Peter Smith, *Introduction to Formal Logic* (CUP, 2nd edition)

Exercises 18: The truth-functional conditional

(a) Suppose we are working in a PL language where 'P' means *Putnam is a philosopher*, 'Q' means *Quine is a philosopher*, etc. Translate the following as best you can:

(1) If either Quine or Putnam is a philosopher, so is Russell.
(2) Only if Putnam is a philosopher is Russell one too.
(3) Quine and Russell are both philosophers only if Sellars is.
(4) Russell's being a philosopher is a necessary condition for Quine's being one.
(5) Russell's being a philosopher is a sufficient condition for Quine's being one.
(6) Putnam is a philosopher if and only if Quine isn't.
(7) Provided that Quine is a philosopher, Russell is one too.
(8) Quine is not a philosopher unless Russell is one.
(9) Only if either Putnam or Russell is a philosopher are both Quine and Sellars philosophers.

(b) Assuming that we are dealing with a suitable PL language. Which of the following arguments *ought* to come out valid, assuming that '$\rightarrow$' is a reasonably good surrogate for 'if ..., then ...'? Which is tautologically valid?

(1) $\mathsf{P},\ (\mathsf{P} \rightarrow \mathsf{Q}),\ (\mathsf{Q} \rightarrow \mathsf{R})\ \therefore\ \mathsf{R}$
(2) $\neg\mathsf{R},\ (\mathsf{P} \rightarrow \mathsf{R}),\ (\mathsf{Q} \rightarrow \mathsf{P})\ \therefore\ \neg\mathsf{Q}$
(3) $(\mathsf{P} \rightarrow \neg(\mathsf{Q} \vee \mathsf{R})),\ (\mathsf{Q} \rightarrow \mathsf{R}),\ (\neg\mathsf{R} \rightarrow \mathsf{P})\ \therefore\ (\mathsf{P} \rightarrow \mathsf{R})$
(4) $(\mathsf{P} \vee \mathsf{Q}),\ (\mathsf{P} \rightarrow \mathsf{R}), \neg(\mathsf{Q} \wedge \neg\mathsf{R})\ \therefore\ \mathsf{R}$
(5) $(\mathsf{R} \rightarrow (\neg\mathsf{P} \vee \mathsf{Q})),\ (\mathsf{P} \wedge \neg\mathsf{R}) \therefore \neg(\neg\mathsf{R} \vee \mathsf{Q})$
(6) $(\neg\mathsf{P} \vee \mathsf{Q}),\ \neg(\mathsf{Q} \wedge \neg\mathsf{R})\ \therefore\ (\mathsf{P} \rightarrow \mathsf{R})$
(7) $(\mathsf{P} \wedge \neg\mathsf{R}),\ (\mathsf{Q} \rightarrow \mathsf{R})\ \therefore\ \neg(\mathsf{P} \rightarrow \mathsf{Q})$
(8) $\neg(\neg\mathsf{S} \rightarrow (\neg\mathsf{Q} \wedge \mathsf{R})),\ (\mathsf{P} \vee \neg\neg\mathsf{Q}),\ (\mathsf{R} \vee (\mathsf{S} \rightarrow \mathsf{P}))\ \therefore\ (\mathsf{P} \rightarrow \mathsf{S})$

(c) Which of the following are true for all α, β, γ in a PL language and why? Which of the true claims correspond to true claims about the vernacular (bi)conditional?

(1) If $\alpha, \beta \vDash \gamma$ then $\alpha \vDash (\beta \rightarrow \gamma)$.
(2) $((\alpha \wedge \beta) \rightarrow \gamma) \mathrel{=\!\!|\!\vDash} (\alpha \rightarrow (\beta \rightarrow \gamma))$.
(3) $((\alpha \vee \beta) \rightarrow \gamma) \mathrel{=\!\!|\!\vDash} ((\alpha \rightarrow \gamma) \vee (\beta \rightarrow \gamma))$.
(4) If $\vDash (\alpha \rightarrow \beta)$ and $\vDash (\beta \rightarrow \gamma)$, then $\vDash (\alpha \rightarrow \gamma)$.
(5) If $\vDash (\alpha \rightarrow \beta)$ and $\vDash (\alpha \rightarrow \neg\beta)$, then $\vDash \neg\alpha$.
(6) $\vDash (\alpha \leftrightarrow \alpha)$
(7) $(\alpha \leftrightarrow \beta) \vDash (\beta \leftrightarrow \alpha)$.
(8) $(\alpha \leftrightarrow \beta), (\beta \leftrightarrow \gamma) \vDash (\alpha \leftrightarrow \gamma)$.
(9) If $\vDash \alpha \leftrightarrow \beta$ then α and β are tautologically consistent.
(10) If $\vDash \alpha \leftrightarrow \neg\beta$ then α and β are tautologically inconsistent.

(d*) On alternative languages for propositional logic:

(1) Suppose the language PL_1 has just the connectives $\rightarrow$ and $\neg$ (with the same interpretation as before). Show that disjunction and conjunction can be expressed in PL_1. Conclude that PL_1 has an expressively adequate set of built-in connectives.
(2) Consider too the variant language PL_2 whose only logical constants are $\rightarrow$ and the absurdity constant $\bot$. Show that in PL_2 we can introduce a negation connective so that $\neg\alpha$ is shorthand for $(\alpha \rightarrow \bot)$. Conclude that PL_2 is also expressively adequate.