Peter Smith, *Introduction to Formal Logic* (CUP, 2nd edition)

Exercises 30: More QL translations

In QL_2, the proper names with their interpretations are

m: Maldwyn,
n: Nerys,
o: Owen;

and the predicates are

F: ① is a man,
G: ① is a woman,
L: ① loves ②,
M: ① is married to ②,
R: ① is a child of ② and ③.

The domain of quantification: people (living people, for definiteness).

(a) Translate the following into QL_2:

(1) Maldwyn loves anyone who loves Owen.

$\simeq \forall \mathsf{x}(\mathsf{Lxo} \to \mathsf{Lmx})$

You can of course use another variable! And we won't keep repeating this in the answers below!!

(2) Everyone loves whoever they are married to.

$\simeq \forall \mathsf{x} \forall \mathsf{y}(\mathsf{Mxy} \to \mathsf{Lxy})$

(3) Some man is a child of Owen and someone or other.

$\simeq \exists \mathsf{x}(\mathsf{Fx} \land \exists \mathsf{y}\, \mathsf{Rxoy})$
$\simeq \exists \mathsf{x} \exists \mathsf{y}(\mathsf{Fx} \land \mathsf{Rxoy})$

(4) Whoever is a child of Maldwyn and Nerys loves them both.

$\simeq \forall \mathsf{x}(\mathsf{Rxmn} \to (\mathsf{Lxm} \land \mathsf{Lxn}))$

(5) Owen is a child of Nerys and someone who loves Nerys.

$\simeq \exists \mathsf{x}(\mathsf{Ronx} \land \mathsf{Lxn})$

Or of course we could the conjuncts the other way about. Again we won't keep pointing out examples where the ordering of conjuncts is pretty arbitrary.

(6) Some men do not love those who they are married to.

$\simeq \exists \mathsf{x}(\mathsf{Fx} \land \forall \mathsf{y}(\mathsf{Mxy} \to \neg \mathsf{Lxy}))$

The following is equivalent:

$\simeq \exists \mathsf{x} \forall \mathsf{y}(\mathsf{Fx} \land (\mathsf{Mxy} \to \neg \mathsf{Lxy}))$

However, the first translation is to be clearly preferred. Why? Because it keeps together the '$\exists \mathsf{x}(\mathsf{Fx} \land \ldots)$' which renders the restricted existential quantification 'some men', and keeps together the '$\forall \mathsf{y}(\mathsf{Mxy} \to \ldots)$ which renders the restricted universal expressed by 'those who they are married to'.

(7) Every man who loves Nerys loves someone who is married to Owen.

$\simeq \forall \mathsf{x}((\mathsf{Fx} \land \mathsf{Lxn}) \to \exists \mathsf{y}(\mathsf{Myo} \land \mathsf{Lxy}))$

(8) No woman is loved by every married man.

$\simeq$ (Every woman is such that)[it is not the case that (every man who is married to someone is such that) he loves her
$\simeq \forall x(Gx \rightarrow \neg\forall y((Fy \wedge \exists z\, Myz) \rightarrow Lyx))$

Or alternatively,

$\simeq$ It is not the case that (some woman is such that)(every man who is married to someone is such that) he loves her
$\simeq \neg\exists x(Gx \wedge \forall y((Fy \wedge \exists z\, Myz) \rightarrow Lyx))$

(9) Everyone who loves Maldwyn loves no one who loves Owen.

$\simeq \forall x(Lxm \rightarrow \forall y(Lyo \rightarrow \neg Lxy))$
$\simeq \forall x(Lxm \rightarrow \neg\exists y(Lyo \wedge Lxy))$

(10) Whoever loves Maldwyn loves a man only if the latter loves Maldwyn too.

$\simeq \forall x(Lxm \rightarrow \forall y((Fy \wedge Lxy) \rightarrow Lxm))$

(11) Only if Maldwyn loves every woman does he love whoever is married to Owen.

$\simeq$ [Maldwyn loves whoever is married to Owen] only if [Maldwyn loves every woman]
$\simeq (\forall x(Mxo \rightarrow Lmx) \rightarrow \forall y(Gy \rightarrow Lmy))$

(12) No one loves anyone who has no children.

A a warm up, let's first translate 'Maldwyn has no children' – i.e. there is no one who is a child of Maldwyn and someone (or a bit less naturally: anyone you pick is *not* a child of Maldwyn and someone):

$\simeq \neg\exists z\exists w\, Rzmw$,
(or a bit less naturally) $\simeq \forall z\neg\exists w\, Rzmw$

With that in mind, consider (12) which is equivalent to the Loglish

$\simeq$ (Everyone x is such that)(anyone y is such that)[if y has no children then x doesn't love y]:

this can then be rendered

$\simeq \forall x\forall y(\neg\exists z\exists wRzyw \rightarrow \neg Lxy)$

Alternatively we can note that (12) is equivalent to

$\simeq$ It isn't the case that (there is someone x such that)(there is someone y such that)[y has no children and yet x does love y]:

and this can then be rendered

$\simeq \neg\exists x\exists y(\neg\exists z\exists wRzyw \wedge Lxy)$

(b) Now consider the language QL_3 whose quantifiers range over the positive integers, with the following glossary:

n: one,
F: ① is odd,
G: ① is even,
H: ① is prime,
L: ① is less than ②,
R: ① is the sum of ② and ③.

Then translate the following into natural English (they are not all true!) – and we will allow ourselves to be quite generously relaxed in departing from strictly following the surface form of the wffs:

(1) $\forall x\forall y\exists zRzxy$

$\simeq$ Any two numbers have a (unique) sum.

(2) $\exists y \forall x \mathsf{Lxy}$

> $\simeq$ There's a number such that every number is less than it. (False of course!!)

(3) $\forall x \exists y(\mathsf{Lxy} \wedge \mathsf{Hy})$

Oops, the version of this exercise in the printed book lacked the brackets and is not a wff. So a bonus point if you answered that the expression couldn't be translated because not a wff. But with the brackets inserted as shown, it renders

> $\simeq$ For any number you chose, there is a larger prime number.

(4) $\forall x(\mathsf{Hx} \to \exists y(\mathsf{Lxy} \wedge \mathsf{Hy}))$

> $\simeq$ Given any prime, there is a larger one.

Which tells us that there is an infinity of primes, proved by Euclid.

(5) $\forall x \forall y((\mathsf{Fx} \wedge \mathsf{Ryxn}) \to \neg \mathsf{Fy})$

> $\simeq$ The result of adding one to any odd number is not odd.

(6) $\forall x \exists y((\mathsf{Gx} \wedge \mathsf{Fy}) \wedge \mathsf{Rxyy})$

This is an exercise in reading what is front of you! It says

> $\simeq$ Every number is even(!) and there exists an odd number such that the former is the sum of latter with itself.

A more sensible wff would have been e.g.

> $\forall x(\mathsf{Gx} \to \exists y(\mathsf{Fy} \wedge \mathsf{Rxyy}))$

which says that every even number is twice an odd number – though that's still false! Some truths in the neighbourhood are:

> $\exists x \exists y((\mathsf{Gx} \wedge \mathsf{Fy}) \wedge \mathsf{Rxyy})$
> $\forall x(\mathsf{Fx} \to \exists y(\mathsf{Gy} \wedge \mathsf{Ryxx}))$

(7) $\forall x \forall y(\exists z(\mathsf{Rzxn} \wedge \mathsf{Ryzn}) \to (\mathsf{Gx} \to \mathsf{Gy}))$

Think for a moment. Suppose we had names j and k denoting the numbers j and k; then $\exists z(\mathsf{Rzjn} \wedge \mathsf{Rkzn})$ says that there's a number z such that $z = j + 1$ and $k = z + 1$; so k is two more than j. Hence (7) says

> $\simeq$ Given any two numbers, if the second is two more than the first, then if the first is even, so is the second.

(8) $\forall x \forall y \forall z(((\mathsf{Fx} \wedge \mathsf{Fy}) \wedge \mathsf{Rzxy}) \to \mathsf{Gz})$

> $\simeq$ The sum of two odd numbers is even.

(9) $\forall x((\mathsf{Gx} \wedge \neg \mathsf{Rxnn}) \to \exists y \exists z((\mathsf{Hy} \wedge \mathsf{Hz}) \wedge \mathsf{Rxyz}))$

> $\simeq$ Any even number other than two is the sum of two primes.

(Relying on the triviality that two is result of adding one and one!) This is Goldbach's conjecture – one of the great open problems of number theory.

(10) $\forall x \exists y((\mathsf{Hy} \wedge \mathsf{Lxy}) \wedge \exists w \exists z((\mathsf{Rwyn} \wedge \mathsf{Rzwn}) \wedge \mathsf{Hz}))$

This is another of the great open problems, the Twin Prime conjecture – there is no end to the pairs of primes which differ by two. In this version:

> $\simeq$ For any number, you can find a larger prime and another prime which differs by two.