

Peter Smith, *Introduction to Formal Logic* (CUP, 2nd edition)

Exercises 19: More on ‘if’s and ‘ \rightarrow ’s

(a) Show that the following hold for PL languages with the relevant atoms:

- (1) $\models ((P \rightarrow Q) \vee (Q \rightarrow P))$
- (2) $((P \wedge Q) \rightarrow R) \models ((P \rightarrow R) \vee (Q \rightarrow R))$
- (3) $((P \rightarrow Q) \rightarrow R), \neg P \models R$

Do these results raise more problems for equating ‘ \rightarrow ’ with ‘if’?

More generally, the pattern of inference $\gamma \therefore (\alpha \rightarrow \gamma)$ is valid. What about a corresponding ordinary-language inference of the form *C; so if A then C*?

The pattern of inference $(\beta \rightarrow \gamma) \therefore ((\alpha \wedge \beta) \rightarrow \gamma)$ is also valid. What about a corresponding ordinary-language inference of the form *C, if B; so C, if A and B*?

(b) Assess the following arguments for validity:

- (1) Beth is a logician, provided that Amy is one too. And given that Beth is a logician, Chloe is a philosopher. But, hold on! – Chloe *isn't* a philosopher. So Amy isn't a logician.
- (2) Jo went up the hill. Why so? Because Jo went up the hill if Jack and Jill did. While Jack went up the hill if Jane did, and either Jane didn't go or Jill did. But Jane *did* go up the hill!
- (3) If Angharad speaks Welsh, so does either Bryn or Carys. But Bryn doesn't speak Welsh. So if Carys doesn't speak Welsh, neither does Angharad.
- (4) Veronika is a violinist only if Peter plays the cello. Marek is a violinist if and only if Jiří plays the viola. Veronika doesn't play the violin if Marek doesn't. Jiří doesn't play the viola only if Peter doesn't play the cello. Hence either Veronika isn't a violinist or Marek isn't.
- (5) It isn't the case that Popper is a good philosopher and Quine isn't. But in fact, Quine isn't a good philosopher, unless Russell is one too. And if Russell is merely a wordy fool, then he isn't a good philosopher. Hence if Popper is a good philosopher, then it isn't the case that either Russell is a wordy fool or isn't himself a good philosopher.
- (6) If the switch had been flipped, the light would have been on. If the light had been on, the burglar would have been discovered. So, if the switch had been flipped, the burglar would have been discovered.