expression  $((\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha))$ .

(c) Outline a proof that 'one and two makes three' (i.e. show that *There is one F* and *There are two Gs* (where *none of the Fs are Gs*), then *There are three things which are F-or-G*).